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Class X practice studio

CLASS X · 2026-27 · MODEL PAPER

Class X Mathematics Standard (Code 041) Model Paper 4

Worked answers & explanations

Maximum marks: 80

Time: 3 hours

Subject code: 041

Practice, understand, improve.

Instructions

All 38 questions are compulsory, with one selected option for each internal choice.

Section A has 18 MCQs and 2 assertion-reason questions; Section E has 1+1+2-mark case subparts.

Calculators are not allowed; draw labelled figures where needed.

Section A

Question 1

1 mark

Mcq

Choose the correct answer.

(a) Given class intervals 10–20 (f=5), 20–30 (f=8), 30–40 (f=6), the modal class is: [1 mark]

A. 10–20
B. 20–30
C. 30–40
D. 15–25

Answer

B. 20–30

Step by step

1. **Select and justify the option:** The modal class is the one with the highest frequency, which is 20–30 (f=8). Why: Modal class is identified by having the maximum frequency among all classes.

Topic and concept

Mode of grouped data — modal class; mode formula. The mode in grouped data is estimated within the modal class by applying Mode = $l+((f_1-f_0)/(2f_1-f_0-f_2))\times h$.

How marks are earned

1. The modal class is the one with the highest frequency, which is 20–30 (f=8). (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 13, §13.3: Modal class is identified by having the maximum frequency among all classes.

Question 2

1 mark

Mcq

Choose the correct answer.

(a) A toy is made by joining a right circular cone to a hemisphere of the same base radius r. If l is the slant height of the cone, its total painted surface area is [1 mark]

A. πrl + 2πr²

B. πrl + πr²

C. 3πr²

D. πrl + 3πr²

Answer

A. πrl + 2πr²

Step by step

1. **Add only exposed curved surfaces:** The common circular base is hidden, so painted area = cone CSA + hemisphere CSA = πrl + 2πr² = πr(l+2r), which is option A. Why: The flat faces are joined internally and receive no paint; adding either base circle would overcount the external area.

Topic and concept

Surface area and volume of combinations — cone + hemisphere, cylinder + hemisphere, etc.. For a cone joined to a hemisphere of the same radius, the common circular face is hidden; total exposed area is πrl+2πr².

How marks are earned

1. Exclude the hidden common base and add cone CSA πrl and hemisphere CSA 2πr² to get πrl+2πr². (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 12, §12.2: When a cone and a hemisphere of equal base radius are joined at their flat faces, the total surface area is the sum of their curved surface areas.

Question 3

1 mark

Mcq

Choose the correct answer.

(a) A 90° sector of radius 5 cm has its triangular region (formed by the radii) removed. What is the remaining area? [1 mark]

A. 25π/2 − 25/2 cm²

B. 25π/4 − 25/2 cm²

C. 25π/4 + 25/2 cm²

D. 25π/4 − 5 cm²

Answer

B. The remaining area is 25π/4 − 25/2 cm².

Step by step

1. **Compute the sector area, compute the triangle area, and subtract the triangle from the sector:** Sector area = (90/360)·π·5² = (1/4)·25π = 25π/4 cm². The two radii are perpendicular, so the triangle is right-angled with legs 5 cm and 5 cm, and its area is ½ × 5 × 5 = 25/2 cm². Remaining area = 25π/4 − 25/2 cm², which is option B. Why: The region left after cutting the triangle off a sector is the sector area minus the triangle area, which is the segment-area idea in NCERT §11.2 and §11.3. Option B is the only choice that has 25π/4 as the sector part and 25/2 as the subtracted triangle part.

Topic and concept

Combination of plane figures (triangle + sector, etc.). A segment is the part of a sector left after the triangle formed by the two radii and the chord is removed. Its area is (sector area) − (triangle area). For a 90° sector of radius 5 cm, the sector is a quarter circle with area 25π/4 cm², and the triangle is a right triangle with area 25/2 cm².

How marks are earned

1. Calculate sector area 25π/4 cm², subtract triangle area 25/2 cm², and choose unique option B. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 11, §11.2 Summary: Area of a segment is sector area minus triangle area.

Question 4

1 mark

Mcq

Choose the correct answer.

(a) In the fair example, if Akhila played Hoopla y times and took x rides, which relation represents “the number of Hoopla games is half the number of rides”? [1 mark]

A. y = 2x

B. y = x/2

C. x = y/2

D. x + y = 2

Answer

B. The number of Hoopla games y is half the number of rides x, so y=x/2.

Step by step

1. **Translate the statement 'Hoopla games are half the number of rides' into an algebraic relation, with y as games and x as rides.:** Half of the rides x is x/2, and this quantity equals the number of Hoopla games y, so y = x/2. Numeric check: for x = 6 rides, y = 6/2 = 3 games, and 3 is indeed half of 6. This matches option B. Why: The quantity described as 'half of' something is set equal to that something divided by 2. Here the games y are the half and the rides x are the whole. Option A, y = 2x, would make games double the rides (6 rides would give 12 games). Option C, x = y/2, makes rides half the games, which reverses the relationship. Option D, x + y = 2, is a sum and gives no proportional relation.

Topic and concept

Formulation and solution of word problems using pair of linear equations. Translating a verbal comparison into a linear equation means identifying which quantity is the 'part' and which is the 'whole'. 'y is half of x' gives y = x/2, which is the same as x = 2y. The NCERT fair example in Ch. 3 §3.1 uses y = x/2 for Hoopla games and rides. Its pair of linear equations is formed from such relations.

How marks are earned

1. Select unique option B and translate “y is half of x” as y=x/2. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 3, §3.1: The number of Hoopla games y is half the number of rides x, giving y = x/2.

Question 5

1 mark

Mcq

Choose the correct answer.

(a) If M is the midpoint of segment AB, which of the following is always true? [1 mark]

A. AM = MB

B. A, M, B are not collinear

C. AM = AB + MB

D. AB = AM − MB

Answer

A. AM = MB

Step by step

1. **Select and justify the option:** By definition of midpoint, AM = MB. Why: The midpoint divides a segment into two equal parts.

Topic and concept

Midpoint as a special case of internal division. The midpoint of segment PQ is obtained by setting m=n=1 in the section formula, giving $((x_1+x_2)/2,(y_1+y_2)/2)$.

How marks are earned

1. By definition of midpoint, AM = MB. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 7, §7.4 Summary: The midpoint divides a segment into two equal parts.

Question 6

1 mark

Mcq

Choose the correct answer.

(a) A cone and a hemisphere of equal radius are joined at their flat circular faces. Which listed surface is hidden at the joint and therefore excluded from the exposed total surface area? [1 mark]

A. Curved surface of the cone

B. Flat circular base of the cone

C. Curved surface of the hemisphere

D. Both curved surfaces

Answer

B. Flat circular base of the cone.

Step by step

1. **Identify the hidden face at the joint:** The cone's flat circular base meets the hemisphere's flat circular face inside the solid. The cone base is therefore hidden and corresponds to option B; both curved surfaces remain exposed. Why: Total surface area of a joined toy counts only its outer surfaces. The common flat faces are internal and are omitted, while the curved surfaces remain visible.

Topic and concept

Surface area and volume of combinations — cone + hemisphere, cylinder + hemisphere, etc.. A cone and hemisphere joined at equal-radius flat faces expose their two curved surfaces. The touching flat faces are internal, so they do not contribute to the outside surface area.

How marks are earned

1. Choose B: the cone's flat circular base is joined to the hemisphere's flat face and is hidden inside the toy. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 12, §12.2: Hidden faces where two solids meet are not counted in the total surface area.

Question 7

1 mark

Mcq

Choose the correct answer.

(a) In the median formula Median = l + ((n/2 − cf)/f) × h, what does 'cf' represent? [1 mark]

A. frequency of median class

B. cumulative frequency of class preceding median class

C. class width

D. lower class boundary

Answer

B. cumulative frequency of class preceding median class

Step by step

1. **Select and justify the option:** 'cf' denotes the cumulative frequency of all classes before the median class, as per formula definition. Why: The term 'cf' is defined as cumulative frequency before the median class.

Topic and concept

Median of grouped data — algebraic method. For grouped data, the median is located by finding the class whose cumulative frequency just exceeds n/2, then interpolating within that class as Median = l+ ((n/2−cf)/f)×h.

How marks are earned

1. 'cf' denotes the cumulative frequency of all classes before the median class, as per formula definition. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 13, §13.4: The term 'cf' is defined as cumulative frequency before the median class.

Question 8

1 mark

Mcq

Choose the correct answer.

(a) In Fig. 10.6(iii), from external point P two tangents PT_1 and PT_2 meet the circle. Which relation holds? [1 mark]

A. $PT_1 = PT_2$

B. $PT_1 > PT_2$

C. $PT_1 < PT_2$

D. $PT_1 + PT_2$ equals the diameter

Answer

A. $PT_1 = PT_2$

Step by step

1. **Select option A by applying the equal-tangent-lengths theorem to external point P and its two tangents PT_1 and PT_2.:** Join the centre O to P, T_1 and T_2. Then $OT_1 = OT_2$ (radii), OP is common, and $\angle OT_1P = \angle OT_2P = 90°$ because a tangent is perpendicular to the radius at the point of contact. So right triangles OT_1P and OT_2P are congruent by RHS, which gives $PT_1 = PT_2$, so option A is correct; B and C would need unequal lengths, and D compares the sum of the tangents with the diameter, which the theorem never states. Why: Congruent right triangles, sharing the hypotenuse OP and having equal radii, force the two tangent segments from one external point to be equal. This is the theorem that the lengths of the two tangents drawn from an external point to a circle are equal.

Topic and concept

Number of tangents from an external point; lengths of two tangents from an external point are equal (theorem + proof). From an external point P, exactly two tangents can be drawn to a circle, touching it at T_1 and T_2. The tangent-length theorem states that $PT_1 = PT_2$. It follows from the fact that a tangent is perpendicular to the radius at the point of contact. This makes OT_1P and OT_2P right triangles with a common hypotenuse OP and equal radii.

How marks are earned

1. The lengths of the two tangents drawn from an external point to a circle are equal. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 10, §10.3: The lengths PT_1 and PT_2 from external point P to the circle are equal.

Question 9

1 mark

Mcq

Choose the correct answer.

(a) Find the area of a 90° sector in a circle of radius 7 cm. [1 mark]

A. 49π/2 cm²

B. 49π/4 cm²

C. 7π/2 cm²

D. 14π cm²

Answer

B. 49π/4 cm²

Step by step

1. **Select and justify the option:** Area = (90/360)×π×7² = (1/4)×49π = 49π/4 cm². Why: Area of a sector = (θ/360)×πr².

