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Class X practice studio

CLASS X · 2026-27 · MODEL PAPER

Class X Mathematics Standard (Code 041) Model Paper 5

Worked answers & explanations

Maximum marks: 80

Time: 3 hours

Subject code: 041

Practice, understand, improve.

Instructions

All 38 questions are compulsory, with one selected option for each internal choice.

Section A has 18 MCQs and 2 assertion-reason questions; Section E has 1+1+2-mark case subparts.

Calculators are not allowed; draw labelled figures where needed.

Section A

Question 1

1 mark

Mcq

Choose the correct answer.

(a) If sin θ = 3/5 for an acute angle θ, what is tan θ? [1 mark]

A. 3/4
B. 4/3
C. 5/3
D. 3/5

Answer

A. 3/4

Step by step

1. **Derive cos θ from sin θ = 3/5 and then compute tan θ = sin θ/cos θ to select option A:** Take a right triangle with opposite side 3 and hypotenuse 5. The adjacent side is $\sqrt{(5^2 - 3^2)} = \sqrt{(25 - 9)} = \sqrt{16} = 4$, so cos θ = 4/5; equivalently cos θ = $\sqrt{(1 - 9/25)}$ = 4/5, taken positive because θ is acute. Then tan θ = sin θ/cos θ = (3/5)/(4/5) = 3/4, which is option A. Check: opposite/adjacent = 3/4 in the 3-4-5 triangle gives the same value. Why: tan A is defined as sin A divided by cos A, so cos θ is needed and the Pythagoras relation $\sin^2\theta + \cos^2\theta = 1$ supplies it. Because θ is acute, cos θ is positive, so tan θ cannot be negative.

Topic and concept

Properties of trigonometric ratios (their ranges). For an acute angle, tan θ = sin θ/cos θ = opposite/adjacent. When one ratio such as sin θ = 3/5 is known, the sides of a right triangle follow from the Pythagoras theorem, giving sides 3, 4, 5 and so tan θ = 3/4.

How marks are earned

1. tan θ = sin θ/cos θ = (3/5)/(4/5) = 3/4. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 8, §8.5: tan A is defined as sin A divided by cos A.

Question 2

1 mark

Mcq

Choose the correct answer.

(a) Consider the system 2x+3y=5 and 4x+6y=10. Which statement is correct about its solutions? [1 mark]

A. Exactly one solution

B. No solution

C. Infinitely many solutions

D. Two solutions

Answer

C. Infinitely many solutions

Step by step

1. **Select and justify the option:** Here $a_1/a_2 = 2/4 = 1/2$, $b_1/b_2 = 3/6 = 1/2$ and $c_1/c_2 = 5/10 = 1/2$, so all three ratios are equal indicating infinitely many solutions. Why: If $a_1/a_2 = b_1/b_2 = c_1/c_2$, the pair of equations has infinitely many solutions.

Topic and concept

Graphical representation of pair of linear equations; consistent, inconsistent, dependent conditions. The ratios of coefficients determine whether a linear pair has one, infinitely many, or no solutions.

How marks are earned

1. Here $a_1/a_2 = 2/4 = 1/2$, $b_1/b_2 = 3/6 = 1/2$ and $c_1/c_2 = 5/10 = 1/2$, so all three ratios are equal indicating infinitely many solutions. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 3, §3.2: If $a_1/a_2 = b_1/b_2 = c_1/c_2$, the pair of equations has infinitely many solutions.

Question 3

1 mark

Mcq

Choose the correct answer.

(a) Tangents PT_1 and PT_2 from an external point P touch a circle with centre O at T_1 and T_2. Which pair of right triangles is compared by RHS to prove $PT_1=PT_2$? [1 mark]

A. ΔOPT_1 and ΔOPT_2

B. ΔOPT_1 and ΔPT_1T_2

C. ΔOPT_2 and ΔPT_1T_2

D. ΔOT_1T_2 and ΔPT_1T_2

Answer

A. ΔOPT_1 and ΔOPT_2

Step by step

1. **Compare the two right triangles:** Tangents PT_1 and PT_2 touch the circle at T_1 and T_2, so $OT_1 \perp PT_1$ and $OT_2 \perp PT_2$. The right triangles ΔOPT_1 and ΔOPT_2 have common hypotenuse OP and equal radii $OT_1=OT_2$, hence are congruent by RHS; choose A. Why: Corresponding tangent sides PT_1 and PT_2 are equal after RHS congruence, which is the theorem the question asks students to identify.

Topic and concept

Number of tangents from an external point; lengths of two tangents from an external point are equal (theorem + proof). Radii to tangent contact points are perpendicular to the tangents. The two right triangles share OP as hypotenuse and have equal radii, so RHS proves equal tangent segments.

How marks are earned

1. ΔOPT_1 and ΔOPT_2 are right triangles with common hypotenuse OP and equal radii $OT_1=OT_2$, so RHS gives $PT_1=PT_2$. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 10, §10.4: The proof uses congruent triangles formed by the radii and the tangents' segments.

Question 4

1 mark

Mcq

Choose the correct answer.

(a) Which of the following correctly identifies a line that meets a circle at exactly one point? [1 mark]

A. Tangent

B. Secant

C. Chord

D. Non-intersecting line

Answer

A. Tangent

Step by step

1. **Select and justify the option:** A line that intersects the circle at exactly one point is called a tangent. Why: A tangent intersects the circle at one point, a secant at two, and a non-intersecting line at none.

Topic and concept

Tangent to a circle; tangent at any point is perpendicular to the radius (theorem + proof). A tangent to a circle is a line that touches the circle at exactly one point, unlike a secant which meets it at two points or a non-intersecting line which meets it at none.

How marks are earned

1. A line that intersects the circle at exactly one point is called a tangent. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 10, §10.1: A tangent intersects the circle at one point, a secant at two, and a non-intersecting line at none.

Question 5

1 mark

Mcq

Choose the correct answer.

(a) How many terms are there in the AP 7, 14, 21, ..., 70? [1 mark]

A. 9

B. 10

C. 11

D. 12

Answer

B. 10

Step by step

1. **Select and justify the option:** Using $l = a + (n-1)d$ gives $70 = 7 + (n-1)\times 7 \Rightarrow n-1 = 9 \Rightarrow n = 10$. Why: The last term l satisfies $l = a + (n-1)d$.

Topic and concept

Problems on finding n, a, d or S given partial information. In a finite AP the first term a, common difference d, number of terms n, and last term l satisfy $l = a + (n - 1)d$.

How marks are earned

1. Using $l = a + (n-1)d$ gives $70 = 7 + (n-1)\times 7 \Rightarrow n-1 = 9 \Rightarrow n = 10$. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 5, §5.5: The last term l satisfies $l = a + (n-1)d$.

Question 6

1 mark

Mcq

Choose the correct answer.

(a) Which of the following best describes the angle of depression of an object? [1 mark]

A. The angle between the horizontal and the line of sight when looking at a point lower than the observer's eye level

B. The angle between the vertical and the line of sight when looking at a point lower than the observer's eye level

C. The angle between the horizontal and the line of sight when looking at a point higher than the observer's eye level

D. The angle between the vertical and the line of sight when looking at a point higher than the observer's eye level

Answer

A. The angle between the horizontal and the line of sight when looking at a point lower than the observer's eye level

Step by step

1. **Select and justify the option:** Angle of depression is the angle formed by the observer's line of sight with the horizontal when the object is below the horizontal level. Why: Angle of depression is the angle formed by the line of sight with the horizontal when the point viewed is below the horizontal level.

Topic and concept

Angle of elevation and angle of depression; applications to heights and distances. Using tan θ = height / horizontal distance allows calculation of unknown heights or distances when the angle of elevation or depression is known.

How marks are earned

1. Angle of depression is the angle formed by the observer's line of sight with the horizontal when the object is below the horizontal level. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 9, Heights and Distances: Angle of depression is the angle formed by the line of sight with the horizontal when the point viewed is below the horizontal level.

Question 7

1 mark

Mcq

Choose the correct answer.

(a) In constructing a factor tree for 462, which prime is used immediately after dividing by 2? [1 mark]

A. 2

B. 3

C. 5

D. 7

Answer

B. 3

Step by step

1. **Divide 462 by 2, then test 231 for the next smallest prime divisor to choose the option:** 462 ÷ 2 = 231. Since 231 is odd, 2 does not divide it. Its digit sum is 2+3+1 = 6, which is divisible by 3, so 3 divides 231: 231 ÷ 3 = 77. Then 77 = 7 × 11, so the tree gives 462 = 2 × 3 × 7 × 11 and the prime used right after 2 is 3, which is option B. Why: A factor tree divides a composite number by its smallest prime divisor at each stage. After 2 fails on the odd number 231, the next prime to try is 3, and the digit-sum test shows it works. This is why 3 comes next, and 5 and 7 are not the answer.

Topic and concept

Fundamental Theorem of Arithmetic — statement, illustration and applications through prime factorisation. A composite number is broken down into its prime factors by repeatedly dividing by the smallest possible prime until only primes remain. For 462, this gives 2 first, then 3 (from 231), then 7 and 11, so 462 = 2 × 3 × 7 × 11.

How marks are earned

1. 462÷2=231 and the smallest prime dividing 231 is 3. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 1, §1.2: A composite number is broken down into its prime factors by repeatedly dividing by the smallest possible prime.

Question 8

1 mark

Mcq

Choose the correct answer.

(a) In the section formula $P((m\cdot x_2 + n\cdot x_1)/(m + n), (m\cdot y_2 + n\cdot y_1)/(m + n))$, the ratio PA:PB equals which of the following? [1 mark]

A. m:n

B. n:m

C. m:(m+n)

D. (m+n):m

Answer

A. m:n

Step by step

1. **Read the segment ratio from the section formula:** In $P=((m\,x_2+n\,x_1)/(m+n),(m\,y_2+n\,y_1)/(m+n))$, the coefficient m multiplies B's coordinates and n multiplies A's coordinates. Therefore PA:PB=m:n, option A. If m=n, P is the midpoint and the ratio is 1:1. Why: A larger m puts P closer to B and makes PA longer, so m belongs to PA and n to PB. Option C compares m with the total m+n instead of with n and is generally different from m:n.

Topic and concept

Section formula for internal division. For an internal division PA:PB=m:n, the section formula is the weighted average $P=((m\,x_2+n\,x_1)/(m+n),(m\,y_2+n\,y_1)/(m+n))$. The weight on B corresponds to PA.

How marks are earned

1. Choose A: the internal section formula weights the coordinates of B by m and of A by n, corresponding to PA:PB=m:n. (1 mark)

Source check: NCERT Class 10 maths, Ch. 7, §7.3 Section Formula gives PA:PB=m:n for the stated weighted-average formula.

Question 9

1 mark

Mcq

Choose the correct answer.

(a) The discriminant of the equation $3x^2 + 2x + 1 = 0$ is: [1 mark]

A. −8

B. 1

C. 4

D. 5

Answer

A. −8

Step by step

1. **Select and justify the option:** $D = b^2 - 4ac = 2^2 - 4 \times 3 \times 1 = 4 - 12 = -8$. Why: The discriminant is defined as $D = b^2 - 4ac$ to determine root nature.

Topic and concept

Nature of roots from discriminant (b^2-4ac): two distinct real, two equal, no real roots. The discriminant $D = b^2 - 4ac$ indicates root type: $D > 0$ yields two distinct real roots, $D = 0$ two equal real roots, and $D < 0$ no real roots.

How marks are earned

1. $D = b^2 - 4ac = 2^2 - 4 \times 3 \times 1 = 4 - 12 = -8$. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 4, §4.4 Nature of Roots: The discriminant is defined as $D = b^2 - 4ac$ to determine root nature.

Question 10

1 mark

Mcq

Choose the correct answer.

(a) Which expression represents cot θ in terms of sin θ and cos θ? [1 mark]

A. cos θ/sin θ

B. sin θ/cos θ

C. 1/(sin θ·cos θ)

D. tan θ

Answer

A. cos θ/sin θ

Step by step

1. **Select and justify the option:** cot θ = adjacent/opposite = cos θ/sin θ. Why: cot A is expressed as cos A divided by sin A.

Topic and concept

Simple identities derived from $\sin^2\theta + \cos^2\theta = 1$. By dividing the Pythagorean relation by other sides squared, we obtain $1+\tan^2\theta=\sec^2\theta$ and $1+\cot^2\theta=\operatorname{cosec}^2\theta$.

How marks are earned

1. cot θ = adjacent/opposite = cos θ/sin θ. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 8, §8.2: cot A is expressed as cos A divided by sin A.

Question 11

1 mark

Mcq

Choose the correct answer.

(a) A student on level ground looks up at the top of a tower. In tanθ = tower height / horizontal distance, θ is the: [1 mark]

A. Angle of elevation

B. Angle of depression

C. Angle between height and base

D. Angle of incidence

Answer

A. Angle of elevation, measured upward from the student's horizontal line.

Step by step

1. **Use the stated observer position:** A student standing on level ground looks upward to the tower top, so θ is measured above the horizontal and is an angle of elevation; choose A. Why: The contextual direction of the line of sight removes the original ambiguity with an angle of depression.

Topic and concept

Angle of elevation and angle of depression; applications to heights and distances. For an observer below a target, the angle between the upward line of sight and the horizontal is the angle of elevation; tanθ=height/horizontal distance.

How marks are earned

1. Select A because a ground-level observer looking upward at the tower top measures an angle of elevation. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 9, §9.1 Heights and Distances: The height and distance model tan θ = height/horizontal distance applies for angle of elevation.

Question 12

1 mark

Mcq

Choose the correct answer.