Topic and concept

Area of a sector and length of an arc. The area of a sector with central angle θ in a circle of radius r is (θ/360°)×πr².

How marks are earned

1. Area = (90/360)×π×7² = (1/4)×49π = 49π/4 cm². (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 11, §11.2 Summary: Area of a sector = (θ/360)×πr².

Question 10

1 mark

Mcq

Choose the correct answer.

(a) When solving a problem with two observation points A and B looking at the top of the same tower, what is the first construction step? [1 mark]

A. Draw two right-angled triangles sharing the same vertical height from the tower to the two observation points

B. Draw a single triangle using only the combined horizontal distance to one point

C. Measure the angle of depression instead of elevation

D. Use the sine ratio directly without constructing triangles

Answer

A. Draw two right-angled triangles sharing the same vertical height from the tower to the two observation points

Step by step

1. **Choose option A by deciding what must be drawn first: two right triangles on a common vertical tower side.:** Let the tower be the vertical segment from its foot T to its top P. Take A and B on level ground in line with T. Join T to A and T to B along the ground. Join P to A and P to B. This gives right triangles PTA and PTB, each with the right angle at T. They share the vertical side PT = h. If the angles of elevation at A and B are α and β, then tan α = h/TA and tan β = h/TB. With TA = x and TB = x + d, these become tan α = h/x and tan β = h/(x+d). Why: Trigonometric ratios such as tan apply only to the sides of a right-angled triangle. So the figure with right triangles must be drawn before any ratio is written. Because the tower height h is the same side in both triangles, it links the two equations, and the unknowns can then be solved together. Option B uses a single triangle with a combined distance and loses this link. Option C changes the angle being measured. Option D uses a ratio with no triangle, so there is nothing for the ratio to refer to.

Topic and concept

Problems involving two right triangles and two angles of elevation/depression. In heights and distances problems, the real situation is turned into a figure of right triangles so that trigonometric ratios can be applied (Ch. 9 Summary: draw right triangles and apply trig ratios). With two observation points A and B on the ground in line with the tower, the lines of sight to the top form two right triangles sharing the vertical side, which is the tower's height. Their bases are the ground distances from the foot to A and B. Each triangle gives one ratio equation in the same height h.

How marks are earned

1. First, draw two right triangles that share the vertical side representing the tower's height and have horizontal bases to points A and B. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 9, Summary: To determine height, draw right triangles and apply trigonometric ratios.

Question 11

1 mark

Mcq

Choose the correct answer.

(a) In the system 3x+2y=6 and 5x+2y=10, which operation will eliminate y? [1 mark]

A. Add the two equations

B. Subtract the first from the second

C. Multiply the first by 2

D. Multiply the second by 3

Answer

B. Subtract the first from the second

Step by step

1. **Select and justify the option:** Subtracting (3x+2y=6) from (5x+2y=10) gives 2x=4, eliminating y. Why: Elimination removes one variable by matching coefficients and subtracting or adding equations.

Topic and concept

Algebraic methods: substitution and elimination. Elimination method removes one variable by matching coefficients through addition or subtraction of equations.

How marks are earned

1. Subtracting (3x+2y=6) from (5x+2y=10) gives 2x=4, eliminating y. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 3, §3.3.2: Elimination removes one variable by matching coefficients and subtracting or adding equations.

Question 12

1 mark

Mcq

Choose the correct answer.

(a) When an observer at the top of a tower looks down at a point on the ground, the angle of depression is 30°. The angle of elevation of the tower from that point on the ground is: [1 mark]

A. 30°

B. 60°

C. 45°

D. 90°

Answer

A. 30°

Step by step

1. **Choose option A by using the equality of alternate angles between the horizontal at the observer and the line of sight to the ground point.:** Let the observer be at the top P of the tower and the ground point be A. At P, draw a horizontal line. The line of sight PA makes the angle of depression 30° with this horizontal. The ground through A is also horizontal, so the two horizontals are parallel and PA is a transversal. The angle of elevation of P from A is measured between the ground line at A and the line AP. It is the alternate interior angle to the angle of depression, so it equals 30°. Hence the answer is 30°, option A. Why: When a transversal cuts two parallel lines, alternate interior angles are equal. Here the two parallel lines are the horizontal at the observer's eye level and the horizontal ground at the point. The line of sight is the transversal, so depression and elevation are equal. Option B (60°) would be the complement, 90° − 30°, which is the angle between the line of sight and the tower, not the elevation. Option C (45°) has no basis in the data. Option D (90°) would mean the observer is directly above the point.

Topic and concept

Problems involving two right triangles and two angles of elevation/depression. The angle of depression is measured downward from the horizontal through the observer's eye. The angle of elevation is measured upward from the horizontal at the object being observed. Since the two horizontals are parallel and the line of sight is a transversal, the alternate interior angles are equal. This is the Ch. 9 §9.1 result that the angle of depression equals the corresponding angle of elevation.

How marks are earned

1. The angle of elevation from the ground point equals the angle of depression due to alternate interior angles formed by parallel horizontal lines. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 9, §9.1 Heights and Distances: Angle of depression and corresponding angle of elevation are equal when measured from parallel horizontals.

Question 13

1 mark

Mcq

Choose the correct answer.

(a) What is the volume of a solid obtained by joining a right circular cylinder of radius r and height h to a hemisphere of radius r? [1 mark]

A. $\pi r^2h + (2/3)\pi r^3$

B. $\pi r^2h - (2/3)\pi r^3$

C. $2\pi r^3 + \pi r^2h$

D. $(4/3)\pi r^3 + \pi r^2h$

Answer

A. $\pi r^2h + (2/3)\pi r^3$

Step by step

1. **Select and justify the option:** Volume = volume of cylinder (πr^2h) + volume of hemisphere ($(2/3)\pi r^3$) = $\pi r^2h + (2/3)\pi r^3$. Why: The volume of a solid formed by joining basic solids is the sum of their individual volumes.

Topic and concept

Surface area and volume of combinations — cone + hemisphere, cylinder + hemisphere, etc.. The volume of a solid formed by joining two basic solids equals the sum of their individual volumes.

How marks are earned

1. Volume = volume of cylinder (πr^2h) + volume of hemisphere ($(2/3)\pi r^3$) = $\pi r^2h + (2/3)\pi r^3$. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 12, §12.3: The volume of a solid formed by joining basic solids is the sum of their individual volumes.

Question 14

1 mark

Mcq

Choose the correct answer.

(a) A bag contains 4 red balls and 1 blue ball. One ball is drawn at random. What is the probability of drawing a red ball? [1 mark]

A. 1/5

B. 4/5

C. 1/4

D. 2/3

Answer

B. 4/5

Step by step

1. **Select and justify the option:** There are 4 favourable red balls out of 5 total, so probability = 4/5. Why: Use P(E)=favourable outcomes/total equally likely outcomes.

Topic and concept

Probability of sure and impossible events; 0 ≤ P(E) ≤ 1. A certain event has probability 1 and an impossible event has probability 0.

How marks are earned

1. There are 4 favourable red balls out of 5 total, so probability = 4/5. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 14, §Summary: Use P(E)=favourable outcomes/total equally likely outcomes.

Question 15

1 mark

Mcq

Choose the correct answer.

(a) The reciprocal of tan θ is: [1 mark]

A. cot θ

B. sec θ

C. cosec θ

D. cos θ

Answer

A. cot θ

Step by step

1. **Select and justify the option:** cot θ is defined as the reciprocal of tan θ. Why: cot A is defined as 1/tan A.

Topic and concept

Relationships between trigonometric ratios. Cosec θ, sec θ and cot θ are the reciprocals of sin θ, cos θ and tan θ respectively.

How marks are earned

1. cot θ is defined as the reciprocal of tan θ. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 8, §8.2: cot A is defined as 1/tan A.

Question 16

1 mark

Mcq

Choose the correct answer.

(a) If an arc length equals half the circumference of its circle, what is the measure of the central angle? [1 mark]

A. 90°

B. 180°

C. 120°

D. 60°

Answer

B. 180°

Step by step

1. **Select and justify the option:** Arc length = (θ/360)×circumference = (θ/360)×2πr; setting this = ½×2πr gives θ = 180°. Why: Arc length = (θ/360)×2πr.

Topic and concept

Area of a sector and length of an arc. The area of a sector with central angle θ in a circle of radius r is (θ/360°)×πr².

How marks are earned

1. Arc length = (θ/360)×circumference = (θ/360)×2πr; setting this = ½×2πr gives θ = 180°. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 11, §11.2 Summary: Arc length = (θ/360)×2πr.

Question 17

1 mark

Mcq

Choose the correct answer.

(a) Shanta's shed is a cuboid of length 15 m, breadth 7 m, height 8 m, with a half cylinder of diameter 7 m and length 15 m on top. Which expression gives the shed's total internal volume? [1 mark]

A. 15×7×8 + ½×π×(3.5)²×15

B. 15+7+8 + ½×π×7²×15

C. 15×7×8 + π×(3.5)²×15

D. 15×7×8 + ½×π×7×15

Answer

A. 15×7×8 + ½×π×(3.5)²×15

Step by step

1. **Select and justify the option:** Volume = volume of cuboid (15×7×8) + half the volume of cylinder (½×π×3.5²×15). Why: A shed shaped as a cuboid surmounted by a half cylinder has volume equal to the sum of the cuboid's volume and half the cylinder's volume.

Topic and concept

Surface area and volume of combinations — cone + hemisphere, cylinder + hemisphere, etc.. A shed shaped as a cuboid with a half cylinder on top has volume = volume_cuboid + ½×volume_cylinder.

How marks are earned

1. Volume = volume of cuboid (15×7×8) + half the volume of cylinder (½×π×3.5²×15). (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 12, §12.3: A shed shaped as a cuboid surmounted by a half cylinder has volume equal to the sum of the cuboid's volume and half the cylinder's volume.

Question 18

1 mark

Mcq

Choose the correct answer.

(a) A fair six-sided die is rolled once. What is the probability of obtaining a prime number? [1 mark]

A. 1/2

B. 1/3

C. 1/6

D. 1/4

Answer

A. 1/2

Step by step

1. **Select and justify the option:** Favourable outcomes are {2,3,5} (3 faces) out of 6 total, so probability = 3/6 = 1/2. Why: P(E) = Number of outcomes favourable to E / Total number of equally likely outcomes.

Topic and concept

Probability of sure and impossible events; 0 ≤ P(E) ≤ 1. A certain event has probability 1 and an impossible event has probability 0.

How marks are earned

1. Favourable outcomes are {2,3,5} (3 faces) out of 6 total, so probability = 3/6 = 1/2. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 14, §Summary: P(E) = Number of outcomes favourable to E / Total number of equally likely outcomes.

Question 19

1 mark

Assertion Reason

Choose the correct answer.

(a) Assertion (A): If $a_1/a_2 = b_1/b_2 = c_1/c_2$ then the pair of equations has infinitely many solutions. Reason (R): Coincident lines share every point in common. [1 mark]

A. Both A and R true and R explains A

B. Both A and R true but R does not explain A

C. A true, R false

D. A false, R true

Answer

A. Both A and R true and R explains A

Step by step

1. **Test A, test R, then check whether R explains A:** If $a_1/a_2 = b_1/b_2 = c_1/c_2$, both equations are multiples of one another and represent the same line, so the lines coincide and A is true. Coincident lines share every point, so R is true. Every point on that common line satisfies both equations, which gives infinitely many solutions. Hence both A and R are true and R explains A, which is option A. Why: The ratio condition for a pair of linear equations (NCERT §3.2) is a test for coincident lines. Since the solutions of a pair are the common points of its two lines, sharing every point is the reason there are infinitely many solutions. This makes R the direct explanation of A, not just an extra true statement.

Topic and concept

Graphical representation of pair of linear equations; consistent, inconsistent, dependent conditions. For $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, the ratios of coefficients decide the number of solutions. Equal ratios $a_1/a_2 = b_1/b_2 = c_1/c_2$ mean the lines coincide and the pair is consistent with infinitely many solutions. Here the reason, that coincident lines share every point, is exactly why the assertion holds.

How marks are earned

1. Both A and R are true. When all ratios are equal, the lines coincide, giving infinitely many common solutions. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 3, §3.2: Equal ratios $a_1/a_2 = b_1/b_2 = c_1/c_2$ indicate coincident lines with infinite solutions.

Question 20

1 mark

Assertion Reason

Choose the correct answer.

(a) Assertion (A): Exactly two tangents can be drawn to a circle from a point outside it. Reason (R): The two tangent segments from an external point have equal lengths. [1 mark]

A. Both A and R true and R explains A

B. Both A and R true but R does not explain A

C. A true, R false

D. A false, R true

Answer

B. Both A and R true but R does not explain A.

Step by step

1. **Check A and R separately, then test whether R explains A:** A is true because a point outside a circle has exactly two tangents to it. R is true because the two tangent segments from an external point are equal in length. However, equal lengths describe two tangents that have already been drawn. They do not show why there are exactly two, so R does not explain A, which is option B. Why: The number of tangents depends on where the point lies relative to the circle: zero inside, one on the circle, two outside (NCERT §10.3). Equal tangent lengths are a separate property (NCERT §10.4). A statement about a property of the two tangents cannot be the cause of how many tangents exist.

Topic and concept

Number of tangents from an external point; lengths of two tangents from an external point are equal (theorem + proof). A point outside a circle has two tangents, which is a statement about the count. The two tangent segments from that point are equal in length, which is a statement about their length. Both are true, but the second does not account for the first.

How marks are earned

1. Choose B: an external point has two tangents and their lengths are equal, but equality of already drawn segments does not prove the number of tangents. (1 mark)

Source check: NCERT Class 10 maths, Ch. 10, §10.3 gives the number of tangents by point location; §10.4 gives equality of the two external tangent lengths.

Section B

Question 21

2 marks

Short Answer

Attempt one option.

Option A

Answer the question with working and reasons where needed.

(a) A tree casts a shadow of 15 m when the angle of elevation of the sun is 30°. Find the height of the tree. [2 marks]

Answer

Using the trigonometric relation tan 30° = opposite/adjacent = height of the tree/shadow length, we substitute tan 30° = 1/√3 and the given shadow length of 15 m to set up (1/√3)=h/15, which upon solving gives h=15×(1/√3)=5√3 metres above the ground. Therefore, by calculating and simplifying the expression obtained from the tangent ratio, we conclude that the height of the tree is exactly 5√3 metres.

Step by step

1. **Model the situation as a right triangle and write the tangent ratio.:** Let the tree be the vertical side AB = h m (opposite to the 30° angle at the shadow's tip). The shadow BC = 15 m is the horizontal side adjacent to it. The sun's rays make the 30° angle of elevation at C. Then tan 30° = opposite/adjacent = h/15, with tan 30° = 1/√3. Why: The tree stands vertically on level ground, so with its shadow it forms a right triangle. The angle of elevation is measured at the shadow's tip C. tan θ links the opposite side (height) to the adjacent side (shadow), which are the unknown and the given length.
2. **Solve for h and rationalise the result.:** 1/√3 = h/15, so h = 15/√3 = (15 × √3)/(√3 × √3) = 15√3/3 = 5√3 m. Check: 5√3 ≈ 5 × 1.732 = 8.66 m, and 8.66/15 ≈ 0.577 = 1/√3, which agrees. Why: Multiplying the numerator and denominator by √3 removes the surd from the denominator without changing the value. The height is a length, so the answer must carry the unit of metres. The numerical check confirms that the ratio is correct.

Topic and concept

Angle of elevation and angle of depression; applications to heights and distances. In a right triangle formed by a vertical object, its horizontal shadow or ground distance, and the line of sight, tan θ = height/horizontal distance. Knowing the angle of elevation and one side lets us find the other side. Here the angle is 30° and the shadow is 15 m, so the height is 5√3 m.

How marks are earned

1. Using the trigonometric relation tan 30° = opposite/adjacent = height of the tree/shadow length, we substitute tan 30° = 1/√3 and the given shadow length of 15 m to set up (1/√3)=h/15, which upon solving gives h=15×(1/√3)=5√3 metres above the ground. (1 mark)
2. Therefore, by calculating and simplifying the expression obtained from the tangent ratio, we conclude that the height of the tree is exactly 5√3 metres. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 9, §9.2 Summary point 2: Heights and distances can be determined using trigonometric ratios.

OR

Option B

Answer the question with working and reasons where needed.

(a) From a point on level ground, the angle of elevation to the top of a pole is 45°, and the pole's height is 20 m. Calculate the horizontal distance of the point from the base of the pole. [2 marks]

Answer

Using tan 45° = height / distance gives 1 = 20 / d, so d = 20 m. Hence, the horizontal distance from the point to the pole's base is 20 metres.

Step by step

1. **Set up the right triangle and apply tan 45° = height / distance:** The pole is vertical and the ground is level, so the pole, the ground and the line of sight form a right triangle. The angle at the observer's point is 45°. The pole (20 m) is opposite this angle and the horizontal distance d is adjacent to it. So tan 45° = 20/d, and since tan 45° = 1, we get 1 = 20/d, so d = 20 m. Why: Tangent relates the side opposite the angle of elevation to the adjacent side. The height is the opposite side and the horizontal distance is the adjacent side, so tan θ = height / distance is the right ratio (NCERT §9.2). Using sine or cosine would need the line of sight, which is not given.
2. **State the horizontal distance with its unit and verify it:** The horizontal distance of the point from the base of the pole is 20 m. Check: in a right triangle with a 45° angle, the other acute angle is also 45°, so the two legs are equal. The opposite side is 20 m, so the adjacent side is also 20 m. Why: A right triangle with a 45° angle is isosceles, so the legs are equal. This agrees with tan 45° = 1 and confirms the answer independently. Writing the unit in metres shows that d is a length.

Topic and concept

Angle of elevation and angle of depression; applications to heights and distances. When the angle of elevation of the top of a vertical object is known, a right triangle is formed. tan θ = height / horizontal distance links the known height and angle to the unknown distance. For θ = 45°, tan 45° = 1, so height and distance are equal.

How marks are earned

1. Using tan 45° = height / distance gives 1 = 20 / d, so d = 20 m. (1 mark)

2. Hence, the horizontal distance from the point to the pole's base is 20 metres. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 9, §9.2 Summary point 2: Heights and distances can be determined using trigonometric ratios.

Question 22

2 marks

Short Answer

Attempt one option.

Option A

Answer the question with working and reasons where needed.

(a) Why does the mode formula for grouped data involve the frequencies f_0, f_1, and f_2 of the modal class and its two adjacent classes? [2 marks]

Answer

f_1 is frequency of the modal class, while f_0 and f_2 are frequencies of the classes just before and after it; their differences (f_1-f_0) and (f_1-f_2) measure the skew of data around the modal class. The ratio $(f_1-f_0)/(2f_1-f_0-f_2)$ scales this skew within the class width h, so l+[…]×h locates the most frequent value inside the modal class.

Step by step

1. **Identify what f_1, f_0 and f_2 stand for and what the differences mean.:** f_1 is the frequency of the modal class (the highest frequency), f_0 is the frequency of the class just before it and f_2 of the class just after it. The differences $f_1 - f_0$ and $f_1 - f_2$ show how much the modal class exceeds each neighbour. For example, with $f_0 = 8$, $f_1 = 12$, $f_2 = 4$: $f_1 - f_0 = 4$ and $f_1 - f_2 = 8$, so the larger gap is on the right. Why: The mode is the value with the greatest concentration of observations. The neighbouring frequencies show on which side of the modal class the observations are more crowded. The mode is therefore not simply the midpoint of the class.
2. **Show how the ratio places the mode inside the modal class using h.:** Mode = l + $[(f_1-f_0)/(2f_1-f_0-f_2)]$ × h. For the example, with l = 20 and h = 10: (12−8)/(24−8−4) = 4/12 = 1/3, so mode = 20 + (1/3)×10 ≈ 23.3, which lies in the lower part of the class. In general, if $f_0 > f_2$ (a bigger lower neighbour), the mode is pulled towards the lower half of the class; if $f_2 > f_0$, it is pulled to the upper half. Why: The ratio is a fraction between 0 and 1 for a modal class. Multiplying by the class width h turns it into a distance above the lower limit l. The mode is thus shifted towards the side of the larger neighbouring frequency but stays within the class. This follows from the grouped-data mode formula.

Topic and concept

Mode of grouped data — modal class; mode formula. For grouped data we cannot read the mode directly, so we find the modal class and use the frequencies of the classes on either side. The formula Mode = l + $[(f_1-f_0)/(2f_1-f_0-f_2)]$ × h uses these to estimate where the highest concentration lies within the modal class. A larger neighbouring frequency shifts the mode towards that neighbour's side.

How marks are earned

1. f_1 is frequency of the modal class, while f_0 and f_2 are frequencies of the classes just before and after it; their differences (f_1-f_0) and (f_1-f_2) measure the skew of data around the modal class. (1 mark)
2. The ratio $(f_1-f_0)/(2f_1-f_0-f_2)$ scales this skew within the class width h, so $l+[\ldots]\times h$ locates the most frequent value inside the modal class. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 13, §13.3 Mode of Grouped Data: Mode for grouped data is given by $l+((f_1-f_0)/(2f_1-f_0-f_2))\times h$ using modal and adjacent frequencies.