(a) In triangle ABC right-angled at B, which expression represents cos A? [1 mark]

A. AB/AC

B. BC/AC

C. AB/BC

D. AC/AB

Answer

A. AB/AC

Step by step

1. **Select and justify the option:** cos A is defined as adjacent side (AB) over hypotenuse (AC), so cos A = AB/AC. Why: cos A is defined as the length of the side adjacent to A divided by the hypotenuse.

Topic and concept

Values at 0°, 30°, 45°, 60°, 90°. The exact values of sin θ, cos θ, tan θ and their reciprocals at θ = 0°, 30°, 45°, 60°, 90° follow from standard triangle constructions.

How marks are earned

1. cos A is defined as adjacent side (AB) over hypotenuse (AC), so cos A = AB/AC. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 8, §8.2: cos A is defined as the length of the side adjacent to A divided by the hypotenuse.

Question 13

1 mark

Mcq

Choose the correct answer.

(a) Which of the following cannot be a valid probability value? [1 mark]

A. –0.2

B. 0

C. 0.75

D. 1

Answer

A. –0.2

Step by step

1. **Select and justify the option:** A probability must lie between 0 and 1 inclusive, so –0.2 is invalid. Why: 0 ≤ P(E) ≤ 1 for any event E.

Topic and concept

Complementary events and their probability relation. For any event E, the sum of its probability and the probability of its complement is 1: P(E)+P(Ē)=1.

How marks are earned

1. A probability must lie between 0 and 1 inclusive, so –0.2 is invalid. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 14, §Summary: 0 ≤ P(E) ≤ 1 for any event E.

Question 14

1 mark

Mcq

Choose the correct answer.

(a) Which of the following best defines the angle of elevation of an object from an observer's eye?
[1 mark]

A. The angle between the line of sight and the horizontal when looking upward at an object above the horizontal.

B. The angle between the line of sight and the vertical when looking upward.

C. The angle between the line of sight and the horizontal when looking downward.

D. The angle between the object and its base on the ground.

Answer

A. The angle between the line of sight and the horizontal when looking upward at an object above the horizontal.

Step by step

1. **Select and justify the option:** The angle of elevation is the angle formed by the line of sight with the horizontal when viewing a point above the horizontal. Why: Angle of elevation is defined as the angle formed by the line of sight with the horizontal when viewing an object above the horizontal.

Topic and concept

Angle of elevation and angle of depression; applications to heights and distances. Using tan θ = height / horizontal distance allows calculation of unknown heights or distances when the angle of elevation or depression is known.

How marks are earned

1. The angle of elevation is the angle formed by the line of sight with the horizontal when viewing a point above the horizontal. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 9, §9.1 Heights and Distances: Angle of elevation is defined as the angle formed by the line of sight with the horizontal when viewing an object above the horizontal.

Question 15

1 mark

Mcq

Choose the correct answer.

(a) In right triangle PQR with $\angle Q = 90°$, which side is the hypotenuse? [1 mark]

A. PQ

B. QR

C. PR

D. Not enough information

Answer

C. PR

Step by step

1. **Select and justify the option:** PR is the side opposite the right angle at Q and is therefore the hypotenuse. Why: The hypotenuse is defined as the side opposite the right angle and is the longest side.

Topic and concept

Trigonometric ratios — sin, cos, tan, cot, sec, cosec — defined for acute angles in a right triangle. In a right triangle, for an acute angle θ, sin θ, cos θ and tan θ are defined as ratios of specific sides, and sec θ, cosec θ, cot θ are their reciprocals.

How marks are earned

1. PR is the side opposite the right angle at Q and is therefore the hypotenuse. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 8, §8.2: The hypotenuse is defined as the side opposite the right angle and is the longest side.

Question 16

1 mark

Mcq

Choose the correct answer.

(a) Which configuration best describes advanced heights and distances problems involving two right triangles? [1 mark]

A. Two right triangles sharing a common vertical side (height)

B. Two obtuse triangles sharing the base

C. A right triangle and an acute triangle sharing an angle

D. Two isosceles triangles with equal base lengths

Answer

A. Two right triangles sharing a common vertical side (height)

Step by step

1. **Compare each configuration with the setup of two sight angles to the same vertical object to select option A:** Suppose an observer measures two angles of elevation α and β, e.g. from two points at distances x and x + d from the foot of a tower of height h. Each sight line forms a right triangle with the same vertical side h, so h = x tan α and h = (x + d) tan β. Both triangles are right-angled and share the vertical height, which is option A. A sketch of the tower with two sight lines shows this common side. Why: Trigonometric ratios such as tan apply only in right triangles, and the shared vertical side links the two triangles, so the equations can be solved together. Obtuse, isosceles or mixed triangles do not provide this right-angle and common-height structure.

Topic and concept

Problems involving two right triangles and two angles of elevation/depression. When angles of elevation or depression are measured from the same location or along the same line to an object, two right triangles share a common vertical side. Applying a trigonometric ratio to each triangle gives equations in the shared height.

How marks are earned

1. Advanced heights and distances problems involve two right triangles sharing the same vertical height to apply trigonometric ratios independently. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 9, §9.1 Heights and Distances: Advanced problems form two right triangles with a common height.

Question 17

1 mark

Mcq

Choose the correct answer.

(a) Which pair of equations represents two parallel lines? [1 mark]

A. 2x+3y=5 and 4x+6y=10

B. 2x+3y=5 and 4x+6y=8

C. x–y=2 and 2x–2y=4

D. x+y=3 and 2x+2y=6

Answer

B. 2x+3y=5 and 4x+6y=8

Step by step

1. **Identify the coefficient ratios and eliminate coincident-line distractors.:** For option B, the pair 2x+3y=5 and 4x+6y=8 gives $a_1/a_2=2/4=1/2$ and $b_1/b_2=3/6=1/2$, but $c_1/c_2=5/8$, so $a_1/a_2=b_1/b_2\neq c_1/c_2$; options A, C, and D instead have $a_1/a_2=b_1/b_2=c_1/c_2$, so they represent coincident lines, not the required distinct parallel lines. Why: For a pair of linear equations in two variables, equal ratios of the x- and y-coefficients give the same direction or slope, and an unequal constant-term ratio shifts the line to a different parallel position rather than making it the same line.

Topic and concept

Graphical representation of pair of linear equations; consistent, inconsistent, dependent conditions. Distinct parallel lines in a pair of linear equations have no common solution, and this is identified by the condition $a_1/a_2=b_1/b_2\neq c_1/c_2$.

How marks are earned

1. For the pair 2x+3y=5 and 4x+6y=8, $a_1/a_2 = 2/4 = 1/2$ and $b_1/b_2 = 3/6 = 1/2$ while $c_1/c_2 = 5/8 \neq 1/2$, so the lines are parallel. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 3, §3.2: Parallel lines occur when $a_1/a_2 = b_1/b_2 \neq c_1/c_2$.

Question 18

1 mark

Mcq

Choose the correct answer.

(a) In the Sridharacharya formula for roots of $ax^2 + bx + c = 0$, what does the expression under the square root represent? [1 mark]

A. Sum of the roots

B. Product of the roots

C. Discriminant

D. Coefficient ratio

Answer

C. Discriminant

Step by step

1. **Select and justify the option:** The expression under the square root, $b^2 - 4ac$, is called the discriminant of the quadratic equation. Why: The formula $x = (-b \pm \sqrt{(b^2 - 4ac)}) / 2a$ shows that the term under the square root is the discriminant.

Topic and concept

Solution by quadratic formula $x = (-b \pm \sqrt{(b^2-4ac)}) / 2a$. The Sridharacharya formula $x = (-b \pm \sqrt{(b^2 - 4ac)}) / 2a$ gives the roots of $ax^2 + bx + c = 0$ when the discriminant is non-negative.

How marks are earned

1. The expression under the square root, $b^2 - 4ac$, is called the discriminant of the quadratic equation. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 4, §4.4: The formula $x = (-b \pm \sqrt{(b^2 - 4ac)}) / 2a$ shows that the term under the square root is the discriminant.

Question 19

1 mark

Assertion Reason

Choose the correct answer.

(a) Assertion (A): When x_i is less than the assumed mean a, the deviation $d_i = x_i - a$ is negative. Reason (R): By definition $d_i = x_i - a$, so if $x_i < a$ then subtracting a from x_i yields a negative value. [1 mark]

A. Both A and R true and R explains A

B. Both A and R true but R does not explain A

C. A true, R false

D. A false, R true

Answer

A. Both A and R true and R explains A

Step by step

1. **Test A and R with a numeric example and decide whether R explains A to select option A:** By definition $d_i = x_i - a$. Example: for $x_i = 15$ and $a = 25$, $d_i = 15 - 25 = -10$, which is negative, so A is true. R states the same rule: if $x_i < a$, subtracting the larger a from x_i gives a negative number, so R is true. Check: R gives the reason for the sign in A, so R explains A and option A holds. Why: To judge whether R explains A, ask whether R's statement is the cause of A's claim. Here the sign of d_i follows directly from the definition and the order $x_i < a$, so R is the explanation and option B is not correct.

Topic and concept

Mean of grouped data — direct method, assumed mean method, step-deviation method. In the assumed mean method, each class mark x_i is compared with an assumed mean a using the deviation $d_i = x_i - a$. Values below a give negative deviations, values above a give positive deviations, and the mean is then found by adding the average deviation to a.

How marks are earned

1. Since $d_i = x_i - a$, when $x_i < a$, d_i is negative; the reason directly explains this. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 13, §13.2: Assumed mean method defines $d_i = x_i - a$.

Question 20

1 mark

Assertion Reason

Choose the correct answer.

(a) Assertion (A): The angle of depression is measured between the vertical and the line of sight. Reason (R): The angle of depression is the angle formed by the line of sight with the horizontal when the point viewed is below the horizontal level. [1 mark]

A. Both A and R true and R explains A

B. Both A and R true but R does not explain A

C. A true, R false

D. A false, R true

Answer

D. A false, R true

Step by step

1. **Select and justify the option:** The assertion is false because the angle of depression is measured with respect to the horizontal, not the vertical; the reason correctly states its definition.
   Why: Angle of depression is defined as the angle formed by the line of sight with the horizontal when looking down.

Topic and concept

Angle of elevation and angle of depression; applications to heights and distances. The angle of depression is the angle between the horizontal line through the observer's eye and the line of sight when looking at a point below the horizontal.

How marks are earned

1. The assertion is false because the angle of depression is measured with respect to the horizontal, not the vertical; the reason correctly states its definition. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 9, §9.1 Heights and Distances: Angle of depression is defined as the angle formed by the line of sight with the horizontal when looking down.

Section B

Question 21

2 marks

Short Answer

Attempt one option.

Option A

Answer the question with working and reasons where needed.

(a) In an AP, the 15th term is 70 and the common difference is 4. Determine the first term. [2 marks]

Answer

Using $a_n = a + (n-1)d$ with $a_{15} = 70$, $n = 15$ and $d = 4$ gives $70 = a + 14\times4$. Solving, $a = 70 - 56 = 14$, so the first term of the AP is 14.

Step by step

1. **Substitute the given values into the nth term formula:** The formula is $a_n = a + (n-1)d$. With $a_{15} = 70$, $n = 15$ and $d = 4$, the substitution gives $70 = a + (15-1)\times4 = a + 14\times4 = a + 56$. Why: The factor is $n-1 = 14$, not 15, because the first term a is already the 1st term. Only 14 common differences are added to reach the 15th term, so the formula counts the steps taken from a.
2. **Solve for a and verify:** Subtracting 56 from both sides gives $a = 70 - 56 = 14$. Check: $a_{15} = 14 + 14\times4 = 14 + 56 = 70$, which matches the given term. Why: The formula is linear in a, so one subtraction isolates it. Substituting the answer back into the formula and recovering 70 confirms that the arithmetic and the use of $n-1$ are right.

Topic and concept

nth term formula: $a_n = a + (n-1)d$. In an AP each term is the previous one plus the common difference d. So the nth term is the first term a plus $(n-1)$ increments of d: $a_n = a + (n-1)d$. If any three of a_n, a, n and d are known, the fourth can be found by rearranging, so here the formula is solved for a.

How marks are earned

1. Using $a_n = a + (n-1)d$ with $a_{15} = 70$, $n = 15$ and $d = 4$ gives $70 = a + 14\times4$. (1 mark)
2. Solving, $a = 70 - 56 = 14$, so the first term of the AP is 14. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 5, §5.3: The nth term formula can be rearranged to find the first term when other parameters are known.

OR

Option B

Answer the question with working and reasons where needed.

(a) How many terms are there in the AP 7, 14, 21, …, 105? [2 marks]

Answer

For a finite AP, the last term l satisfies l = a + (n−1)d; here 105 = 7 + (n−1)×7. Solving gives n−1 = (105−7)/7 = 14, so n = 15 terms in the progression.

Step by step

1. **Identify the AP parameters and form the last-term equation.:** In the AP 7, 14, 21, …, 105, the first term is a = 7, the common difference is d = 14 − 7 = 7, and the last term is l = 105. Using l = a + (n − 1)d, substitute: 105 = 7 + (n − 1) × 7. Why: The finite AP relation connects the last term directly to the first term, common difference and number of terms, so it sets up an equation for n.
2. **Solve the equation for the number of terms.:** 105 = 7 + 7(n − 1) ⇒ 105 − 7 = 7(n − 1) ⇒ 98 = 7(n − 1) ⇒ n − 1 = 98/7 = 14 ⇒ n = 15. Therefore, there are 15 terms in the progression. Why: The expression n − 1 counts the number of equal jumps from the first term to the last term, so adding 1 gives the actual number of terms.

Topic and concept

Problems on finding n, a, d or S given partial information. In a finite AP the first term a, common difference d, number of terms n, and last term l satisfy l = a + (n − 1)d.