OR

Option B

Answer the question with working and reasons where needed.

(a) Define the class mark of an interval and give its formula. [2 marks]

Answer

The class mark of an interval is the value at its midpoint, representing all observations in that class. For a class with lower limit L and upper limit U, the class mark is calculated as (L + U)/2.

Step by step

1. **Define the class mark as the midpoint of the class:** The class mark of an interval is the value at its midpoint. It is a single number that stands for all the observations in that class. For example, for the class 10–20 the class mark is 15, which lies exactly halfway between 10 and 20. Why: In grouped data the individual values are not known, so each class is represented by one value. The mean of grouped data assumes that the frequency of a class is centred at its midpoint (NCERT §13.2). That is why the midpoint, called the class mark, is used as x_i.
2. **Write the formula and test it on the example:** For a class with lower limit L and upper limit U, class mark = (L + U)/2. For 10–20 this gives (10 + 20)/2 = 30/2 = 15, which matches the midpoint found above. Why: The midpoint of two numbers is their average, which is the sum divided by 2. This gives the formula, and the example confirms it. A check is that the class mark always lies between L and U.

Topic and concept

Mean of grouped data — direct method, assumed mean method, step-deviation method. Grouped data gives only class intervals and frequencies. Each class is represented by its class mark, the midpoint (L + U)/2. These class marks are the x_i values used in the grouped-data mean, which is $\Sigma f_i x_i$ divided by Σf_i.

How marks are earned

1. The class mark of an interval is the value at its midpoint, representing all observations in that class. (1 mark)

2. For a class with lower limit L and upper limit U, the class mark is calculated as (L + U)/2. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths Ch. 13 §13.2: The mean of grouped data assumes the frequency of a class is centred at its mid-point or class mark.

Question 23

2 marks

Short Answer

Answer the question with working and reasons where needed.

(a) Define the angle of elevation and the angle of depression, highlighting the difference between them. [2 marks]

Answer

The angle of elevation is the angle formed by the observer's line of sight with the horizontal when looking at a point above the horizontal level. The angle of depression is the angle formed by the observer's line of sight with the horizontal when looking at a point below the horizontal level.

Step by step

1. **Define the angle of elevation relative to the horizontal through the observer's eye.:** Draw a horizontal line through the observer's eye. If the object is above this line, the line of sight to it rises above the horizontal. The angle between the horizontal and the line of sight, measured upward, is the angle of elevation. Why: Both angles are measured from the horizontal through the observer's eye and not from the vertical or from the ground. For an object above eye level, the sight line is raised, so the angle is called elevation. This is the convention used in NCERT's heights and distances.
2. **Define the angle of depression and state how it differs.:** If the object is below the horizontal through the observer's eye, the line of sight goes down. The angle between the horizontal and this line, measured downward, is the angle of depression. The difference: elevation is measured upward to a point above eye level, depression is measured downward to a point below eye level. Why: The only distinction is the direction in which the line of sight is turned from the same horizontal. Looking up gives elevation and looking down gives depression. For the same two points, the two angles are equal because of alternate angles on parallel horizontal lines.

Topic and concept

Angle of elevation and angle of depression; applications to heights and distances. The angle of elevation and the angle of depression are both angles between the observer's line of sight and the horizontal through the eye. Elevation applies when the observed point is above that horizontal, and depression when it is below. Once drawn, each angle sits in a right triangle where tan θ = height/horizontal distance can be applied.

How marks are earned

1. The angle of elevation is the angle formed by the observer's line of sight with the horizontal when looking at a point above the horizontal level. (1 mark)
2. The angle of depression is the angle formed by the observer's line of sight with the horizontal when looking at a point below the horizontal level. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 9, §9.1: Angle of elevation is when looking up; angle of depression is when looking down.

Question 24

2 marks

Short Answer

Answer the question with working and reasons where needed.

(a) Find the 20th term of the AP 7, 12, 17, …. [2 marks]

Answer

We recall that the general term of an arithmetic progression is given by $a_n = a + (n-1)d$, where a denotes the first term and d the common difference. Substituting a = 7, d = 5, and n = 20 into the formula yields $a_{20} = 7 + 19\times5 = 7 + 95 = 102$, so the twentieth term of the progression is 102.

Step by step

1. **Identify a and d and write the nth term formula:** The first term is a = 7. The common difference is d = 12 − 7 = 5, and 17 − 12 = 5 confirms it. The nth term of an AP is $a_n = a + (n - 1)d$, and here n = 20. Why: An AP has a constant difference between consecutive terms, so d is found by subtracting any term from the next. The nth term formula a + (n − 1)d (NCERT §5.3) adds d exactly (n − 1) times to a, because the first term needs no addition.
2. **Substitute and calculate a_{20}:** $a_{20} = 7 + (20 - 1) \times 5 = 7 + 19 \times 5 = 7 + 95 = 102$. Check: the terms rise by 5 each step, and 19 steps from the first term add 95, so the 20th term is 102. Why: There are 19 steps of size d from term 1 to term 20, which is why 19 and not 20 multiplies d. Using the formula avoids listing 20 terms. The check by counting steps confirms the arithmetic.

Topic and concept

nth term formula: $a_n = a + (n-1)d$. To find a specific term of an AP, find a and d, then substitute them with n into $a_n = a + (n - 1)d$. Here (n − 1) is the number of common differences added to the first term to reach the nth term.

How marks are earned

1. We recall that the general term of an arithmetic progression is given by $a_n = a + (n-1)d$, where a denotes the first term and d the common difference. (1 mark)
2. Substituting a = 7, d = 5, and n = 20 into the formula yields $a_{20} = 7 + 19\times5 = 7 + 95 = 102$, so the twentieth term of the progression is 102. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 5, §5.3 nth Term of an AP: The general term of an AP is a + (n − 1)d.

Question 25

2 marks

Short Answer

Answer the question with working and reasons where needed.

(a) Use prime factorisation to find the least common multiple (LCM) of 84 and 126. [2 marks]

Answer

By applying successive divisions, we obtain the prime factorization of 84 as 84 = 2^2 × 3^1 × 7^1, clearly listing each prime factor with its exponent. By comparing the highest exponent of each prime in 84 = 2^2 × 3^1 × 7^1 and 126 = 2^1 × 3^2 × 7^1, we compute the LCM as 2^2 × 3^2 × 7^1 = 252.

Step by step

1. **Find the prime factorisations of both 84 and 126 by successive division.:** 84 ÷ 2 = 42, 42 ÷ 2 = 21, 21 ÷ 3 = 7, 7 ÷ 7 = 1, so 84 = 2² × 3 × 7. For 126: 126 ÷ 2 = 63, 63 ÷ 3 = 21, 21 ÷ 3 = 7, 7 ÷ 7 = 1, so 126 = 2 × 3² × 7. Why: Every composite number has a unique prime factorisation, and the LCM is built from these prime powers. Dividing by the smallest primes in turn guarantees that no factor is missed. Both numbers must be factorised so their powers can be compared.
2. **Take the highest power of each prime and multiply to get the LCM.:** Primes present: 2 (powers 2² and 2¹, highest 2²), 3 (powers 3¹ and 3², highest 3²), 7 (7¹ in both, so 7¹). LCM = 2² × 3² × 7 = 4 × 9 × 7 = 252. Check: 252 ÷ 84 = 3 and 252 ÷ 126 = 2, both whole numbers. Why: The LCM must be divisible by both numbers, so it needs each prime at least as many times as it appears in either number. That means the highest exponent of each prime. Taking the lowest exponents would give the HCF instead.

Topic and concept

Fundamental Theorem of Arithmetic — statement, illustration and applications through prime factorisation. To find the LCM by prime factorisation, factorise each number and for every prime that appears take its highest exponent, then multiply. For 84 = 2² × 3 × 7 and 126 = 2 × 3² × 7, this gives 2² × 3² × 7 = 252. The lowest exponents would give the HCF.

How marks are earned

1. By applying successive divisions, we obtain the prime factorization of 84 as 84 = 2^2 × 3^1 × 7^1, clearly listing each prime factor with its exponent. (1 mark)
2. By comparing the highest exponent of each prime in 84 = 2^2 × 3^1 × 7^1 and 126 = 2^1 × 3^2 × 7^1, we compute the LCM as 2^2 × 3^2 × 7^1 = 252. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 1, §1.2: A composite number can be expressed uniquely as a product of primes, which we use for LCM.

Section C

Question 26

3 marks

Short Answer

Answer the question with working and reasons where needed.

(a) A circle has centre O and radius 7 cm. From a point P such that OP = 25 cm, two tangents PA and PB are drawn. Calculate the length PA. [3 marks]

Answer

In right triangle OPA, $OP^2 = OA^2 + PA^2$ by the Pythagorean relation, so $PA^2 = OP^2 - OA^2$. Substituting OP = 25 cm and OA = 7 cm gives $PA^2 = 25^2 - 7^2 = 625 - 49 = 576$, so $PA = \sqrt{576}$. Therefore, PA = 24 cm, and both tangents from P have this length.

Step by step

1. **Justify the right angle and write the Pythagoras relation in ΔOPA:** The tangent PA touches the circle at A, so it is perpendicular to the radius OA, giving $\angle OAP = 90°$. In right triangle OPA the hypotenuse is OP, so $OP^2 = OA^2 + PA^2$, which rearranges to $PA^2 = OP^2 - OA^2$. Why: A tangent is perpendicular to the radius at its point of contact. This is what makes ΔOPA right-angled, and Pythagoras applies only to right triangles.
2. **Substitute the given lengths and simplify:** OP = 25 cm and OA = 7 cm (radius), so $PA^2 = 25^2 - 7^2 = 625 - 49 = 576$ cm², and $PA = \sqrt{576}$ cm. Why: The radius OA is a leg of the right triangle and OP is the hypotenuse. Subtracting OA^2 from OP^2 isolates the square of the remaining leg, the tangent length.
3. **Take the square root, state the unit and use the equal-tangent property:** $\sqrt{576} = 24$, so PA = 24 cm. Check: $7^2 + 24^2 = 49 + 576 = 625 = 25^2$. Since the two tangents from P are equal, PB = 24 cm too. Why: The lengths of the two tangents drawn from an external point to a circle are equal, so the length found for PA also holds for PB. The check confirms the sides fit Pythagoras.

Topic and concept

Number of tangents from an external point; lengths of two tangents from an external point are equal (theorem + proof). A tangent is perpendicular to the radius at the point of contact. For an external point P this gives the right triangle OAP, so $PA^2 = OP^2 - r^2$. The two tangents from P are equal in length.