How marks are earned

1. For a finite AP, the last term l satisfies l = a + (n−1)d; here 105 = 7 + (n−1)×7. (1 mark)
2. Solving gives n−1 = (105−7)/7 = 14, so n = 15 terms in the progression. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 5, Summary: In a finite AP, last term l = a + (n−1)d.

Question 22

2 marks

Short Answer

Attempt one option.

Option A

Answer the question with working and reasons where needed.

(a) Which similarity criterion applies when two angles of one triangle are respectively equal to two angles

of another triangle? [2 marks]

Answer

When two angles of one triangle are equal to two angles of another triangle, the triangles are similar by the AA criterion. This follows because equal corresponding angles ensure proportionality of all sides.

Step by step

1. **Match the given condition to a similarity criterion:** Two angles of one triangle equal two angles of the other, say $\angle A = \angle D$ and $\angle B = \angle E$. This is the AA (angle–angle) condition, so $\Delta ABC \sim \Delta DEF$ by the AA criterion. Why: The AA criterion says that two pairs of equal angles are enough to make two triangles similar. No side lengths need to be known, so the condition in the question points directly to AA.
2. **Explain why proportional sides follow:** The angle sum of a triangle is 180°, so $\angle C = 180° - \angle A - \angle B$ and $\angle F = 180° - \angle D - \angle E$. Hence $\angle C = \angle F$, and all three angle pairs are equal. Equal corresponding angles give $AB/DE = BC/EF = AC/DF$. Why: Equal angles make the triangles the same shape, though not necessarily the same size. Same shape means the corresponding sides are in one constant ratio, which is the definition of similarity. This is why only two angle pairs need to be checked.

Topic and concept

Criteria for similarity of triangles — AA, SSS, SAS. Two triangles are similar when their corresponding angles are equal and their corresponding sides are proportional. The AA criterion is a shortcut. Because the angle sum is 180°, two equal angle pairs force the third pair to be equal, and then the sides are proportional.

How marks are earned

1. When two angles of one triangle are equal to two angles of another triangle, the triangles are similar by the AA criterion. (1 mark)
2. This follows because equal corresponding angles ensure proportionality of all sides. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 6, §6.4: AA criterion: two pairs of equal angles imply triangles are similar.

OR

Option B

Answer the question with working and reasons where needed.

(a) In triangles ABC and DEF, $\angle A = \angle D$ and $\angle B = \angle E$. State the steps to prove that $\Delta ABC \sim \Delta DEF$.

[2 marks]

Answer

Compute ∠C = 180° – (∠A + ∠B) and ∠F = 180° – (∠D + ∠E) to show ∠C = ∠F. (By angle sum in a triangle.) Conclude that ΔABC ~ ΔDEF by the AA similarity criterion since two pairs of angles are equal.

Step by step

1. **Use the angle sum property to find the third angles of both triangles:** In ΔABC, ∠C = 180° – (∠A + ∠B). In ΔDEF, ∠F = 180° – (∠D + ∠E). Since ∠A = ∠D and ∠B = ∠E, the sums ∠A + ∠B and ∠D + ∠E are equal, so ∠C = ∠F. Why: The angles of any triangle add up to 180°, so the third angle is fixed by the other two. This step turns the two given angle pairs into a third equal pair, which the similarity criterion needs.
2. **Conclude similarity by the AA criterion:** Two pairs of angles are equal (∠A = ∠D and ∠B = ∠E), and the third pair is also equal (∠C = ∠F). So the triangles are equiangular, and therefore ΔABC ~ ΔDEF by AA similarity. Why: The AA criterion states that two triangles with two pairs of equal angles are similar, which means their corresponding sides are in proportion. Only the angle equalities are needed, and no side lengths are used.

Topic and concept

Criteria for similarity of triangles — AA, SSS, SAS. If two angles of one triangle are equal to two angles of another, the triangles are similar (AA criterion). Equal angles guarantee that the corresponding sides are in proportion. The angle sum property links the two given pairs to the equal third pair.

How marks are earned

1. Compute ∠C = 180° – (∠A + ∠B) and ∠F = 180° – (∠D + ∠E) to show ∠C = ∠F. (By angle sum in a triangle.) (1 mark)
2. Conclude that ΔABC ~ ΔDEF by the AA similarity criterion since two pairs of angles are equal. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 6, §6.4: If two angles of one triangle are equal to two angles of another, the triangles are similar (AA criterion).

Question 23

2 marks

Short Answer

Answer the question with working and reasons where needed.

(a) Explain why 1/40 has a terminating decimal expansion, while 1/18 does not, using the Fundamental

Theorem of Arithmetic. [2 marks]

Answer

Prime factorisation of 40 = 2^3 × 5; since its prime factors are only 2 and 5, 1/40 terminates. Prime factorisation of 18 = 2 × 3^2; presence of prime 3 means 1/18 has a non-terminating repeating decimal.

Step by step

1. **Factorise 40 and show that 1/40 terminates:** 40 = $2^3 \times 5$, so its only prime factors are 2 and 5. Multiply numerator and denominator by 5^2 to get 1/40 = 25/1000 = 0.025, a terminating decimal. Check: 40 × 0.025 = 1. Why: A rational number in lowest terms terminates if and only if its denominator has the form 2^m5^n. By the Fundamental Theorem of Arithmetic the factorisation is unique, so no other primes are hidden in 40.
2. **Factorise 18 and show that 1/18 does not terminate:** 18 = 2×3^2. Since 1/18 is already in lowest terms and 3 is a prime factor of the denominator other than 2 or 5, 1/18 = 0.0555… is a non-terminating repeating decimal. Check: no multiplication by powers of 2 and 5 can cancel the factor 3^2 to give a power of 10. Why: By unique factorisation, the factor 3 cannot be removed, so the denominator can never become a power of 10. Hence the decimal expansion does not terminate.

Topic and concept

Fundamental Theorem of Arithmetic — statement, illustration and applications through prime factorisation. The Fundamental Theorem of Arithmetic says every integer greater than 1 factorises into primes in exactly one way, apart from order. A rational number p/q in lowest terms terminates exactly when q = 2^m5^n, so the prime factors of q decide the result.

How marks are earned

1. Prime factorisation of 40 = 2^3 × 5; since its prime factors are only 2 and 5, 1/40 terminates. (1 mark)
2. Prime factorisation of 18 = 2 × 3^2; presence of prime 3 means 1/18 has a non-terminating repeating decimal. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 1, §1.1 Introduction: Decimal expansion of a rational number terminates iff denominator's prime factors are 2 or 5.

Question 24

2 marks

Short Answer

Answer the question with working and reasons where needed.

(a) Briefly justify, using congruent triangles, why the lengths of two tangents from an external point are

equal. [2 marks]

Answer

Let O be the centre, P the external point and A, B the two contacts. OA=OB, OP is common, and ∠OAP=∠OBP=90°. Thus right triangles OAP and OBP are congruent by RHS, so PA=PB.

Step by step

1. **Identify the right triangles:** Let O be the centre, P the external point and A, B the tangent contacts. OA=OB as radii, OP is common, and ∠OAP=∠OBP=90°. Why: A radius is perpendicular to its tangent at the contact point, giving two right triangles with the same hypotenuse OP.
2. **Apply RHS and match sides:** Triangles OAP and OBP are congruent by RHS: equal hypotenuse OP and one equal leg OA=OB. Therefore corresponding tangent sides PA and PB are equal. Why: Corresponding parts of congruent triangles are equal, so the two tangent lengths match.

Topic and concept

Number of tangents from an external point; lengths of two tangents from an external point are equal (theorem + proof). For tangents PA and PB from P, radii OA and OB meet the tangents at right angles. Right triangles OAP and OBP have common hypotenuse OP and equal radius legs, so RHS congruence gives PA=PB.

How marks are earned

1. Identify triangles OAP and OBP, equal radii OA=OB, common hypotenuse OP, and the right angles at contacts A and B. (1 mark)
2. Apply RHS congruence to right triangles OAP and OBP and conclude corresponding tangent sides PA and PB are equal. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 10, §10.3 Number of Tangents from a Point on a Circle: The two tangents from an external point have equal lengths, established via congruent right-angled triangles.

Question 25

2 marks

Short Answer

Answer the question with working and reasons where needed.

(a) Write the formula for calculating the mean by the assumed mean method and define the term d_i. [2 marks]

Answer

The formula for the assumed mean method is $\bar{x} = a + (\Sigma f_i d_i / \Sigma f_i)$. Here, $d_i = x_i - a$ represents the deviation of each class mark from the assumed mean a.

Step by step

1. **State the assumed mean formula:** For values or class marks x_i with frequencies f_i and chosen assumed mean a, calculate $\bar{x} = a + (\Sigma f_i d_i)/(\Sigma f_i)$. Why: The weighted deviations correct the assumed mean to the actual mean.
2. **Define the deviations:** For every value or class mark x_i, set $d_i = x_i - a$, its signed distance from the assumed mean. Why: The formula uses these deviations rather than the original x_i directly.

Topic and concept

Mean of grouped data — direct method, assumed mean method, step-deviation method. The assumed mean method chooses a convenient a, computes $d_i = x_i - a$ and takes the frequency-weighted mean of the deviations: $\bar{x} = a + \Sigma f_i d_i / \Sigma f_i$.

How marks are earned

1. The formula for the assumed mean method is $\bar{x} = a + (\Sigma f_i d_i / \Sigma f_i)$. (1 mark)
2. Here, $d_i = x_i - a$ represents the deviation of each class mark from the assumed mean a. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths Ch. 13 §13.2: The assumed mean method formula expresses the mean as a shift from a chosen reference value a using $\Sigma f_i d_i / \Sigma f_i$.

Section C

Question 26

3 marks

Short Answer

Answer the question with working and reasons where needed.

(a) A cube of side a is joined to a hemisphere of radius a/2 by attaching one face of the cube to the base of the hemisphere. Find the total surface area of the resulting solid in terms of a. [3 marks]

Answer

The cube contributes $6a^2-\pi(a/2)^2$ because only the circular contact patch is hidden. The hemisphere contributes curved area $2\pi(a/2)^2=\pi a^2/2$. Hence total external surface area is $6a^2-\pi a^2/4+\pi a^2/2=6a^2+\pi a^2/4$.

Step by step

1. **Count the exposed cube area:** The cube has area $6a^2$, but the hemisphere covers only a circular patch of radius a/2 on the top face, so exposed cube area is $6a^2-\pi(a/2)^2=6a^2-\pi a^2/4$. Why: The remaining part of the top square is visible; subtracting the entire top face would undercount the external surface.
2. **Add the curved hemisphere:** The hemisphere contributes curved area $2\pi r^2=2\pi(a/2)^2=\pi a^2/2$. Why: Its flat base is attached to the cube and hidden, so only the curved hemispherical surface is counted.
3. **Combine and simplify:** Total external surface area is $6a^2-\pi a^2/4+\pi a^2/2=6a^2+\pi a^2/4$. Why: Subtract the shared circular area once from the cube and add only the hemisphere's curved area to avoid double counting.

Topic and concept

Combinations are restricted to any two cubes, cuboids, spheres, hemispheres and right circular cylinders or cones. For joined solids, count every exposed face once. Here the hemisphere covers a circle of area $\pi a^2/4$ on one cube face, while its curved area is $\pi a^2/2$.

How marks are earned

1. Find exposed cube area $6a^2-\pi(a/2)^2=6a^2-\pi a^2/4$, retaining the uncovered part of the top face. (1 mark)
2. Find the curved area of the hemisphere as $2\pi(a/2)^2=\pi a^2/2$. (1 mark)
3. Add exposed areas and simplify to $6a^2+\pi a^2/4$. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 12, §12.4: Composite solids combine exactly two basic solids; only exposed surfaces count for TSA.

Question 27

3 marks

Short Answer

Answer the question with working and reasons where needed.

(a) A letter is chosen at random from the letters of the word ‘MATHEMATICS’. Find the probability that it is a vowel. [3 marks]

Answer

The word ‘MATHEMATICS’ contains 11 letters in total, with certain letters like A appearing multiple times, each considered a distinct outcome when selecting one letter. The vowels present in the word are A, E, A, and I, which together appear in four distinct positions out of the total eleven letters. Consequently, the probability of choosing a vowel at random from the letters of ‘MATHEMATICS’ is P(vowel) = 4/11, reflecting the ratio of vowel occurrences to the total letters.

Step by step

1. **Count all possible outcomes by listing the letters of MATHEMATICS:** The letters are M, A, T, H, E, M, A, T, I, C, S. Counting each position as one outcome gives 11 equally likely outcomes, because repeated letters such as M, A and T each occur twice and are counted at each position. Why: The probability formula needs the total number of equally likely outcomes. Each position in the word is equally likely to be picked, so repeated letters must be counted every time they appear.
2. **Count the favourable outcomes, which are the vowel positions:** The vowels are A (2nd letter), E (5th letter), A (7th letter) and I (9th letter). That gives 2 A's + 1 E + 1 I = 4 favourable outcomes. Why: The event 'the letter is a vowel' is the set of positions holding a, e, i, o or u. Counting each repeated A separately keeps the favourable count consistent with the total count of 11.
3. **Apply P(E) = favourable outcomes / total outcomes:** P(vowel) = 4/11. The value is between 0 and 1, and 4 < 11, so it is a valid probability. Why: Theoretical probability is the number of outcomes favourable to the event divided by the total number of equally likely outcomes. A valid answer must lie between 0 and 1, which gives a quick check.

Topic and concept

Simple real-life problems on finding the probability of an event. Theoretical probability is P(E) = (number of favourable outcomes) / (total number of equally likely outcomes). Here each of the 11 letter positions in MATHEMATICS is equally likely to be chosen, and the vowels occupy 4 of them.