How marks are earned

1. In right triangle OPA, $OP^2 = OA^2 + PA^2$ by the Pythagorean relation, so $PA^2 = OP^2 - OA^2$. (1 mark)
2. Substituting OP = 25 cm and OA = 7 cm gives $PA^2 = 25^2 - 7^2 = 625 - 49 = 576$, so $PA = \sqrt{576}$. (1 mark)

3. Therefore, PA = 24 cm, and both tangents from P have this length. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 10, §10.4 Summary: The relation $PT^2 = OP^2 - r^2$ gives the length of a tangent from an external point.

Question 27

3 marks

Short Answer

Answer the question with working and reasons where needed.

(a) In ΔABC and ΔDEF, ∠A=∠D and ∠B=∠E. If AB = 5 cm, BC = 7 cm and DE = 15 cm, find EF. [3 marks]

Answer

Since two pairs of corresponding angles ∠A = ∠D and ∠B = ∠E are equal in triangles ABC and DEF, by the AA criterion we conclude the triangles are similar. From the similarity of the triangles, the corresponding sides are proportional, so we can write AB/DE = BC/EF, which becomes 5/15 = 7/EF when substituting the known lengths. Solving the proportion 5/15 = 7/EF through cross-multiplication and division gives EF = (15 × 7)/5, which simplifies to EF = 21 cm.

Step by step

1. **Establish similarity by AA and fix the vertex correspondence:** Given ∠A = ∠D and ∠B = ∠E, two angles of ΔABC equal two angles of ΔDEF, so ΔABC ~ ΔDEF by AA. The correspondence is A↔D, B↔E and C↔F, so ∠C = ∠F as well. Why: The AA criterion says two equal pairs of angles are enough for similarity. Writing the correspondence fixes which sides match: AB with DE, BC with EF.
2. **Write the proportion of corresponding sides and substitute:** Corresponding sides are proportional: AB/DE = BC/EF. Substituting AB = 5, DE = 15 and BC = 7 gives 5/15 = 7/EF. Why: In similar triangles all pairs of corresponding sides have the same ratio. AB and DE lie between the same angle pairs (A,B and D,E), so they correspond, as do BC and EF.
3. **Solve for EF and check the ratio:** 5/15 = 1/3, so EF = 7 × 3 = 21 cm (equivalently EF = 15 × 7 / 5 = 21 cm). Check: BC/EF = 7/21 = 1/3 = AB/DE, so the scale ratio is 1:3. Why: Cross-multiplying the proportion isolates the unknown side. The check confirms both pairs of sides give the same ratio 1:3, as similarity requires.

Topic and concept

Criteria for similarity of triangles — AA, SSS, SAS. If two angles of one triangle equal two angles of another, the triangles are similar (AA). Their corresponding sides are then in the same ratio, so one unknown side can be found from a proportion.

How marks are earned

1. Since two pairs of corresponding angles ∠A = ∠D and ∠B = ∠E are equal in triangles ABC and DEF, by the AA criterion we conclude the triangles are similar. (1 mark)

2. From the similarity of the triangles, the corresponding sides are proportional, so we can write AB/DE = BC/EF, which becomes 5/15 = 7/EF when substituting the known lengths. (1 mark)
3. Solving the proportion 5/15 = 7/EF through cross-multiplication and division gives EF = (15 × 7)/5, which simplifies to EF = 21 cm. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 6, §6.4: AA criterion states that two equal angles imply similarity and proportional sides.

Question 28

3 marks

Short Answer

Attempt one option.

Option A

Answer the question with working and reasons where needed.

(a) The incomes of two persons are in the ratio 9:7 and their expenditures are in the ratio 4:3. Each saves ₹2000. Find their monthly incomes. [3 marks]

Answer

Let the incomes be 9x and 7x, and their expenditures be 4y and 3y respectively. Since each saves ₹2000, we form 9x − 4y = 2000 and 7x − 3y = 2000. Eliminating y by multiplying and subtracting gives x = 2000 and then incomes are 9×2000 = ₹18000 and 7×2000 = ₹14000.

Step by step

1. **Assume the unknowns using the given ratios:** Let the incomes be 9x and 7x (rupees) and the expenditures be 4y and 3y (rupees), so the first person has income 9x and expenditure 4y, and the second has 7x and 3y. Why: A ratio such as 9:7 means the quantities are a common multiple of 9 and 7. Using x for incomes and y for expenditures gives two unknowns for a pair of linear equations.
2. **Form the equations from the savings:** Savings = income − expenditure = ₹2000 for each, so 9x − 4y = 2000 ...(1) and 7x − 3y = 2000 ...(2). Why: Each person saves the same amount, so each gives one linear equation in x and y. Two equations in two unknowns can be solved.
3. **Eliminate y, solve and verify:** Multiply (1) by 3: 27x − 12y = 6000. Multiply (2) by 4: 28x − 12y = 8000. Subtracting gives x = 2000. Incomes: 9 × 2000 = ₹18000 and 7 × 2000 = ₹14000. Check: from (1), 18000 − 4y = 2000, so y = 4000. Expenditures 16000 and 12000 give savings 18000 − 16000 = 2000 and 14000 − 12000 = 2000. Why: Elimination makes the y-coefficients equal (both 12), so subtracting removes y and leaves one equation in x. Substituting back and recomputing the savings confirms the solution satisfies both equations.

Topic and concept

Formulation and solution of word problems using pair of linear equations. Word problems with ratios can be turned into a pair of linear equations by using a common multiplier. The elimination method then matches coefficients to remove one variable and find the answer.

How marks are earned

1. Let the incomes be 9x and 7x, and their expenditures be 4y and 3y respectively. (1 mark)
2. Since each saves ₹2000, we form 9x − 4y = 2000 and 7x − 3y = 2000. (1 mark)
3. Eliminating y by multiplying and subtracting gives x = 2000 and then incomes are 9×2000 = ₹18000 and 7×2000 = ₹14000. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 3, §3.3.2 Elimination Method: The excerpt uses elimination by matching coefficients in a ratio-based income problem.

OR

Option B

Answer the question with working and reasons where needed.

(a) For each of the following pairs, determine whether the system is dependent (infinitely many solutions) or inconsistent (no solution), and justify using coefficient ratios: (i) x − 2y = 3 and 2x − 4y = 6 (ii) x − 2y = 3 and 2x − 4y = 8 [3 marks]

Answer

For (i) $a_1/a_2 = 1/2$, $b_1/b_2 = (-2)/(-4) = 1/2$, and $c_1/c_2 = 3/6 = 1/2$, so all ratios are equal indicating a dependent system with infinitely many solutions. For (ii) $a_1/a_2 = 1/2$, $b_1/b_2 = 1/2$, but $c_1/c_2 = 3/8 \neq 1/2$, so the system is inconsistent with no solution. These conclusions follow the condition that a dependent system has $a_1/a_2 = b_1/b_2 = c_1/c_2$, whereas an inconsistent system has $a_1/a_2 = b_1/b_2 \neq c_1/c_2$.

Step by step

1. **Compare all three coefficient ratios for pair (i):** For x − 2y = 3 and 2x − 4y = 6 we have a_1=1, b_1=−2, c_1=3 and a_2=2, b_2=−4, c_2=6. So a_1/a_2 = 1/2, b_1/b_2 = (−2)/(−4) = 1/2 and c_1/c_2 = 3/6 = 1/2; all three ratios are equal, so the system is dependent with infinitely many solutions. As a check, the second equation is exactly 2 times the first. Why: When $a_1/a_2 = b_1/b_2 = c_1/c_2$, one equation is a constant multiple of the other, so both describe the same line. Every point of that line satisfies both equations, which is why the solutions are infinite.
2. **Compare the ratios for pair (ii) and spot the mismatch in constants:** For x − 2y = 3 and 2x − 4y = 8 we have a_1/a_2 = 1/2 and b_1/b_2 = (−2)/(−4) = 1/2, but c_1/c_2 = 3/8, and 3/8 ≠ 1/2. The coefficients of x and y are in the same ratio but the constants are not, so the system is inconsistent with no solution. Check: doubling the first equation gives 2x − 4y = 6, which contradicts 2x − 4y = 8. Why: Equal a- and b-ratios mean the two lines have the same slope, so they are parallel. A different c-ratio shifts one line away from the other, so they never meet and no (x, y) satisfies both.

3. **State the three cases and the geometric meaning that justify both conclusions:** Unique solution needs $a_1/a_2 \neq b_1/b_2$ (lines intersect at one point). Dependent needs $a_1/a_2 = b_1/b_2 = c_1/c_2$ (coincident lines). Inconsistent needs $a_1/a_2 = b_1/b_2 \neq c_1/c_2$ (parallel lines). Pair (i) fits the second case and pair (ii) fits the third. Why: The ratio test is the algebraic form of the graphical method in §3.2. Whether two lines intersect, coincide or are parallel decides the number of common solutions, so comparing ratios gives the answer without graphing.

Topic and concept

Graphical representation of pair of linear equations; consistent, inconsistent, dependent conditions. For $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, comparing a_1/a_2, b_1/b_2 and c_1/c_2 tells whether the lines intersect (one solution), coincide (infinitely many) or are parallel (none). Here both pairs share the ratio 1/2 for x and y, so only the constant term separates (i) from (ii).

How marks are earned

1. For (i) $a_1/a_2 = 1/2$, $b_1/b_2 = (-2)/(-4) = 1/2$, and $c_1/c_2 = 3/6 = 1/2$, so all ratios are equal indicating a dependent system with infinitely many solutions. (1 mark)
2. For (ii) $a_1/a_2 = 1/2$, $b_1/b_2 = 1/2$, but $c_1/c_2 = 3/8 \neq 1/2$, so the system is inconsistent with no solution. (1 mark)
3. These conclusions follow the condition that a dependent system has $a_1/a_2 = b_1/b_2 = c_1/c_2$, whereas an inconsistent system has $a_1/a_2 = b_1/b_2 \neq c_1/c_2$. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 3, §3.2 Graphical Method of Solution of a Pair of Linear Equations: The excerpt outlines that coincident lines (dependent) satisfy $a_1/a_2 = b_1/b_2 = c_1/c_2$, while parallel lines (inconsistent) satisfy $a_1/a_2 = b_1/b_2 \neq c_1/c_2$.

Question 29

3 marks

Short Answer

Answer the question with working and reasons where needed.

(a) Prove that the tangent at any point of a circle is perpendicular to the radius at the point of contact. [3 marks]

Answer

Let XY be the tangent at P to the circle with centre O, and choose any other Q on XY. Since Q is outside the circle, OQ>OP, so OP is the shortest segment from O to XY. The shortest distance from a point to a line is perpendicular; therefore OP⊥XY at P.

Step by step

1. **Set up the figure and show that any other point on the tangent lies outside the circle:** Draw a circle with centre O and a tangent XY touching it at P. Take any point Q on XY other than P. If Q were on or inside the circle, the line XY would meet the circle at a second point, so XY would be a secant, not a tangent. Hence Q lies outside the circle. Why: A tangent has exactly one point in common with the circle. This single-contact property is what forces every other point of the tangent to lie outside the circle.
2. **Compare OQ with the radius OP:** Q is outside the circle, so OQ is greater than the radius: OQ > OP. This holds for every point Q on XY other than P, so among all segments from O to points of XY, OP is the shortest. Why: A point is outside a circle exactly when its distance from the centre exceeds the radius. OP is a radius, so it is the minimum distance from O to the line XY.
3. **Use the shortest-distance property to conclude perpendicularity:** The shortest distance from a point to a line is the perpendicular from that point to the line. OP is the shortest segment from O to XY and meets XY at P, so OP $\perp$ XY, that is, $\angle OPX = 90°$. Why: For any point not on a line, the perpendicular segment is shorter than every slanting segment (in a right triangle the hypotenuse is the longest side). So the shortest segment must be the perpendicular, which gives the right angle at the point of contact.

Topic and concept

Tangent to a circle; tangent at any point is perpendicular to the radius (theorem + proof). A tangent meets the circle at exactly one point P, so all its other points are outside the circle and farther from the centre than the radius OP. The radius to the point of contact is therefore the shortest distance from the centre to the tangent, and the shortest distance from a point to a line is the perpendicular.

How marks are earned

1. Let XY be tangent at P and take any other point Q on XY; Q lies outside the circle. (1 mark)
2. Use OQ>OP to show OP is the shortest distance from the centre O to the tangent line XY. (1 mark)
3. Conclude that OP is perpendicular to XY because the shortest distance from a point to a line is perpendicular. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 10, §10.2 Tangent to a Circle: The radius to the point of contact is the shortest distance to the tangent, establishing a right angle.

Question 30

3 marks

Short Answer

Answer the question with working and reasons where needed.

(a) Shakila deposits ₹100 on her daughter's first birthday and increases the deposit by ₹50 each subsequent birthday. How much money does she deposit in total by the time of the daughter's 15th

birthday? Show all steps. [3 marks]

Answer

The deposits form an AP with first term a = ₹100, common difference d = ₹50 and number of terms n = 15. Using the sum formula $S_n = n/2[2a + (n−1)d]$ gives $S_{15} = 15/2[2×100 + 14×50]$. Thus, $S_{15} = 7.5[200 + 700] = 7.5×900 = ₹6750$ is the total deposited by the 15th birthday.

Step by step

1. **Identify the AP and count the terms:** The deposits are ₹100, ₹150, ₹200, ... Each is ₹50 more than the previous, so they form an AP with a = ₹100 and d = ₹50. The 1st to 15th birthdays give n = 15 deposits, one per birthday. Why: A constant increase each year is the defining feature of an AP. Counting from the 1st to the 15th birthday inclusive gives 15 terms, one deposit per birthday.
2. **Write the sum formula and substitute:** $S_n = n/2\ [2a + (n − 1)d]$, so $S_{15} = 15/2\ [2 × 100 + (15 − 1) × 50] = 15/2\ [200 + 14 × 50]$. Why: The sum formula adds the first n terms of an AP without listing them. It needs only a, d and n, which are all known here.
3. **Evaluate, give the unit and check:** 14 × 50 = 700, so $S_{15} = 15/2 × (200 + 700) = 7.5 × 900 = ₹6750$. Check: the 15th term is 100 + 14 × 50 = ₹800, and the average of the first and last terms is (100 + 800)/2 = 450, so 15 × 450 = ₹6750. Why: For an AP, the sum is the number of terms times the average of the first and last terms. Getting the same value both ways confirms the arithmetic.

Topic and concept

Sum of first n terms: $S_n = n/2(2a+(n−1)d) = n/2(a+l)$. When a quantity rises by a fixed amount each time, the values form an AP. The total of its first n terms is found with $S_n = n/2[2a + (n−1)d]$ instead of adding 15 deposits one by one.

How marks are earned

1. The deposits form an AP with first term a = ₹100, common difference d = ₹50 and number of terms n = 15. (1 mark)
2. Using the sum formula $S_n = n/2[2a + (n−1)d]$ gives $S_{15} = 15/2[2×100 + 14×50]$. (1 mark)
3. Thus, $S_{15} = 7.5[200 + 700] = 7.5×900 = ₹6750$ is the total deposited by the 15th birthday. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 5, §5.4: Money deposits increasing by a fixed amount each birthday form an AP whose sum can be found using the formula.

Question 31

3 marks

Short Answer

Attempt one option.

Option A

Answer the question with working and reasons where needed.

(a) Find the area of the minor segment of a circle with radius 7 cm when the central angle is 60°. Take π = 22/7. Show all calculations clearly. [3 marks]

Answer

Area of the sector = (60/360) × (22/7) × 7² = (1/6) × 154 = 25.67 cm². Area of the equilateral triangle = (√3/4) × 7² = (1.732/4) × 49 = 21.217 cm². Subtracting yields segment area = 25.67 − 21.217 = 4.453 cm², approximately 4.45 cm².

Step by step

1. **State the segment formula and find the sector area:** Area of minor segment = area of sector − area of triangle formed by the two radii and the chord. Sector area = (60/360) × (22/7) × 7² = (1/6) × (22/7) × 49 = (1/6) × 154 = 25.67 cm². Why: The segment is the part of the sector that lies outside the triangle OAB. Removing the triangle from the sector leaves the segment. The sector is the fraction 60/360 of the full circle.
2. **Show the triangle is equilateral and find its area:** OA = OB = 7 cm (radii) and ∠AOB = 60°, so the other two angles are (180° − 60°)/2 = 60° each. Triangle OAB is therefore equilateral with side 7 cm. Area = (√3/4) × 7² = (1.732/4) × 49 = 84.868/4 = 21.217 cm². Why: Equal radii make the triangle isosceles, so its base angles are equal. The angle sum of 180° then makes all angles 60°, so all sides are equal and the equilateral-triangle area formula (√3/4)a² can be used.
3. **Subtract and state the answer with units:** Segment area = 25.67 − 21.217 = 4.453 cm², which is approximately 4.45 cm². Why: Subtracting the triangle from the sector follows the segment formula. Both areas are in cm², so the result is in cm². It is small and positive, as a thin minor segment should be.

Topic and concept

Area of segment of a circle. The area of a circle segment equals the sector area minus the area of the triangle formed by its radii and chord. With a central angle of 60° and equal radii, that triangle is equilateral.

How marks are earned

1. Area of the sector = (60/360) × (22/7) × 7² = (1/6) × 154 = 25.67 cm². (1 mark)
2. Area of the equilateral triangle = (√3/4) × 7² = (1.732/4) × 49 = 21.217 cm². (1 mark)
3. Subtracting yields segment area = 25.67 − 21.217 = 4.453 cm², approximately 4.45 cm². (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 11, §11.2 Summary: The summary states that segment area is sector area minus triangle area.

OR

Option B

Answer the question with working and reasons where needed.

(a) In a circle of radius 10 cm, calculate (a) the length of the arc and (b) the area of the sector subtended by a central angle of 144°. Use π = 3.14. Show your steps. [3 marks]

Answer

Arc length: l = (144/360) × 2π × 10 = 0.4 × 2 × 3.14 × 10 = 25.12 cm. Sector area: A = (144/360) × π × 10² = 0.4 × 3.14 × 100 = 125.6 cm². Thus, the arc length is 25.12 cm and the sector area is 125.6 cm².

Step by step

1. **Find the fraction of the circle and apply the arc length formula:** The fraction of the circle is 144/360 = 2/5 = 0.4. Arc length l = (θ/360) × 2πr = 0.4 × 2 × 3.14 × 10 cm = 0.4 × 62.8 cm = 25.12 cm. Why: An arc subtending θ at the centre is the fraction θ/360 of the whole circumference 2πr, because arc length is proportional to the central angle. Radius 10 cm makes the unit of length cm.
2. **Apply the sector area formula with the same fraction:** Area of sector A = (θ/360) × πr² = 0.4 × 3.14 × 10² cm² = 0.4 × 314 cm² = 125.6 cm². Why: A sector occupies the same fraction θ/360 of the full circle area πr². The units are cm² because area is length squared, so the result is not in cm.
3. **Verify and state both results with units:** Check: the full circumference is 62.8 cm and 0.4 × 62.8 = 25.12 cm. The full area is 314 cm² and 0.4 × 314 = 125.6 cm². So the arc length is 25.12 cm and the sector area is 125.6 cm². Why: The same fraction 2/5 applies to both circumference and area, so both values must be 40% of the full-circle quantities. Recomputing them this way catches arithmetic slips.

Topic and concept

Area of a sector and length of an arc. A sector of central angle θ in a circle of radius r is the fraction θ/360 of the whole circle. So its arc length is (θ/360) × 2πr and its area is (θ/360) × πr². Here θ = 144° gives the fraction 2/5 for both.

How marks are earned

1. Arc length: l = (144/360) × 2π × 10 = 0.4 × 2 × 3.14 × 10 = 25.12 cm. (1 mark)
2. Sector area: A = (144/360) × π × 10² = 0.4 × 3.14 × 100 = 125.6 cm². (1 mark)
3. Thus, the arc length is 25.12 cm and the sector area is 125.6 cm². (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 11, §11.2 Summary: The summary lists formulas for both arc length and sector area.

Section D

Question 32

5 marks

Long Answer

Answer both connected parts. Show the required method, explanation and units.

(a) Find the 20th term of the arithmetic progression 5, –1, –7, …. [2 marks]

Answer

The given sequence 5, -1, -7,… has first term a = 5 and common difference d = (−1) − 5 = −6, so applying a_{20} = a + (20−1)d yields a_{20} = 5 + 19×(−6). Calculating 19×(−6) = −114 and adding to 5 gives a_{20} = 5 − 114 = −109; hence, the twentieth term of the arithmetic progression is -109.