How marks are earned

1. The word ‘MATHEMATICS’ contains 11 letters in total, with certain letters like A appearing multiple times, each considered a distinct outcome when selecting one letter. (1 mark)

2. The vowels present in the word are A, E, A, and I, which together appear in four distinct positions out of the total eleven letters. (1 mark)
3. Consequently, the probability of choosing a vowel at random from the letters of ‘MATHEMATICS’ is P(vowel) = 4/11, reflecting the ratio of vowel occurrences to the total letters. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 14, §14.2: Probability is determined by dividing the number of favourable equally likely outcomes by the total number of equally likely outcomes.

Question 28

3 marks

Short Answer

Attempt one option.

Option A

Answer the question with working and reasons where needed.

(a) Find the HCF and LCM of 84 and 120 by prime factorisation, and verify that their product equals 84 × 120. [3 marks]

Answer

By factorising 84 = 2^2 × 3^1 × 7^1 and 120 = 2^3 × 3^1 × 5^1, and comparing common primes, we compute HCF = 2^2 × 3^1 = 12. By selecting the highest exponent for each prime from both factorizations, we calculate LCM = 2^3 × 3^1 × 5^1 × 7^1 = 840 as their least common multiple. Verifying the product rule, we multiply HCF and LCM to get 12 × 840 = 10080, which matches 84 × 120 = 10080, confirming the relationship.

Step by step

1. **Factorise 84 and 120 into primes and find the HCF:** 84 ÷ 2 = 42, 42 ÷ 2 = 21, 21 ÷ 3 = 7, so 84 = $2^2 \times 3 \times 7$. 120 ÷ 2 = 60, 60 ÷ 2 = 30, 30 ÷ 2 = 15, 15 ÷ 3 = 5, so 120 = $2^3 \times 3 \times 5$. The common primes are 2 and 3. Their smaller powers are 2^2 and 3^1, so HCF = $2^2 \times 3 = 4 \times 3 = 12$. Why: The HCF is the largest number that divides both numbers. A divisor can use each prime only as many times as it occurs in both numbers, so we take the minimum exponent of each common prime. 5 and 7 each appear in only one number, so they are left out.
2. **Find the LCM using maximum exponents:** Take every prime that appears in either number with its highest power: 2^3, 3^1, 5^1 and 7^1. LCM = $8 \times 3 \times 5 \times 7 = 24 \times 35 = 840$. Why: The LCM is the smallest number that both 84 and 120 divide. It must contain all the prime powers of each number, so we take the maximum exponent of every prime. This includes the primes 5 and 7, which are not common to both.
3. **Verify the product relation HCF × LCM = product of the numbers:** HCF × LCM = 12 × 840 = 10080. Also 84 × 120 = 84 × 100 + 84 × 20 = 8400 + 1680 = 10080. The two products are equal, so the results are consistent. Why: For each prime, the minimum and maximum exponents together use up the exponents from both numbers, so HCF × LCM equals the product of the two numbers. Getting the same value on both sides is a check on the HCF and LCM found above.

Topic and concept

Fundamental Theorem of Arithmetic — statement, illustration and applications through prime factorisation. By unique prime factorisation, each number is a product of primes in exactly one way. Comparing prime powers gives the HCF (lowest power of each common prime) and the LCM (highest power of every prime present). For two numbers, HCF × LCM equals their product.

How marks are earned

1. By factorising 84 = 2^2 × 3^1 × 7^1 and 120 = 2^3 × 3^1 × 5^1, and comparing common primes, we compute HCF = 2^2 × 3^1 = 12. (1 mark)
2. By selecting the highest exponent for each prime from both factorizations, we calculate LCM = 2^3 × 3^1 × 5^1 × 7^1 = 840 as their least common multiple. (1 mark)
3. Verifying the product rule, we multiply HCF and LCM to get 12 × 840 = 10080, which matches 84 × 120 = 10080, confirming the relationship. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 1, §1.2: Comparing prime-factor exponents lets us compute HCF and LCM to verify their product.

OR

Option B

Answer the question with working and reasons where needed.

(a) Prove that √3 is irrational by contradiction using prime factorisation. [3 marks]

Answer

Assume √3 = p/q in lowest terms, so $p^2 = 3q^2$ by squaring both sides. Since 3 divides p^2, by unique prime factorisation 3 divides p, so p = 3k. Substituting gives $q^2 = 3k^2$, so 3 divides q; this contradicts that p/q is in lowest terms.

Step by step

1. **Assume the opposite and square to get a relation between p and q:** Suppose √3 is rational. Then √3 = p/q, where p and q are integers, q ≠ 0, and p/q is in lowest terms, so p and q have no common factor other than 1. Squaring both sides gives $3 = p^2/q^2$, so $p^2 = 3q^2$. Why: Proof by contradiction starts by assuming the opposite of what we want to prove. Writing the fraction in lowest terms matters, because the contradiction will come from p and q sharing the factor 3.
2. **Show that 3 divides p using unique prime factorisation:** From $p^2 = 3q^2$, 3 divides p^2. If 3 did not divide p, then 3 would not appear in the prime factorisation of p, so it could not appear in that of p^2 either. Hence 3 divides p, and we write p = 3k for some integer k. Why: By the Fundamental Theorem of Arithmetic, the prime factorisation is unique. The primes in p^2 are exactly the primes in p, repeated. So if the prime 3 divides p^2, then 3 must already be in p.

3. **Substitute p = 3k, show that 3 divides q, and state the contradiction and conclusion:** Substituting p = 3k gives $(3k)^2 = 3q^2$, so $9k^2 = 3q^2$, which gives $q^2 = 3k^2$. So 3 divides q^2 and, by the same argument, 3 divides q. Now 3 divides both p and q, which contradicts p/q being in lowest terms. So our assumption is wrong, and √3 is irrational. Why: A common factor of 3 contradicts the assumption that p and q have no common factor. The only assumption we made was that √3 is rational, so that assumption must be false. This proves that √3 is irrational.

Topic and concept

Proofs of irrationality of √2, √3, √5 and similar surds. Proof by contradiction assumes the opposite of what we want to prove and uses unique prime factorisation to reach an impossibility. For √3, assuming √3 = p/q in lowest terms leads to 3 dividing both p and q, which cannot be true.

How marks are earned

1. Assume √3 = p/q in lowest terms, so $p^2 = 3q^2$ by squaring both sides. (1 mark)
2. Since 3 divides p^2, by unique prime factorisation 3 divides p, so p = 3k. (1 mark)
3. Substituting gives $q^2 = 3k^2$, so 3 divides q; this contradicts that p/q is in lowest terms. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 1, §1.3: The contradiction arises because both numerator and denominator become divisible by the same prime.

Question 29

3 marks

Short Answer

Answer the question with working and reasons where needed.

(a) Find the exact values of sin 60°, cos 30° and tan 45° with brief justification for each. [3 marks]

Answer

sin 60° = √3/2, obtained from a 30°–60°–90° triangle where the side opposite 60° is √3 and hypotenuse is 2. cos 30° = √3/2, since in the same triangle the side adjacent to 30° is √3 and hypotenuse is 2. tan 45° = 1, because in a 45°–45°–90° triangle the legs are equal, making opposite and adjacent sides the same.

Step by step

1. **Build a 30°–60°–90° triangle and find sin 60°:** Take an equilateral triangle ABC of side 2 and draw the altitude AD to BC. D is the midpoint of BC, so BD = 1 and ∠BAD = 30°. In right triangle ABD, $AD^2 = AB^2 - BD^2 = 4 - 1 = 3$, so AD = √3. The sides are 1, √3 and 2. For ∠B = 60°, the opposite side is AD = √3 and the hypotenuse is AB = 2, so sin 60° = √3/2. Why: Each angle of an equilateral triangle is 60°. The altitude bisects the base and the apex angle, which gives a right triangle with angles 30°, 60° and 90°. Pythagoras then gives the exact length √3, so the ratio is exact and not a decimal approximation.

2. **Read cos 30° from the same triangle:** At the 30° angle (∠BAD) in triangle ABD, the adjacent side is AD = √3 and the hypotenuse is AB = 2. So cos 30° = adjacent/hypotenuse = √3/2. Why: The side opposite 60° is the side adjacent to 30°, because the two acute angles share the same two sides. This is why sin 60° and cos 30° have the same value, and it avoids relying on memory.
3. **Use an isosceles right triangle for tan 45°:** Take a right triangle with angles 45°, 45° and 90°. The sides opposite the equal angles are equal, say both a. So tan 45° = opposite/adjacent = a/a = 1. Why: Equal angles have equal opposite sides, so the two legs are equal. The tangent is the ratio of the legs, so any equal pair gives exactly 1, whatever the size of the triangle.

Topic and concept

Proof and application of $\sin^2\theta + \cos^2\theta = 1$. Trigonometric ratios depend only on the angle, not on the size of the right triangle. The exact values for 30°, 45° and 60° come from two standard triangles. Halving an equilateral triangle gives sides 1 : √3 : 2. An isosceles right triangle has equal legs.

How marks are earned

1. sin 60° = √3/2, obtained from a 30°–60°–90° triangle where the side opposite 60° is √3 and hypotenuse is 2. (1 mark)
2. cos 30° = √3/2, since in the same triangle the side adjacent to 30° is √3 and hypotenuse is 2. (1 mark)
3. tan 45° = 1, because in a 45°–45°–90° triangle the legs are equal, making opposite and adjacent sides the same. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 8, §8.3: Values of sin 60° and cos 30° are derived from a 30°–60°–90° triangle, and tan 45° from a 45°–45°–90° triangle.

Question 30

3 marks

Short Answer

Answer the question with working and reasons where needed.

(a) A composite solid consists of a right circular cylinder of radius 5 cm and height 20 cm with two hemispheres of the same radius attached to its ends. Find the volume of this solid. (Use π = 3.14). [3 marks]

Answer

First, the volume of the cylindrical part is calculated as $V_{cyl} = \pi r^2 h = 3.14 \times 5^2 \times 20 = 3.14 \times 25 \times 20 = 1570\ cm^3$. Next, the volume of a single hemisphere of radius 5 cm is $V_{hemi} = (2/3)\pi r^3 = (2/3) \times 3.14 \times 125 \approx 261.67\ cm^3$, so for two hemispheres $V_{hemis} = 2 \times 261.67 \approx 523.33\ cm^3$. Finally, adding the volumes of the cylinder and both hemispheres gives the total volume $V_{total} = 1570\ cm^3 + 523.33\ cm^3 = 2093.33\ cm^3$.

Step by step

1. **Write the cylinder formula and compute its volume:** $V_{cyl} = \pi r^2 h$. With r = 5 cm, h = 20 cm and π = 3.14, $V_{cyl} = 3.14 \times 5^2 \times 20 = 3.14 \times 25 \times 20 = 3.14 \times 500 = 1570$ cm³. Why: The cylinder is the middle part of the solid, and its volume is base area times height. The base area is πr^2 and the height is the cylinder's own height of 20 cm. The hemispheres add to the length of the solid but not to this height.
2. **Find the volume of the two hemispheres:** Each hemisphere has radius 5 cm, the same as the cylinder, and $V_{hemi} = (2/3)\pi r^3$. Two hemispheres make one sphere, so the total is $2 \times (2/3)\pi r^3 = (4/3)\pi r^3 = (4/3) \times 3.14 \times 125 = 1570/3 \approx 523.33$ cm³. This also equals 2 × 261.67 = 523.33 cm³. Why: The two hemispheres have equal radii, so together they form one full sphere of radius 5 cm. Using $(4/3)\pi r^3$ once saves a step and gives a check on the 2 × 261.67 route. The radius is the same as the cylinder's, so the pieces join with no gap.
3. **Add the volumes of the parts:** $V_{total} = V_{cyl} + V_{hemispheres} = 1570 + 523.33 = 2093.33$ cm³. Why: The solid is formed by joining the cylinder and the hemispheres without overlap. So its volume is the sum of the parts' volumes. Volume is a cubic quantity, so the answer is in cm³.

Topic and concept

Surface area and volume of combinations — cone + hemisphere, cylinder + hemisphere, etc.. When basic solids are joined, the volume of the combined solid is the sum of their volumes. A cylinder with a hemisphere on each end is a cylinder plus a full sphere of the same radius. The volume is therefore $\pi r^2 h + (4/3)\pi r^3$.

How marks are earned

1. First, the volume of the cylindrical part is calculated as $V_{cyl} = \pi r^2 h = 3.14 \times 5^2 \times 20 = 3.14 \times 25 \times 20 = 1570$ cm³. (1 mark)
2. Next, the volume of a single hemisphere of radius 5 cm is $V_{hemi} = (2/3)\pi r^3 = (2/3) \times 3.14 \times 125 \approx 261.67$ cm³, so for two hemispheres $V_{hemis} = 2 \times 261.67 \approx 523.33$ cm³. (1 mark)
3. Finally, adding the volumes of the cylinder and both hemispheres gives the total volume $V_{total} = 1570$ cm³ + 523.33 cm³ = 2093.33 cm³. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 12, §12.3: Volume of composite solid with cylinder and two hemispheres equals sum of individual volumes.

Question 31

3 marks

Short Answer

Attempt one option.

Option A

Answer the question with working and reasons where needed.

(a) For each of the following pairs, determine whether the system is dependent (infinitely many solutions) or inconsistent (no solution), and justify using coefficient ratios: (i) $x - 2y = 3$ and $2x - 4y = 6$ (ii) $x - 2y = 3$ and $2x - 4y = 8$ [3 marks]

Answer

For (i) $a_1/a_2 = 1/2$, $b_1/b_2 = (-2)/(-4) = 1/2$, and $c_1/c_2 = 3/6 = 1/2$, so all ratios are equal indicating a dependent system with infinitely many solutions. For (ii) $a_1/a_2 = 1/2$, $b_1/b_2 = 1/2$, but $c_1/c_2 = 3/8 \neq 1/2$, so the system is inconsistent with no solution. These conclusions follow the condition that a dependent system has $a_1/a_2 = b_1/b_2 = c_1/c_2$, whereas an inconsistent system has $a_1/a_2 = b_1/b_2 \neq c_1/c_2$.

Step by step

1. **Compare all three coefficient ratios for pair (i).:** For $x - 2y = 3$ and $2x - 4y = 6$, take $a_1 = 1$, $b_1 = -2$, $c_1 = 3$ and $a_2 = 2$, $b_2 = -4$, $c_2 = 6$. Then $a_1/a_2 = 1/2$, $b_1/b_2 = (-2)/(-4) = 1/2$, and $c_1/c_2 = 3/6 = 1/2$. Since $a_1/a_2 = b_1/b_2 = c_1/c_2$, the system is dependent with infinitely many solutions. Why: Equal ratios of both variable coefficients and constants mean the two equations represent coincident lines, giving infinitely many common points.
2. **Compare the coefficient ratios for pair (ii).:** For $x - 2y = 3$ and $2x - 4y = 8$, take $a_1 = 1$, $b_1 = -2$, $c_1 = 3$ and $a_2 = 2$, $b_2 = -4$, $c_2 = 8$. Then $a_1/a_2 = 1/2$, $b_1/b_2 = (-2)/(-4) = 1/2$, but $c_1/c_2 = 3/8$. Since $3/8 \neq 1/2$, $a_1/a_2 = b_1/b_2 \neq c_1/c_2$, so the system is inconsistent with no solution. Why: Equal variable-coefficient ratios but a different constant ratio indicate parallel distinct lines, so they never intersect.
3. **State the ratio test used to justify both classifications.:** The classification rule is: if $a_1/a_2 = b_1/b_2 = c_1/c_2$, the pair is dependent; if $a_1/a_2 = b_1/b_2 \neq c_1/c_2$, the pair is inconsistent. Pair (i) matches the first condition, while pair (ii) matches the second condition. Why: The coefficient-ratio test determines whether a pair of linear equations has infinitely many solutions or no solution without solving the equations.

Topic and concept

Graphical representation of pair of linear equations; consistent, inconsistent, dependent conditions. The ratios of coefficients determine whether a linear pair has one, infinitely many, or no solutions.

How marks are earned

1. For (i) $a_1/a_2 = 1/2$, $b_1/b_2 = (-2)/(-4) = 1/2$, and $c_1/c_2 = 3/6 = 1/2$, so all ratios are equal indicating a dependent system with infinitely many solutions. (1 mark)
2. For (ii) $a_1/a_2 = 1/2$, $b_1/b_2 = 1/2$, but $c_1/c_2 = 3/8 \neq 1/2$, so the system is inconsistent with no solution. (1 mark)
3. These conclusions follow the condition that a dependent system has $a_1/a_2 = b_1/b_2 = c_1/c_2$, whereas an inconsistent system has $a_1/a_2 = b_1/b_2 \neq c_1/c_2$. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 3, §3.2 Graphical Method of Solution of a Pair of Linear Equations: The excerpt outlines that coincident lines (dependent) satisfy $a_1/a_2 = b_1/b_2 = c_1/c_2$, while parallel lines (inconsistent) satisfy $a_1/a_2 = b_1/b_2 \neq c_1/c_2$.

OR

Option B

Answer the question with working and reasons where needed.

(a) Solve the system of equations by substitution: 4x − y = 5 and 2x + 3y = 13. [3 marks]

Answer

From the first equation 4x − y = 5, isolate y by adding y to both sides and subtracting 5, which gives y = 4x − 5 as an expression in terms of x. Substitute this expression y = 4x − 5 into the second equation 2x + 3y = 13 to obtain 2x + 3(4x − 5) = 13 and simplify it to 14x − 15 = 13, leading to 14x = 28. Finally, divide both sides of 14x = 28 by 14 to get x = 2, and then substitute back into y = 4x − 5 to find y = 4(2) − 5 = 3, giving the solution (2, 3).

Step by step

1. **Isolate y from the first equation.:** Starting with 4x − y = 5: add y to both sides to get 4x = y + 5, then subtract 5 from both sides to get y = 4x − 5. Why: Substitution needs one variable written in terms of the other so that the two-variable system can be reduced to one variable.
2. **Substitute the expression for y into the second equation and simplify.:** Use y = 4x − 5 in 2x + 3y = 13: 2x + 3(4x − 5) = 13 ⇒ 2x + 12x − 15 = 13 ⇒ 14x − 15 = 13 ⇒ 14x = 28. Why: Replacing y by an equal expression involving x only produces a single-variable equation that can be solved directly.
3. **Find x and substitute back to find y.:** From 14x = 28, divide by 14: x = 28/14 = 2. Substitute in y = 4x − 5: y = 4(2) − 5 = 8 − 5 = 3. Hence the solution is (2, 3). Check: 4(2) − 3 = 5 and 2(2) + 3(3) = 4 + 9 = 13. Why: Back-substitution gives the remaining variable, and checking in both original equations confirms that the ordered pair satisfies the system.

Topic and concept

Algebraic methods: substitution and elimination. In the substitution method, rearrange one equation to express one variable in terms of the other, substitute that expression into the second equation, solve the resulting single-variable equation, and substitute back to find the remaining variable. Here y=4x−5 gives x=2 and y=3.

How marks are earned

1. From the first equation 4x − y = 5, isolate y by adding y to both sides and subtracting 5, which gives y = 4x − 5 as an expression in terms of x. (1 mark)
2. Substitute this expression y = 4x − 5 into the second equation 2x + 3y = 13 to obtain 2x + 3(4x − 5) = 13 and simplify it to 14x − 15 = 13, leading to 14x = 28. (1 mark)
3. Finally, divide both sides of 14x = 28 by 14 to get x = 2, and then substitute back into y = 4x − 5 to find y = 4(2) − 5 = 3, giving the solution (2, 3). (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 3, §3.3.1 Substitution Method: The excerpt demonstrates expressing one variable and substituting into the other equation for solution.

Section D

Question 32

5 marks

Long Answer

Answer both connected parts. Show the required method, explanation and units.

(a) For the quadratic polynomial $p(x)=2x^2-4x+1$, find the sum and product of its zeros without solving for the zeros. [2 marks]

Answer

Applying the relation $\alpha+\beta=-b/a$ for the quadratic $p(x)=2x^2-4x+1$, we substitute $b=-4$ and $a=2$ to obtain $\alpha+\beta=-(-4)/2=2$ as the sum of the zeros. Using the formula $\alpha\beta=c/a$ for the same polynomial, where $c=1$ and $a=2$, we compute the product of the zeros as 1/2, which confirms the required result.

Step by step

1. **Use the coefficient formula to find the sum of the zeros.:** For $p(x) = 2x^2 - 4x + 1$, compare with $ax^2 + bx + c$ to get $a = 2$, $b = -4$, $c = 1$. If the zeros are α and β, then $\alpha + \beta = -b/a = -(-4)/2 = 4/2 = 2$. Why: For a quadratic $ax^2 + bx + c$, the sum of the zeros is determined by the coefficients as $-b/a$, so the zeros need not be found separately.
2. **Use the coefficient formula to find the product of the zeros.:** Using $a = 2$ and $c = 1$ for the same polynomial, $\alpha\beta = c/a = 1/2$. Therefore, the product of the zeros is 1/2. Why: The product of the zeros of $ax^2 + bx + c$ is fixed by c/a, which gives the required value directly from the polynomial.

Topic and concept

Relationship between zeros and coefficients for quadratic polynomials. For a quadratic ax^2+bx+c, the sum of its zeros $\alpha+\beta$ equals $-b/a$ and their product $\alpha\beta$ equals c/a.

How marks are earned

1. Applying the relation $\alpha+\beta=-b/a$ for the quadratic $p(x)=2x^2-4x+1$, we substitute $b=-4$ and $a=2$ to obtain $\alpha+\beta=-(-4)/2=2$ as the sum of the zeros. (1 mark)
2. Using the formula $\alpha\beta=c/a$ for the same polynomial, where $c=1$ and $a=2$, we compute the product of the zeros as 1/2, which confirms the required result. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 2, §2.3 Relationship between Zeroes and Coefficients of a Polynomial: For ax^2+bx+c, $\alpha+\beta=-b/a$ and $\alpha\beta=c/a$.

(b) The sum of the reciprocals of the zeros of a monic quadratic is 4/3, and the sum of its zeros is 5.

Find the monic polynomial equation and verify it using the sum and product of its zeros. [3 marks]

Answer

The reciprocal sum gives $(\alpha+\beta)/(\alpha\beta)=4/3$, so $\alpha\beta=15/4$. The required monic equation is $x^2-5x+15/4=0$. Its coefficient sum/product give 5 and 15/4, whose reciprocal sum is 4/3.

Step by step

1. **Find the product of zeros:** Since $1/\alpha+1/\beta=(\alpha+\beta)/(\alpha\beta)=4/3$ and $\alpha+\beta=5$, solve $5/(\alpha\beta)=4/3$ to obtain $\alpha\beta=15/4$. Why: The reciprocal-sum identity relates the given values to the product needed for the polynomial.
2. **Form the monic polynomial:** For roots α and β, a monic quadratic is $x^2-(\alpha+\beta)x+\alpha\beta$, giving $x^2-5x+15/4=0$. Why: The leading coefficient remains one as the prompt requires.
3. **Verify the requested equation:** From $x^2-5x+15/4$, the coefficient relations give root sum 5 and root product 15/4; therefore reciprocal sum $5/(15/4)=4/3$. Why: This check confirms both given conditions without asking for a non-monic equation.

Topic and concept

Relationship between zeros and coefficients for quadratic polynomials. A monic quadratic with roots α and β has equation $x^2-(\alpha+\beta)x+\alpha\beta=0$. Reciprocal sum $(\alpha+\beta)/(\alpha\beta)$ determines the missing product.

How marks are earned

1. Use $(1/\alpha)+(1/\beta)=(\alpha+\beta)/(\alpha\beta)=4/3$ and $\alpha+\beta=5$ to derive $\alpha\beta=15/4$. (1 mark)
2. Form the monic equation $x^2-(\alpha+\beta)x+\alpha\beta=0$ and substitute to get $x^2-5x+15/4=0$. (1 mark)
3. Verify from the monic coefficients that root sum is 5 and reciprocal sum is 4/3; accept an equivalent algebraic check. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 2, §2.3 Relationship between Zeroes and Coefficients of a Polynomial: For ax^2+bx+c, $\alpha+\beta=-b/a$ and $\alpha\beta=c/a$.

Question 33

5 marks

Long Answer

Answer both connected parts. Show the required method, explanation and units.

(a) Check whether ΔPQR with sides 4 cm, 6 cm, 8 cm and ΔXYZ with sides 2 cm, 3 cm, 4 cm are similar, stating the criterion used. [2 marks]

Answer

The ratios of corresponding sides are $PQ/XY=4/2=2$, $QR/YZ=6/3=2$ and $RP/ZX=8/4=2$. Since all three ratios are equal, the triangles are similar by the SSS criterion.

Step by step

1. **Write and simplify the ratios of corresponding sides.:** Match the sides as PQ ↔ XY, QR ↔ YZ and RP ↔ ZX. Then PQ/XY = 4 cm/2 cm = 2, QR/YZ = 6 cm/3 cm = 2, and RP/ZX = 8 cm/4 cm = 2. Why: SSS similarity is tested by comparing all three pairs of corresponding side lengths, and the units cancel in each ratio.
2. **Apply the SSS similarity criterion.:** Since PQ/XY = QR/YZ = RP/ZX = 2, all three pairs of corresponding sides are in the same ratio. Therefore, ΔPQR ~ ΔXYZ by the SSS criterion. Why: When three pairs of corresponding sides of two triangles are proportional, the SSS criterion guarantees that the triangles are similar.

Topic and concept

Criteria for similarity of triangles — AA, SSS, SAS. If three pairs of corresponding sides of two triangles are in proportion, the triangles are similar (SSS criterion).

How marks are earned

1. The ratios of corresponding sides are PQ/XY=4/2=2, QR/YZ=6/3=2 and RP/ZX=8/4=2. (1 mark)
2. Since all three ratios are equal, the triangles are similar by the SSS criterion. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 6, §6.4: SSS criterion: equal ratios of all three pairs of sides imply similarity.

(b) In ΔABC and ΔDEF, AB/DE = AC/DF = 3/4 and ∠A = ∠D. Prove that the triangles are similar. [3 marks]

Answer

AB/DE=AC/DF=3/4 gives two proportional corresponding side pairs, and the included angles ∠A and ∠D are equal. Therefore ΔABC~ΔDEF by SAS similarity.

Step by step

1. **State the hypotheses and the first proportional side pair:** Given: in ΔABC and ΔDEF, AB/DE = 3/4, AC/DF = 3/4 and ∠A = ∠D. The first pair of corresponding sides is AB and DE, with ratio AB/DE = 3/4. Why: SAS similarity needs corresponding sides in proportion, so we first fix which sides correspond. The vertex order A↔D, B↔E, C↔F makes AB correspond to DE. This is a given ratio, not something to be found.
2. **Identify the second side pair and the included angle:** The second pair is AC and DF, with AC/DF = 3/4, the same ratio as AB/DE. The sides AB and AC meet at A, so ∠A is the angle between them. The sides DE and DF meet at D, so ∠D is the angle between them. Given ∠A = ∠D, the equal angles are the included angles of the two proportional pairs. Why: The SAS similarity criterion needs the equal angle to lie between the two proportional sides. An equal angle that is not between them would not be enough. ∠A lies between AB and AC, and ∠D lies between DE and DF, so the condition is met.