Step by step

1. **Find the first term a and the common difference d, then write the formula for a_{20}.:** The first term is a = 5. The common difference is d = a_2 − a_1 = (−1) − 5 = −6. Check: −7 − (−1) = −6, so the difference is constant. For the 20th term n = 20, so n − 1 = 19. Then a_{20} = a + (20 − 1)d = 5 + 19×(−6). Why: An AP has the same difference between each term and the previous one, so d = a_2 − a_1 and the second check confirms the sequence is an AP. A negative d means the terms decrease by 6 each time. The term a_n = a + (n − 1)d uses n − 1 because the first term a already counts as the first, and only 19 further steps of d are added to reach the 20th.
2. **Evaluate 19×(−6) and add it to 5 to get a_{20}.:** 19×(−6) = −114. So a_{20} = 5 + (−114) = 5 − 114 = −109. The twentieth term of the AP is −109. Why: Substituting in a_{20} = a + (n − 1)d reduces to simple arithmetic. Adding a negative number makes the value smaller, so a_{20} is negative, which agrees with the decreasing trend of the terms. The answer is a pure number, so there is no unit.

Topic and concept

nth term formula: a_n = a + (n−1)d. To find a specific term of an AP, substitute a, d and n in a_n = a + (n − 1)d (Ch. 5 §5.3). Here a = 5 and d = −6, and the negative difference means the terms decrease. The formula avoids repeated subtraction of 6 nineteen times.

How marks are earned

1. The given sequence 5, -1, -7,… has first term a = 5 and common difference d = (−1) − 5 = −6, so applying a_{20} = a + (20−1)d yields a_{20} = 5 + 19×(−6). (1 mark)
2. Calculating 19×(−6) = −114 and adding to 5 gives a_{20} = 5 − 114 = −109; hence, the twentieth term of the arithmetic progression is -109. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 5, §5.3: The nth term of an AP is given by a_n = a + (n−1)d.

(b) Reena's annual salary starts at ₹8000 and increases by ₹500 each year. Calculate her salary in the 30th year of employment. Include all steps and the formula used. [3 marks]

Answer

Reena's annual earnings form an arithmetic progression with first term a = ₹8,000 and a constant annual increase of d = ₹500 over each preceding year, up to the thirtieth term. To compute her salary in the thirtieth year we substitute a = ₹8,000, d = ₹500, and n = 30 into the nth term formula a_{30} = a + (n−1)d, yielding a_{30} = 8,000 + 29×500. Evaluating 29×500 = 14,500 and adding to 8,000 gives a_{30} = 22,500; therefore, Reena's salary in the thirtieth year of employment is ₹22,500.

Step by step

1. **Show that the salaries form an AP and identify a and d.:** The salary in year 1 is ₹8,000. Each year it rises by the same amount, ₹500, so the salaries are ₹8,000, ₹8,500, ₹9,000, … The first term is a = ₹8,000 and the common difference is d = ₹500. Why: A sequence with a constant difference between consecutive terms is an AP. Adding a fixed ₹500 every year gives exactly that. This is why the AP formula applies instead of a ratio or other method. The positive d means the salary grows.
2. **Substitute a, d and n = 30 into the nth term formula.:** The formula is $a_n = a + (n - 1)d$. For the 30th year n = 30, so n − 1 = 29. Thus a_{30} = ₹8,000 + 29×₹500. Why: The first year's salary of ₹8,000 has had no raise yet. So by the 30th year there have been 29 raises, not 30, and this is why the formula uses n − 1. The formula replaces adding ₹500 repeatedly, as the NCERT salary example does.
3. **Evaluate the product and the sum, keeping the ₹ unit.:** 29×₹500 = ₹14,500. Then a_{30} = ₹8,000 + ₹14,500 = ₹22,500. Check: 22,500 − 8,000 = 14,500 = 29×500. Her salary in the 30th year is ₹22,500. Why: Working out the 29 raises first and then adding them to the starting salary follows the order in the formula. The reverse check shows the answer equals the starting salary plus 29 equal raises. The unit ₹ is kept because the answer is an amount of money.

Topic and concept

nth term formula: $a_n = a + (n-1)d$. A quantity that rises by a fixed amount each period forms an AP. The nth term $a_n = a + (n - 1)d$ gives the value in any chosen period without listing all earlier ones (Ch. 5 §5.3). In this salary problem a = ₹8,000, d = ₹500 and n = 30.

How marks are earned

1. Reena's annual earnings form an arithmetic progression with first term a = ₹8,000 and a constant annual increase of d = ₹500 over each preceding year, up to the thirtieth term. (1 mark)
2. To compute her salary in the thirtieth year we substitute a = ₹8,000, d = ₹500, and n = 30 into the nth term formula a_{30} = a + (n−1)d, yielding a_{30} = 8,000 + 29×500. (1 mark)
3. Evaluating 29×500 = 14,500 and adding to 8,000 gives a_{30} = 22,500; therefore, Reena's salary in the thirtieth year of employment is ₹22,500. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 5, §5.3: Salary increments form an AP whose terms are given by the nth term formula.

Question 33

5 marks

Long Answer

Answer both connected parts. Show the required method, explanation and units.

(a) State the primary Pythagorean trigonometric identity relating sin θ and cos θ, and briefly explain its origin. [2 marks]

Answer

The Pythagorean trigonometric identity is $\sin^2\theta + \cos^2\theta = 1$ for any acute angle θ in a right triangle. It originates from dividing the Pythagorean theorem (sum of squares of legs equals square of hypotenuse) by the square of the hypotenuse.

Step by step

1. **State the identity:** For an acute angle θ in a right triangle, $\sin^2\theta + \cos^2\theta = 1$, where $\sin^2\theta$ means $(\sin\theta)^2$ and $\cos^2\theta$ means $(\cos\theta)^2$. Why: This is the primary Pythagorean identity. It holds for every acute angle and links the two ratios through their squares, not their plain sum.
2. **Derive it from the Pythagorean theorem:** In a right triangle with perpendicular p, base b and hypotenuse h, $p^2 + b^2 = h^2$. Dividing both sides by h^2 gives $(p/h)^2 + (b/h)^2 = 1$. Since $p/h = \sin\theta$ and $b/h = \cos\theta$, this becomes $\sin^2\theta + \cos^2\theta = 1$. Why: Dividing by h^2 turns the side lengths into the ratios $\sin\theta = p/h$ and $\cos\theta = b/h$. So the identity is the Pythagorean theorem written in trigonometric ratios.

Topic and concept

Determining other trigonometric ratios when one is known. The identity $\sin^2\theta + \cos^2\theta = 1$ follows from the Pythagorean theorem. Dividing $p^2 + b^2 = h^2$ by h^2 turns the side lengths into $\sin\theta$ and $\cos\theta$. This is why it is true for every acute angle θ.

How marks are earned

1. The Pythagorean trigonometric identity is $\sin^2\theta + \cos^2\theta = 1$ for any acute angle θ in a right triangle. (1 mark)
2. It originates from dividing the Pythagorean theorem (sum of squares of legs equals square of hypotenuse) by the square of the hypotenuse. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 8, §8.4: Dividing the Pythagorean relation by the square of the hypotenuse yields $\sin^2 A + \cos^2 A = 1$.

(b) Prove the trigonometric identity $\tan^2 A + 1 = \sec^2 A$ starting from $\sin^2 A + \cos^2 A = 1$. [3 marks]

Answer

Starting from $\sin^2 A + \cos^2 A = 1$, divide every term by $\cos^2 A$ to obtain $\tan^2 A + 1 = \sec^2 A$. This step uses $\tan A = \sin A/\cos A$ and $\sec A = 1/\cos A$ to identify $\tan^2 A = \sin^2 A/\cos^2 A$ and $\sec^2 A = 1/\cos^2 A$. Thus, for all angles where $\cos A \neq 0$, $\tan^2 A + 1 = \sec^2 A$ holds true.

Step by step

1. **Divide every term of $\sin^2 A + \cos^2 A = 1$ by $\cos^2 A$:** Start from $\sin^2 A + \cos^2 A = 1$ and divide each term by $\cos^2 A$, which gives $\sin^2 A/\cos^2 A + \cos^2 A/\cos^2 A = 1/\cos^2 A$. Why: Dividing both sides of an equation by the same non-zero quantity keeps the equality true. Dividing every term, including the right side, is what preserves the identity.
2. **Replace each fraction using the ratio definitions:** $\sin^2 A/\cos^2 A = (\sin A/\cos A)^2 = \tan^2 A$. $\cos^2 A/\cos^2 A = 1$. $1/\cos^2 A = (1/\cos A)^2 = \sec^2 A$. Substituting, $\tan^2 A + 1 = \sec^2 A$. Why: $\tan A = \sin A/\cos A$ and $\sec A = 1/\cos A$ are the definitions of those ratios. Squaring them converts each fraction to the required trigonometric form.
3. **State the condition under which the identity holds:** The division by $\cos^2 A$ is valid only when $\cos A \neq 0$. Therefore $\tan^2 A + 1 = \sec^2 A$ holds for every angle A with $\cos A \neq 0$, for example every acute angle. Why: Division by zero is undefined, and tan A and sec A are themselves undefined when $\cos A = 0$. So the identity is claimed only where both sides have meaning.

Topic and concept

Determining other trigonometric ratios when one is known. One Pythagorean identity can generate another. Since $\sin^2 A + \cos^2 A = 1$ holds for any angle, dividing by $\cos^2 A$ and using the definitions of tan and sec gives the related identity $\tan^2 A + 1 = \sec^2 A$.

How marks are earned

1. Starting from $\sin^2 A + \cos^2 A = 1$, divide every term by $\cos^2 A$ to obtain $\tan^2 A + 1 = \sec^2 A$. (1 mark)
2. This step uses $\tan A = \sin A/\cos A$ and $\sec A = 1/\cos A$ to identify $\tan^2 A = \sin^2 A/\cos^2 A$ and $\sec^2 A = 1/\cos^2 A$. (1 mark)
3. Thus, for all angles where $\cos A \neq 0$, $\tan^2 A + 1 = \sec^2 A$ holds true. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 8, §8.4: Dividing the Pythagorean identity by $\cos^2 A$ yields $\tan^2 A + 1 = \sec^2 A$.

Question 34

5 marks

Long Answer

Attempt one option.

Option A

Answer both connected parts. Show the required method, explanation and units.

(a) Calculate the distance between P(1,2) and Q(4,6). [2 marks]

Answer

Applying the general two-dimensional distance formula to P(1,2) and Q(4,6) gives PQ = √[(4 − 1)² + (6 − 2)²] = √(9 + 16), representing the sum of the squares of the coordinate differences. Evaluating the expression √(9 + 16) simplifies to √25, therefore the exact distance between P(1,2) and Q(4,6) is determined as PQ = 5 units.