3. **Apply the SAS similarity criterion and conclude:** Since AB/DE = AC/DF = 3/4 and the included angles ∠A and ∠D are equal, ΔABC ~ ΔDEF by the SAS similarity criterion, with the correspondence A↔D, B↔E, C↔F. Why: The SAS criterion says that two triangles are similar if two pairs of corresponding sides are proportional and the included angles are equal. All three parts of that condition have been shown from the given data, so the criterion gives the conclusion directly.

Topic and concept

Criteria for similarity of triangles — AA, SSS, SAS. SAS similarity says that two triangles are similar if two pairs of corresponding sides are in the same ratio and the angles between those sides are equal. Here AB↔DE and AC↔DF are in ratio 3/4, and ∠A = ∠D is the angle between them.

How marks are earned

1. Identify AB/DE=3/4 as one proportional corresponding side pair. (1 mark)
2. Identify AC/DF=3/4 as the second proportional pair and ∠A=∠D as their equal included angles. (1 mark)
3. Apply the SAS similarity criterion and conclude ΔABC~ΔDEF. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 6, §6.4: SAS criterion: two sides in proportion and included angle equal imply similarity.

Question 34

5 marks

Long Answer

Attempt one option.

Option A

Answer both connected parts. Show the required method, explanation and units.

(a) Factorize the quadratic polynomial $p(x)=x^2-2x+1$ and state its zero, indicating its multiplicity. [2 marks]

Answer

By recognizing the perfect square trinomial, we can factorize x^2-2x+1 into $(x-1)^2$, showing its decomposition into identical linear factors. Since the factor $(x-1)$ appears twice in the factorization, the polynomial has a single distinct zero at $x=1$ with algebraic multiplicity two.

Step by step

1. **Rewrite the quadratic as a perfect-square product.:** $p(x)=x^2-2x+1=x^2-2(1)x+1^2=(x-1)^2=(x-1)(x-1)$. Why: This expresses the quadratic as linear factors, so the zeros can be read from factors that make the product equal to 0.

2. **Read the zero and its multiplicity from the repeated factor.:** For $(x-1)^2=0$, we need $x-1=0$, so $x=1$. The factor $(x-1)$ occurs two times, so the only distinct zero is 1 with algebraic multiplicity 2. Why: A quadratic has at most two zeros; here both possible zeros coincide because the same linear factor is repeated.

Topic and concept

Zeros of a polynomial; geometric meaning of zeros as x-intercepts. A polynomial of degree n can have at most n real zeros; in particular a quadratic can have at most two zeros.

How marks are earned

1. By recognizing the perfect square trinomial, we can factorize x^2-2x+1 into $(x-1)^2$, showing its decomposition into identical linear factors. (1 mark)
2. Since the factor $(x-1)$ appears twice in the factorization, the polynomial has a single distinct zero at $x=1$ with algebraic multiplicity two. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 2, §2.4 Summary: A quadratic polynomial can have at most two zeros; if both coincide, the zero is repeated.

(b) Prove algebraically that any non-zero linear polynomial $p(x)=ax+b$, with $a\neq0$, has exactly one zero. [3 marks]

Answer

Solving $ax+b=0$ gives $x=-b/a$ because $a\neq0$. Substitution gives $p(-b/a)=0$. If u and v are zeros, subtracting $au+b=0$ and $av+b=0$ gives $a(u-v)=0$; as $a\neq0$, $u=v$. Thus there is exactly one zero.

Step by step

1. **Find the candidate zero:** Set $ax+b=0$ and divide by $a\neq0$ to obtain $x=-b/a$. Why: The non-zero leading coefficient makes the division valid.
2. **Verify existence:** Substitute $x=-b/a$: $p(-b/a)=a(-b/a)+b=-b+b=0$. Why: The candidate is therefore an actual zero.
3. **Prove uniqueness algebraically:** If u and v are zeros, then $au+b=av+b=0$. Subtraction gives $a(u-v)=0$, hence $u=v$ because $a\neq0$. Why: This proves there cannot be two distinct zeros without using a graph.

Topic and concept

Zeros of a polynomial; geometric meaning of zeros as x-intercepts. A zero satisfies $p(x)=0$. Solving $ax+b=0$ gives $-b/a$; substitution proves existence and subtracting two zero equations proves uniqueness.

How marks are earned

1. Set $ax+b=0$ and solve $x=-b/a$ using $a\neq0$. (1 mark)
2. Substitute $x=-b/a$ to verify $p(-b/a)=0$ and establish existence. (1 mark)
3. Assume two zeros u and v, subtract their equations and use $a\neq0$ to conclude $u=v$, proving uniqueness algebraically. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 2, §2.2 Geometrical Meaning of the Zeroes of a Polynomial: The graph of y=ax+b intersects the x-axis at a single point whose x-coordinate is −b/a.

OR

Option B

Answer both connected parts. Show the required method, explanation and units.

(a) For the linear polynomial p(x)=4x−8, determine its zero and the coordinates of the point where the graph of y=p(x) meets the x-axis. [2 marks]

Answer

By applying the definition of a zero of the polynomial, set p(x)=0 which yields the equation 4x−8=0; solving this gives x=2 as the unique zero of p(x). Because the zero of the function corresponds to its x–intercept on the Cartesian plane, the graph of y=4x−8 meets the x–axis at the coordinate point (2,0).

Step by step

1. **Set the linear polynomial equal to zero and solve for x.:** A zero k satisfies p(k)=0. So, 4x−8=0 ⇒ 4x=8 ⇒ x=8/4=2. Check: p(2)=4(2)−8=8−8=0, so the unique zero is x=2. Why: By definition, a real number is a zero of p(x) exactly when substituting it makes the polynomial value 0.
2. **Convert the zero into the x-axis intersection point.:** On the x-axis, y=0. Since p(2)=0, the graph y=4x−8 meets the x-axis at x=2, giving the coordinate point (2,0). Why: The zero of a polynomial gives the x-coordinate where its graph crosses or meets the x-axis.

Topic and concept

Zeros of a polynomial; geometric meaning of zeros as x-intercepts. The zeros of p(x) correspond to the x-coordinates where the graph of y=p(x) crosses the x-axis.

How marks are earned

1. By applying the definition of a zero of the polynomial, set p(x)=0 which yields the equation 4x−8=0; solving this gives x=2 as the unique zero of p(x). (1 mark)
2. Because the zero of the function corresponds to its x–intercept on the Cartesian plane, the graph of y=4x−8 meets the x–axis at the coordinate point (2,0). (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 2, §2.2 Geometrical Meaning of the Zeroes of a Polynomial: The zero of ax+b is the x-coordinate of the point where the graph of y=ax+b intersects the x-axis.

(b) The graph of a quadratic polynomial y=p(x) crosses the x-axis at x=−2 and x=5 and also passes

through the point (1,−8). Determine the explicit form of p(x). [3 marks]

Answer

Since the polynomial crosses the x–axis at x=−2 and x=5, its factorized form can be written as p(x)=k(x+2)(x−5), where k is a nonzero constant. Substituting the point (1,−8) into this form gives −8=k(1+2)(1−5)=k·3·(−4)=−12k, hence solving for k yields k=(−8)/(−12)=2/3. Thus, upon expanding, the explicit form of the quadratic polynomial is p(x)=(2/3)(x+2)(x−5)=(2/3)(x²−3x−10)=(2/3)x²−2x−20/3 as required.

Step by step

1. **Form the quadratic from its two x-intercepts.:** The graph crosses the x-axis at x=−2 and x=5, so p(−2)=0 and p(5)=0. Hence the corresponding factors are x−(−2)=x+2 and x−5. Therefore, write p(x)=k(x+2)(x−5), where k≠0. Why: If p(α)=0, then x−α is a factor of p(x), so the given zeros determine the linear factors of the quadratic.
2. **Use the given point to find the constant multiplier.:** Since the graph passes through (1,−8), substitute x=1 and p(1)=−8: −8=k(1+2)(1−5)=k·3·(−4)=−12k. Therefore, k=(−8)/(−12)=2/3. Why: The factors give the zeros, but the extra point fixes the vertical scale factor k of the particular quadratic.
3. **Substitute k and expand to get the explicit polynomial.:** p(x)=(2/3)(x+2)(x−5). Now, (x+2)(x−5)=x²−5x+2x−10=x²−3x−10. Hence p(x)=(2/3)(x²−3x−10)=(2/3)x²−2x−20/3. Check: p(1)=2/3−2−20/3=−6/3−18/3=−24/3=−8. Why: Expanding converts the factorized form into an explicit quadratic expression while the check confirms it still passes through the given point.

Topic and concept

Zeros of a polynomial; geometric meaning of zeros as x-intercepts. The zeros of p(x) correspond to the x-coordinates where the graph of y=p(x) crosses the x-axis.

How marks are earned

1. Since the polynomial crosses the x–axis at x=−2 and x=5, its factorized form can be written as p(x)=k(x+2)(x−5), where k is a nonzero constant. (1 mark)
2. Substituting the point (1,−8) into this form gives −8=k(1+2)(1−5)=k·3·(−4)=−12k, hence solving for k yields k=(−8)/(−12)=2/3. (1 mark)
3. Thus, upon expanding, the explicit form of the quadratic polynomial is p(x)=(2/3)(x+2)(x−5)=(2/3)(x²−3x−10)=(2/3)x²−2x−20/3 as required. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 2, §2.4 Summary: If p(α)=0 then (x−α) is a factor of p(x), so zeros determine linear factors.

Question 35

5 marks

Long Answer

Attempt one option.

Option A

Answer both connected parts. Show the required method, explanation and units.

(a) A relay tower must divide AB in the ratio 1:2, where A is (0,0) and B is (36,15). Find its coordinates. [2 marks]

Answer

By section formula, P = ((1·36 + 2·0)/(1+2), (1·15 + 2·0)/3) = (12, 5). Thus, the tower should be located at (12, 5) units from the origin.

Step by step

1. **Apply the internal section formula using the ratio 1:2.:** Let P divide A(0,0) and B(36,15) internally in the ratio AP:PB=1:2. Then P=((1·36+2·0)/(1+2),(1·15+2·0)/(1+2))=(36/3,15/3)=(12,5). Why: For internal division in the ratio m:n, the coordinates are weighted averages of the endpoint coordinates.
2. **State the tower location and verify the required division ratio.:** Thus, the tower should be located at (12,5) units from the origin. Check: AP=$\sqrt{((12-0)^2+(5-0)^2)}=\sqrt{(144+25)}=\sqrt{169}=13$ units, and PB=$\sqrt{((36-12)^2+(15-5)^2)}=\sqrt{(24^2+10^2)}=\sqrt{(576+100)}=\sqrt{676}=26$ units. Therefore, AP:PB=13:26=1:2. Why: Using the distance formula confirms that the computed point divides AB in the required ratio, not merely that it lies on the coordinate plane.

Topic and concept

Section formula for internal division. A point P dividing AB internally in the ratio m:n has coordinates $((m\cdot x_2 + n\cdot x_1)/(m+n), (m\cdot y_2 + n\cdot y_1)/(m+n))$.

How marks are earned

1. By section formula, P = ((1·36 + 2·0)/(1+2), (1·15 + 2·0)/3) = (12, 5). (1 mark)
2. Thus, the tower should be located at (12, 5) units from the origin. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 7, §7.3: Coordinates of a point dividing a segment internally in a given ratio are given by weighted averages.

(b) For A(1,2) and B(7,8), the point P divides AB internally at P(4,5). Determine the ratio AP:PB. [3 marks]

Answer

Applying the section formula to the x-coordinates of A(1,2), B(7,8), and P(4,5) gives (m·7 + n·1)/(m + n) = 4, leading to 7m + n = 4m + 4n and hence simplifying to 3m = 3n. Similarly, applying the ratio to the y-coordinates yields (m·8 + n·2)/(m + n) = 5, resulting in 8m + 2n = 5m + 5n and again simplifying to 3m = 3n, confirming consistency from both coordinates. Since both coordinate equations reduce to m = n, the internal division ratio AP:PB is established as 1:1, indicating that P divides AB exactly at its midpoint.

Step by step

1. **Use the x-coordinate section equation for ratio AP:PB=m:n.:** For A(1,2), B(7,8), and P(4,5), the x-coordinate gives $(m\cdot7+n\cdot1)/(m+n)=4$. Hence $7m+n=4(m+n)=4m+4n$, so $7m-4m=4n-n$, giving $3m=3n$. Why: The x-coordinate of an internal division point is the weighted average of the endpoint x-coordinates in the same ratio m:n.
2. **Use the y-coordinate section equation to confirm the same ratio.:** The y-coordinate gives $(m\cdot8+n\cdot2)/(m+n)=5$. Hence $8m+2n=5(m+n)=5m+5n$, so $8m-5m=5n-2n$, giving $3m=3n$. Why: Checking the y-coordinate ensures that the same ratio is consistent for the full point P(4,5), not only for its horizontal position.
3. **Reduce the relation and state AP:PB.:** From $3m=3n$, divide both sides by 3 to get $m=n$. Therefore, AP:PB=m:n=1:1. Also, $AP=\sqrt{((4-1)^2+(5-2)^2)}=\sqrt{(9+9)}=3\sqrt{2}$ and $PB=\sqrt{((7-4)^2+(8-5)^2)}=\sqrt{(9+9)}=3\sqrt{2}$, confirming P is the midpoint. Why: Equal ratio parts mean the point divides the segment into two equal lengths, which is exactly the midpoint condition.

Topic and concept

Section formula for internal division. A point P dividing AB internally in the ratio m:n has coordinates $((m\cdot x_2 + n\cdot x_1)/(m+n), (m\cdot y_2 + n\cdot y_1)/(m+n))$.

How marks are earned

1. Applying the section formula to the x-coordinates of A(1,2), B(7,8), and P(4,5) gives (m·7 + n·1)/(m + n) = 4, leading to 7m + n = 4m + 4n and hence simplifying to 3m = 3n. (1 mark)
2. Similarly, applying the ratio to the y-coordinates yields (m·8 + n·2)/(m + n) = 5, resulting in 8m + 2n = 5m + 5n and again simplifying to 3m = 3n, confirming consistency from both coordinates. (1 mark)
3. Since both coordinate equations reduce to m = n, the internal division ratio AP:PB is established as 1:1, indicating that P divides AB exactly at its midpoint. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 7, §7.3: Equating weighted sums of coordinates yields the division ratio.

OR

Option B

Answer both connected parts. Show the required method, explanation and units.

(a) Town A is at (10,20) km and town B is at (16,28) km on a coordinate grid. Find their straight-line distance in km using the distance formula. [2 marks]

Answer

AB=√[(16−10)²+(28−20)²] km=√(6²+8²) km=√100 km=10 km.

Step by step

1. **Write the distance formula and substitute the coordinates of A(10,20) and B(16,28):** AB = $\sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$ = $\sqrt{[(16 - 10)^2 + (28 - 20)^2]}$ km. The horizontal difference is 6 km and the vertical difference is 8 km. Why: The distance formula comes from applying the Pythagoras theorem to the horizontal and vertical differences of the two points. These differences are the legs of a right triangle, and the straight-line distance is its hypotenuse.
2. **Evaluate the square root and state the distance with its unit:** AB = $\sqrt{(6^2 + 8^2)}$ km = $\sqrt{(36 + 64)}$ km = $\sqrt{100}$ km = 10 km. Check: $6^2 + 8^2 = 10^2$ is a 6–8–10 Pythagorean triple. Why: The coordinates are given in km, so the distance is also in km. Squaring removes the sign of each difference, so the order of subtraction does not affect the result.

Topic and concept

Distance formula between two points. For two points P(x_1, y_1) and Q(x_2, y_2), the distance is PQ = $\sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$. The formula comes from the Pythagoras theorem applied to the horizontal and vertical differences. The coordinates of the towns are given in km, so the result is in km.

How marks are earned

1. Write the distance formula and substitute the coordinate differences: AB=√[(16−10)²+(28−20)²] km. (1 mark)
2. Evaluate √(6²+8²)=√100=10 and state the distance as 10 km. (1 mark)

Source check: Using the NCERT Class 10 maths Ch. 7 distance formula, AB²=6²+8²=100 km², so AB=10 km.

(b) Find the distance between P(3,4) and Q(-1,2), giving your answer in simplest radical form. [3 marks]

Answer

Starting with P(3,4) and Q(−1,2), we compute the horizontal difference Δx = (−1 − 3) = −4 and the vertical difference Δy = (2 − 4) = −2 before applying the distance formula PQ = $\sqrt{(\Delta x^2 + \Delta y^2)}$. Substituting these values into the formula gives PQ = $\sqrt{[(-4)^2 + (-2)^2]}$ = $\sqrt{(16 + 4)}$ = $\sqrt{20}$, which represents the exact distance before simplification. By extracting the factor 4 under the radical, we simplify $\sqrt{20}$ to $2\sqrt{5}$, thus obtaining the distance between P and Q as PQ = $2\sqrt{5}$ units in simplest radical form.

Step by step

1. **Find the horizontal and vertical differences between P and Q and name the distance formula.:** For P(3,4) and Q(−1,2), $\Delta x = x_2 - x_1 = -1 - 3 = -4$ and $\Delta y = y_2 - y_1 = 2 - 4 = -2$, so the formula to use is $PQ = \sqrt{((x_2-x_1)^2 + (y_2-y_1)^2)} = \sqrt{(\Delta x^2 + \Delta y^2)}$. Why: The distance formula comes from applying the Pythagorean theorem to a right triangle whose legs are the horizontal and vertical differences, so both differences must be found first. The signs do not matter, because each difference gets squared.
2. **Substitute the differences into the formula and add the squares.:** $PQ = \sqrt{[(-4)^2 + (-2)^2]} = \sqrt{(16 + 4)} = \sqrt{20}$, since $(-4)^2 = 16$ and $(-2)^2 = 4$. The value $\sqrt{20}$ is exact but not yet in simplest form. Why: Squaring each difference separately gives the squared lengths of the two legs. Their sum is the square of the hypotenuse PQ, so the square root of that sum is the distance. Squaring also makes the negative signs disappear, so the sum is positive.
3. **Simplify $\sqrt{20}$ by taking out the largest perfect-square factor.:** Write $20 = 4 \times 5$, so $\sqrt{20} = \sqrt{(4 \cdot 5)} = \sqrt{4} \times \sqrt{5} = 2\sqrt{5}$. Check: $(2\sqrt{5})^2 = 4 \times 5 = 20$, which matches. The distance is $PQ = 2\sqrt{5}$ units, which is about 4.47 units. Why: The product rule $\sqrt{(ab)} = \sqrt{a} \times \sqrt{b}$ lets the perfect square 4 come out of the radical as 2. Using the largest perfect-square factor leaves 5 with no square factor, so $2\sqrt{5}$ is in simplest radical form. Distance is a length, so the answer is given in units.

Topic and concept

Distance formula between two points. For any two points $P(x_1,y_1)$ and $Q(x_2,y_2)$, the distance is $PQ = \sqrt{((x_2-x_1)^2 + (y_2-y_1)^2)}$. It follows from the Pythagorean theorem applied to the horizontal and vertical differences, which are the legs of a right triangle with PQ as the hypotenuse. Here the differences are −4 and −2, giving $\sqrt{20}$, which simplifies to $2\sqrt{5}$ units.

How marks are earned

1. Starting with P(3,4) and Q(−1,2), we compute the horizontal difference $\Delta x = (-1 - 3) = -4$ and the vertical difference $\Delta y = (2 - 4) = -2$ before applying the distance formula $PQ = \sqrt{(\Delta x^2 + \Delta y^2)}$. (1 mark)
2. Substituting these values into the formula gives $PQ = \sqrt{[(-4)^2 + (-2)^2]} = \sqrt{(16 + 4)} = \sqrt{20}$, which represents the exact distance before simplification. (1 mark)
3. By extracting the factor 4 under the radical, we simplify $\sqrt{20}$ to $2\sqrt{5}$, thus obtaining the distance between P and Q as $PQ = 2\sqrt{5}$ units in simplest radical form. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 7, §7.2: Distance between two general points uses squared differences and square root.

Section E

Question 36

4 marks

Case Based

A circular park has centre O and radius 28 m. A chord AB subtends 60° at O, dividing the park into a minor sector AOB and a major sector. Additionally, a radial line OC bisects the minor sector AOB, creating two equal sectors of 30° each.

(a) Using the scenario below, calculate the length of arc AB in the circular park. [1 mark]

Answer

Arc length AB = (60/360)×2π×28 = (1/6)×56π = (28/3)π m.

Step by step

1. **Use the central angle to calculate the required arc length.:** The length of arc AB is L= (θ/360)×2πr=(60/360)×2π×28=(1/6)×56π=(28/3)π m, and the metre unit is correct because arc length is a length measured along the circumference of the park. Why: An arc with central angle θ occupies θ/360 of the full circumference 2πr, so a 60° arc is one-sixth of the circle's circumference.

Topic and concept

Area and circumference of a circle. Arc length for a central angle θ is (θ/360)×2πr, with θ measured in degrees; here the chord AB subtends 60° at the centre, so the 60° angle is used for arc AB.

How marks are earned

1. Arc length AB = (60/360)×2π×28 = (1/6)×56π = (28/3)π m. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 11 Summary: Length of an arc of a circle is (θ/360)×2πr.

(b) In the same park scenario, find the area of the minor sector AOB. [1 mark]

Answer

Area of sector AOB = (60/360)×π×28² = (1/6)×784π = (392/3)π m².

Step by step

1. **Find the fraction of the circle that the sector covers, then apply the sector area formula:** The sector angle is 60° out of 360°, so it covers 60/360 = 1/6 of the circle. Area of sector AOB = (60/360) × π × 28² = (1/6) × 784π = (392/3)π m². With π = 22/7 this is (392/3) × (22/7) = 1232/3 ≈ 410.67 m². Why: A sector of angle θ takes up θ/360 of the full circle, so its area is that fraction of πr². The 30° line OC only splits the sector into two halves and does not change the area of the whole sector AOB. The answer is in m², because the radius is in m.

Topic and concept

Area of a sector and length of an arc. The area of a sector with central angle θ in a circle of radius r is (θ/360°) × πr². A 60° sector is 1/6 of the full circle area πr², where r = 28 m. This part asks only for the whole minor sector AOB.

How marks are earned

1. Area of sector AOB = (60/360)×π×28² = (1/6)×784π = (392/3)π m². (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 11 Summary: Area of a sector of a circle is (θ/360)×πr².

Attempt one of the final two-mark options.

Option A

(c) (i) Still using the park details, compute the area of one of the two smaller sectors (for example AOC). (ii) Using the same data, determine the area of the minor segment APB formed by chord AB. [2 marks]

Answer

Central angle for sector AOC = 30°, so area = (30/360)×π×28² = (1/12)×784π = (196/3)π m². Minor segment area = area of sector AOB – area of ΔAOB = (392/3)π – ½×28×28×sin60° = (392/3)π – ½×784×(√3/2) = (392/3)π – 196√3 m².

Step by step

1. **Find the area of the 30° sector AOC using the sector formula:** Area = (θ/360)×πr² = (30/360)×π×28² = (1/12)×784π = 784π/12 = (196/3)π m². Check: this is half of the 60° sector AOB, whose area is (60/360)×784π = (392/3)π m², and half of 392π/3 is 196π/3. With π=22/7 the area is about 205.3 m². Why: A sector with central angle θ is the fraction θ/360 of the whole circle, so its area is that fraction of πr². OC bisects the 60° angle, which makes angle AOC 30° and gives the fraction 30/360=1/12. The area is in m² because it is a length times a length.

2. **Find the minor segment area as sector AOB minus triangle AOB:** Sector AOB = (60/360)×π×28² = (392/3)π m². In triangle AOB, OA=OB=28 m and angle AOB=60°. So the other two angles are each (180°−60°)/2=60°, the triangle is equilateral, and its area is ½×28×28×sin60° = ½×784×(√3/2) = 196√3 m². Segment area = (392/3)π − 196√3 m². Numerically, with π=22/7, this is about 410.67 − 339.5 ≈ 71.2 m², which is positive and smaller than the sector, as it should be. Why: The minor segment is the part of sector AOB left after the triangle bounded by the two radii and the chord is removed, so area of segment = area of sector − area of triangle. The sector area uses the fraction 60/360. The triangle area uses ½ab sinC with two radii as sides and the 60° angle between them. Subtraction is correct because the segment lies inside the sector.

Topic and concept

Area of a sector and length of an arc. A sector with central angle θ in a circle of radius r has area (θ/360°)×πr². A segment is the region between a chord and its arc. Its area is the area of the sector minus the area of the triangle formed by the two radii and the chord. Here r=28 m, and a 60° angle between two equal radii makes triangle AOB equilateral.

How marks are earned

1. Central angle for sector AOC = 30°, so area = (30/360)×π×28² = (1/12)×784π = (196/3)π m². (1 mark)
2. Minor segment area = area of sector AOB – area of ΔAOB = (392/3)π – ½×28×28×sin60° = (392/3)π – ½×784×(√3/2) = (392/3)π – 196√3 m². (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 11 Summary: Area of a sector of a circle is (θ/360)×πr². Check the reasoning against NCERT Class 10 maths, Ch. 11 Summary: Area of a segment = area of corresponding sector – area of corresponding triangle.

OR

Option B

(c) Find the area of the major sector of the park left after the 60° minor sector is removed. [2 marks]

Answer

The major central angle is 360°−60°=300°, so its area is (300/360)πr². With r=28 m, the area is (5/6)π(28²)=(1960/3)π m².

Step by step

1. **Find the central angle of the major sector and its fraction of the circle:** The minor sector is 60° and the full angle at O is 360°, so the major sector angle is 360° − 60° = 300°. Its fraction of the circle is 300/360 = 5/6, so its area is (300/360)×πr². Why: The minor and major sectors together make one complete turn about O, so the major sector's angle is the remainder. A sector's area is its angle divided by 360 times the circle's area πr².

2. **Substitute r = 28 m, simplify, and check against full circle minus minor sector:** Area = (5/6)×π×28² = (5/6)×784π = 3920π/6 = (1960/3)π m². Check: full circle = 784π = (2352/3)π, minor sector = (392/3)π, and (2352/3 − 392/3)π = (1960/3)π m², which matches. With π = 22/7 the area is 6160/3 ≈ 2053.33 m². Why: The areas of the two sectors add up to the area of the whole circle, so subtracting the minor sector from πr² must give the same major sector area. Because r² is in m², the answer is in square metres.

Topic and concept

Area of a sector and length of an arc. The park is split into a minor sector (60°) and a major sector (300°). Each sector's area is (θ/360)×πr², and the two areas sum to πr². The major sector is therefore 5/6 of the park.

How marks are earned

1. The major central angle is 360°−60°=300°, so its area is (300/360)πr². (1 mark)
2. With r=28 m, the area is (5/6)π(28²)=(1960/3)π m². (1 mark)

Source check: Check the passage facts and NCERT Class 10 maths, Ch. 11 Summary against both steps.

Question 37

4 marks

Case Based

In a botanical garden, a circular pond has centre O and radius 8 m. A straight rod AB lies along the tangent to the pond at its contact point P. A point Q lies 15 m from O outside the pond. From Q, two ropes touch the pond at T_1 and T_2, and $QT_1 = QT_2$.

(a) According to the passage, what is a tangent to a circle? [1 mark]

Answer

A tangent to a circle is a straight line that touches the circle at exactly one point.

Step by step

1. **State the definition of a tangent by counting how many points the line shares with the circle.:** A tangent to a circle is a straight line that touches the circle at exactly one point. In the passage, the rod AB lies along the tangent to the pond and touches it only at the contact point P, so AB meets the circle at one point. Why: A line and a circle in a plane can meet in three ways: at no point, at two points (a secant), or at exactly one point (a tangent). Counting the common points tells these cases apart, and only exactly one common point gives a tangent.

Topic and concept

Tangent to a circle; tangent at any point is perpendicular to the radius (theorem + proof). A line and a circle in the same plane can relate in three ways. A non-intersecting line meets the circle at no point, a secant meets it at two points, and a tangent touches it at exactly one point. In the passage, the rod AB touching the pond at P is the tangent, and P is the point of contact.

How marks are earned

1. A tangent to a circle is a straight line that touches the circle at exactly one point. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 10, §10.2: A tangent to a circle intersects it at exactly one point.

(b) What relationship between the rod AB at P and the radius OP is described by the theorem referenced in the passage? [1 mark]

Answer

AB is tangent at P, so AB is perpendicular to radius OP at P.

Step by step

1. **Apply the theorem that a tangent is perpendicular to the radius at the point of contact:** The rod AB lies along the tangent to the pond at P, and OP is the radius to P. By the theorem, AB is perpendicular to OP, so the angle between AB and OP at P is 90°. Why: The theorem states that the tangent at any point of a circle is perpendicular to the radius through the point of contact. It applies here because AB touches the pond at exactly one point, P; a line through P that cuts the circle would not be perpendicular to OP.

Topic and concept

Tangent to a circle; tangent at any point is perpendicular to the radius (theorem + proof). A tangent touches a circle at exactly one point. The tangent at any point of a circle is perpendicular to the radius through that point of contact, because the radius is the shortest distance from the centre to the tangent line.

How marks are earned

1. State that rod AB, tangent at P, is perpendicular to radius OP at the contact point. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 10, Summary: At the point of contact, the tangent is perpendicular to the radius drawn to that point.

Attempt one of the final two-mark options.

Option A

(c) (i) How many tangents can be drawn from point Q to the pond, and what are they named in the passage? (ii) Which property of the lengths QT_1 and QT_2, mentioned in the passage, illustrates a theorem about tangents from an external point? [2 marks]

Answer

From the external point Q, exactly two tangents can be drawn to the pond, named QT_1 and QT_2. The lengths of the two tangents drawn from the external point Q to the pond are equal ($QT_1 = QT_2$).

Step by step

1. **Count and name the tangents from the external point Q:** Q is 15 m from O and the pond has radius 8 m, so Q lies outside the pond. Exactly two tangents can be drawn from Q, named QT_1 and QT_2 in the passage, touching the pond at T_1 and T_2. Why: A point outside a circle has exactly two tangents to it, while a point on the circle has one and a point inside has none. Since OQ = 15 m is greater than the radius 8 m, Q is external.
2. **State and justify the equal-length property:** The passage gives $QT_1 = QT_2$, so the two tangent lengths are equal. Reason: $OT_1 \perp QT_1$ and $OT_2 \perp QT_2$, and $OT_1 = OT_2 = 8$ m with common hypotenuse OQ, so the right triangles are congruent. By Pythagoras, $QT_1 = QT_2 = \sqrt{(15^2 - 8^2)} = \sqrt{(225 - 64)} = \sqrt{161}$ m. Why: Tangents from an external point have equal length because each tangent is perpendicular to its radius, giving two right triangles with equal radii and a shared hypotenuse OQ. Pythagoras then gives the same length for both, which illustrates the equal-tangents theorem.

Topic and concept

Number of tangents from an external point; lengths of two tangents from an external point are equal (theorem + proof). From a point outside a circle, exactly two tangents can be drawn, and the lengths of these two tangent segments are equal. In this pond, the tangents from Q are QT_1 and QT_2, and $QT_1 = QT_2$.

How marks are earned

1. From the external point Q, exactly two tangents can be drawn to the pond, named QT_1 and QT_2. (1 mark)
2. The lengths of the two tangents drawn from the external point Q to the pond are equal ($QT_1 = QT_2$). (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 10, §10.3: A point outside the circle has exactly two tangents to the circle. Check the reasoning against NCERT Class 10 maths, Ch. 10, Summary: The lengths of the two tangents from an external point to a circle are equal.

OR

Option B

(c) Calculate the length of each rope QT_1 and QT_2 from the external point Q. [2 marks]

Answer

A radius to a tangent is perpendicular at contact, so OT_1Q is right-angled and $QT_1^2 = OQ^2 - OT_1^2$. $QT_1 = \sqrt{(15^2 - 8^2)} = \sqrt{161}$ m; tangents from Q are equal, so $QT_2 = \sqrt{161}$ m too.

Step by step

1. **Form the right triangle using the radius to the tangent point T_1.:** The rope QT_1 is tangent to the pond at T_1, and OT_1 is the radius to the point of contact. Therefore $OT_1 \perp QT_1$, so $\triangle OT_1Q$ is right-angled at T_1. By Pythagoras, $OQ^2 = OT_1^2 + QT_1^2$, hence $QT_1^2 = OQ^2 - OT_1^2$. Why: The perpendicularity of a radius to the tangent at the point of contact allows the tangent length to be found as one leg of a right triangle.

2. **Substitute the given radius and external distance, then use equal tangents from Q.:** Given OQ = 15 m and OT_1 = 8 m, QT_1 = √(15² − 8²) = √(225 − 64) = √161 m. Since the two tangents drawn from the same external point Q are equal, QT_2 = QT_1 = √161 m. Why: After computing one tangent segment from the right triangle, the equal-tangents property gives the length of the second rope without a separate calculation.

Topic and concept

Number of tangents from an external point; lengths of two tangents from an external point are equal (theorem + proof). A radius meets a tangent at a right angle, so Pythagoras gives each tangent length from OQ and the radius; tangents from Q are equal.

How marks are earned

1. A radius to a tangent is perpendicular at contact, so OT_1Q is right-angled and $QT_1^2=OQ^2-OT_1^2$. (1 mark)
2. $QT_1=\sqrt{(15^2-8^2)}=\sqrt{161}$ m; tangents from Q are equal, so $QT_2=\sqrt{161}$ m too. (1 mark)

Source check: Check the passage facts and NCERT Class 10 maths, Ch. 10, §10.2 against both steps.

Question 38

4 marks

Case Based

In an experiment, two unbiased coins are tossed together. The possible outcomes of this random experiment are equally likely. Consider events based on this experiment.

(a) List the sample space for the experiment described in the passage. [1 mark]

Answer

The sample space is {HH, HT, TH, TT}.

Step by step

1. **Enumerate all outcomes for the two tossed coins.:** For the first coin: H or T. For the second coin: H or T. Therefore the complete sample space is {HH, HT, TH, TT}. Why: The sample space must include every elementary outcome of the experiment, including HT and TH as separate outcomes for two coins tossed together.

Topic and concept

Classical definition of probability — P(E) = number of favourable outcomes / total outcomes. The probability of an event E, assuming equally likely outcomes, is the ratio of favourable outcomes to the total number of outcomes.

How marks are earned

1. The sample space is {HH, HT, TH, TT}. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 14, §14.1 Probability — A Theoretical Approach: The sample space of a random experiment is the set of all possible outcomes.

(b) If E is the event “exactly one head appears,” find P(E). [1 mark]

Answer

Using U7-CH14-F001, P(E) = Number of favourable outcomes (2) / Total outcomes (4) = 2/4 = 1/2.

Step by step

1. **List the sample space, count the favourable outcomes and apply the probability ratio:** Tossing two coins gives the equally likely outcomes HH, HT, TH, TT, so there are 4 in total. Exactly one head occurs in HT and TH, so there are 2 favourable outcomes. P(E) = favourable outcomes ÷ total outcomes = 2/4 = 1/2. Check: the complement (HH or TT) also has probability 2/4 = 1/2, and 1/2 + 1/2 = 1. Why: For equally likely outcomes, the probability of an event is the number of outcomes favourable to it divided by the total number of outcomes. HT and TH are different outcomes because the two coins are distinguishable, so both are counted. The check P(E)+P(not E)=1 confirms the value.

Topic and concept

Probability of sure and impossible events; 0 ≤ P(E) ≤ 1. When all outcomes of an experiment are equally likely, P(E) equals the number of outcomes favourable to E divided by the total number of outcomes in the sample space. For two coins the sample space is {HH, HT, TH, TT}, and the event 'exactly one head' contains HT and TH. The probability lies between 0 and 1, and P(E)+P(not E)=1.

How marks are earned

1. Using U7-CH14-F001, P(E) = Number of favourable outcomes (2) / Total outcomes (4) = 2/4 = 1/2. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 14, §14.1 Probability — A Theoretical Approach: Classical probability is defined as favourable outcomes over total equally likely outcomes.

Attempt one of the final two-mark options.

Option A

(c) (i) For the event F = “at least one head appears,” determine whether it is a sure event, an impossible event, or neither. (ii) Let G be the event “both tosses show tails.” Find P(G′) and verify that P(G)+P(G′)=1. [2 marks]

Answer

P(F)=3/4, which lies strictly between 0 and 1, so F is neither a sure nor an impossible event. P(G)=1/4, so P(G′)=1−1/4=3/4, and thus P(G)+P(G′)=1/4+3/4=1.

Step by step

1. **List the sample space and the outcomes of F to decide its type:** Tossing two coins gives the sample space {HH, HT, TH, TT}, so there are 4 equally likely outcomes. F = at least one head = {HH, HT, TH}, which has 3 outcomes, so P(F) = 3/4. Since 0 < 3/4 < 1, F is neither a sure event nor an impossible event. Why: Probability equals favourable outcomes divided by total equally likely outcomes. A sure event has probability 1 and an impossible event has probability 0, so a value strictly between 0 and 1 means the event can happen but need not.

2. **Find P(G) and P(G′) and verify that they sum to 1:** G = both tails = {TT}, which has 1 outcome, so P(G) = 1/4. G′ = {HH, HT, TH}, so P(G′) = 3/4, which equals 1 − 1/4. Check: P(G) + P(G′) = 1/4 + 3/4 = 1. Why: An event and its complement together cover every outcome of the sample space without overlap, so their probabilities add up to 1. This gives P(G′) = 1 − P(G) and explains why the check works.

Topic and concept

Complementary events and their probability relation. For equally likely outcomes, P(E) is favourable outcomes divided by total outcomes. A sure event has probability 1, an impossible event has probability 0, and for any event E, P(E) + P(E′) = 1. For two coins the sample space is {HH, HT, TH, TT}.

How marks are earned

1. P(F)=3/4, which lies strictly between 0 and 1, so F is neither a sure nor an impossible event. (1 mark)
2. P(G)=1/4, so P(G′)=1−1/4=3/4, and thus P(G)+P(G′)=1/4+3/4=1. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 14, §14.2 Summary: A probability lies between 0 and 1; 0 indicates impossible, 1 indicates sure. Check the reasoning against NCERT Class 10 maths, Ch. 14, §14.2 Summary: For any event E, P(E)+P(E′)=1.

OR

Option B

(c) Find the probability of at least one tail, then the probability of its complementary event. [2 marks]

Answer

At least one tail occurs in HT, TH and TT, so its probability is 3/4. The complementary event is no tail, namely HH, with probability 1/4; the two probabilities sum to 1.

Step by step

1. **List the sample space and count outcomes with at least one tail:** Tossing two coins gives the sample space {HH, HT, TH, TT}, which has 4 equally likely outcomes. The outcomes with at least one tail are HT, TH and TT, so there are 3 favourable outcomes and P(at least one tail) = 3/4. Why: For equally likely outcomes, probability is favourable outcomes divided by total outcomes (CH14-C01). Listing the sample space first ensures that no outcome is missed or counted twice.
2. **Identify the complementary event and verify using P(E)+P(Ē)=1:** The complement of 'at least one tail' is 'no tail', which contains only HH, so P(no tail) = 1/4. Check with the complement rule: 1 − 3/4 = 1/4, and 3/4 + 1/4 = 1. Why: The complement contains exactly the outcomes not in the event, so the two events together cover the whole sample space and their probabilities add to 1 (CH14-C04). The direct count of 1/4 matching 1 − 3/4 confirms that both answers are consistent.

Topic and concept

Complementary events and their probability relation. With two fair coins, the four outcomes HH, HT, TH and TT are equally likely, so P(E) equals favourable outcomes divided by 4. The event 'at least one tail' has the complement 'no tail' (HH), and P(E) + P(Ē) = 1 lets us cross-check the answers 3/4 and 1/4.

How marks are earned

1. At least one tail occurs in HT, TH and TT, so its probability is 3/4. (1 mark)
2. The complementary event is no tail, namely HH, with probability 1/4; the two probabilities sum to 1. (1 mark)

Source check: Check the passage facts and NCERT Class 10 maths, Ch. 14, §14.1 Probability — A Theoretical Approach against both steps.