Step by step

1. **Write the distance formula and substitute the coordinates of P(1,2) and Q(4,6).:** The distance formula is PQ = $\sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$. With $(x_1, y_1) = (1, 2)$ and $(x_2, y_2) = (4, 6)$, the horizontal difference is 4 − 1 = 3 and the vertical difference is 6 − 2 = 4. So PQ = $\sqrt{[(4 - 1)^2 + (6 - 2)^2]} = \sqrt{(3^2 + 4^2)} = \sqrt{(9 + 16)}$. Why: The horizontal difference 3 and the vertical difference 4 are the legs of a right triangle whose hypotenuse is PQ. The distance formula is the Pythagorean theorem applied to these legs. Each difference is squared first, so the sum of squares gives the squared hypotenuse.
2. **Add the squares and take the square root, giving the length in units.:** 9 + 16 = 25, so PQ = √25 = 5 units. Check: $3^2 + 4^2 = 5^2$, which is the 3-4-5 right triangle. Why: The square root is needed because the sum of squares equals PQ^2, not PQ. Taking the positive root gives the length, because distance cannot be negative. The coordinates have no stated unit, so the answer is in units.

Topic and concept

Distance formula between two points. For any two points $P(x_1, y_1)$ and $Q(x_2, y_2)$, the distance is PQ = $\sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$ (Ch. 7 §7.4 Summary). It follows from the Pythagorean theorem applied to the horizontal and vertical differences, which are the legs of a right triangle with PQ as the hypotenuse.

How marks are earned

1. Applying the general two-dimensional distance formula to P(1,2) and Q(4,6) gives PQ = √[(4 − 1)² + (6 − 2)²] = √(9 + 16), representing the sum of the squares of the coordinate differences. (1 mark)
2. Evaluating the expression √(9 + 16) simplifies to √25, therefore the exact distance between P(1,2) and Q(4,6) is determined as PQ = 5 units. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 7, §7.2: Distance between two general points uses Pythagoras theorem.

(b) Find the distance between P(3,4) and Q(-1,2), giving your answer in simplest radical form. [3 marks]

Answer

Starting with P(3,4) and Q(−1,2), we compute the horizontal difference Δx = (−1 − 3) = −4 and the vertical difference Δy = (2 − 4) = −2 before applying the distance formula PQ = √(Δx² + Δy²). Substituting these values into the formula gives PQ = √[(-4)² + (-2)²] = √(16 + 4) = √20, which represents the exact distance before simplification. By extracting the factor 4 under the radical, we simplify √20 to 2√5, thus obtaining the distance between P and Q as PQ = 2√5 units in simplest radical form.

Step by step

1. **Find the horizontal and vertical differences between P(3,4) and Q(−1,2), and state the distance formula.:** $\Delta x = x_2 - x_1 = -1 - 3 = -4$ and $\Delta y = y_2 - y_1 = 2 - 4 = -2$, so the formula to use is $PQ = \sqrt{(\Delta x^2 + \Delta y^2)}$. Why: The horizontal and vertical differences are the two perpendicular sides of a right triangle whose hypotenuse is PQ, so the Pythagorean theorem gives the distance formula.
2. **Substitute the differences into the formula and evaluate under the root.:** $PQ = \sqrt{[(-4)^2 + (-2)^2]} = \sqrt{(16 + 4)} = \sqrt{20}$, which is the exact distance before simplification. Why: Squaring each difference makes it positive, so the negative signs from subtracting coordinates do not change the length. Adding the squares is the Pythagorean step.
3. **Simplify $\sqrt{20}$ by taking out the largest perfect-square factor.:** $\sqrt{20} = \sqrt{(4 \times 5)} = \sqrt{4} \times \sqrt{5} = 2\sqrt{5}$, so $PQ = 2\sqrt{5}$ units. Check: $2\sqrt{5} \approx 2 \times 2.236 = 4.47$, and $\sqrt{20} \approx 4.47$. Why: Simplest radical form means no perfect-square factor stays under the root. Since $20 = 4 \times 5$ and 4 is a perfect square, $\sqrt{4} = 2$ comes out and $\sqrt{5}$ stays.

Topic and concept

Distance formula between two points. For any two points $P(x_1,y_1)$ and $Q(x_2,y_2)$, the distance PQ is $\sqrt{((x_2-x_1)^2+(y_2-y_1)^2)}$. It comes from applying the Pythagorean theorem to the horizontal and vertical differences. Here the points are not on one axis, so both differences are needed. The answer is then written in simplest radical form.

How marks are earned

1. Starting with P(3,4) and Q(−1,2), we compute the horizontal difference $\Delta x = (-1 - 3) = -4$ and the vertical difference $\Delta y = (2 - 4) = -2$ before applying the distance formula $PQ = \sqrt{(\Delta x^2 + \Delta y^2)}$. (1 mark)
2. Substituting these values into the formula gives $PQ = \sqrt{[(-4)^2 + (-2)^2]} = \sqrt{(16 + 4)} = \sqrt{20}$, which represents the exact distance before simplification. (1 mark)
3. By extracting the factor 4 under the radical, we simplify $\sqrt{20}$ to $2\sqrt{5}$, thus obtaining the distance between P and Q as $PQ = 2\sqrt{5}$ units in simplest radical form. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 7, §7.2: Distance between two general points uses squared differences and square root.

OR

Option B

Answer both connected parts. Show the required method, explanation and units.

(a) Town A is at (10,20) km and town B is at (16,28) km on a coordinate grid. Find their straight-line distance in km using the distance formula. [2 marks]

Answer

$AB=\sqrt{[(16-10)^2+(28-20)^2]}$ km$=\sqrt{(6^2+8^2)}$ km$=\sqrt{100}$ km$=10$ km.

Step by step

1. **Write the distance formula and substitute the coordinate differences:** For A(10, 20) and B(16, 28), AB = $\sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$ = $\sqrt{[(16 - 10)^2 + (28 - 20)^2]}$ = $\sqrt{(6^2 + 8^2)}$. Taking the coordinates in km, as the question states them, the differences are 6 km and 8 km. Why: The distance formula applies Pythagoras' theorem to the horizontal difference (6) and vertical difference (8), which are the legs of a right triangle whose hypotenuse is AB.
2. **Evaluate the square root and state the distance with its unit:** AB = $\sqrt{(36 + 64)}$ = $\sqrt{100}$ = 10 km. Check: 6, 8, 10 is a Pythagorean triple, since $6^2 + 8^2 = 36 + 64 = 100 = 10^2$. Why: Adding the squares of the differences gives the square of the straight-line distance, so taking the square root returns a length. It carries the km unit of the coordinates.

Topic and concept

Distance formula between two points. The distance between P(x_1, y_1) and Q(x_2, y_2) is $\sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$, from Pythagoras' theorem applied to the horizontal and vertical differences. Here the coordinates are given in km, so the result is in km.

How marks are earned

1. Write the distance formula and substitute the coordinate differences: AB=$\sqrt{[(16-10)^2+(28-20)^2]}$ km. (1 mark)
2. Evaluate $\sqrt{(6^2+8^2)}=\sqrt{100}=10$ and state the distance as 10 km. (1 mark)

Source check: Using the NCERT Class 10 maths Ch. 7 distance formula, $AB^2=6^2+8^2=100$ km², so AB=10 km.

(b) With A(0,0), B(3,4) and C(3,0), verify by calculation whether triangle ABC is right-angled. [3 marks]

Answer

First, we use the distance formula to find AB = $\sqrt{[(3 - 0)^2 + (4 - 0)^2]}$ = 5 units, AC = $\sqrt{[(3 - 0)^2 + (0 - 0)^2]}$ = 3 units, and BC = $\sqrt{[(3 - 3)^2 + (0 - 4)^2]}$ = 4 units. Next, we calculate the sum of the squares $AC^2 + BC^2 = 3^2 + 4^2 = 9 + 16 = 25$ and compare it with $AB^2 = 5^2 = 25$ to test the Pythagorean relation for a right angle. Since $AC^2 + BC^2$ equals AB^2, triangle ABC satisfies the Pythagorean theorem, confirming that ∠C is a right angle and the triangle is right-angled at vertex C.

Step by step

1. **Use the distance formula to find the three side lengths of triangle ABC.:** AB = $\sqrt{[(3-0)^2 + (4-0)^2]}$ = $\sqrt{(9+16)}$ = $\sqrt{25}$ = 5 units. AC = $\sqrt{[(3-0)^2 + (0-0)^2]}$ = $\sqrt{9}$ = 3 units. BC = $\sqrt{[(3-3)^2 + (0-4)^2]}$ = $\sqrt{16}$ = 4 units. Why: The distance formula gives each side from the coordinates alone, so the test needs no drawing. The side lengths are then compared using the Pythagorean relation.
2. **Compare the square of the longest side with the sum of the squares of the other two.:** The longest side is AB = 5, so test AB^2 against $AC^2 + BC^2$. $AC^2 + BC^2 = 3^2 + 4^2 = 9 + 16 = 25$ and $AB^2 = 5^2 = 25$. Why: If a triangle is right-angled, the hypotenuse is its longest side, so AB^2 is the one to test. The converse of Pythagoras says that if the squares of two sides add to the square of the third, the angle opposite the third side is a right angle.

3. **Conclude which angle is a right angle.:** Since $AC^2 + BC^2 = AB^2 = 25$, triangle ABC is right-angled at C, the vertex opposite AB. Check: A(0,0) and C(3,0) lie on the x-axis, and B(3,4) is directly above C, so AC is horizontal and BC is vertical. Why: The converse of Pythagoras puts the right angle opposite the longest side. The coordinates give the same answer, because a horizontal line and a vertical line meet at 90°.

Topic and concept

Distance formula between two points. The distance formula $\sqrt{((x_2-x_1)^2+(y_2-y_1)^2)}$ comes from the Pythagorean theorem. In coordinate geometry it gives the three side lengths of a triangle. The converse of Pythagoras then tests for a right angle: if the square of the longest side equals the sum of the squares of the other two, the triangle is right-angled opposite that longest side.

How marks are earned

1. First, we use the distance formula to find $AB = \sqrt{[(3 - 0)^2 + (4 - 0)^2]} = 5$ units, $AC = \sqrt{[(3 - 0)^2 + (0 - 0)^2]} = 3$ units, and $BC = \sqrt{[(3 - 3)^2 + (0 - 4)^2]} = 4$ units. (1 mark)
2. Next, we calculate the sum of the squares $AC^2 + BC^2 = 3^2 + 4^2 = 9 + 16 = 25$ and compare it with $AB^2 = 5^2 = 25$ to test the Pythagorean relation for a right angle. (1 mark)
3. Since $AC^2 + BC^2$ equals AB^2, triangle ABC satisfies the Pythagorean theorem, confirming that $\angle C$ is a right angle and the triangle is right-angled at vertex C. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 7, §7.2: Use of Pythagoras theorem in coordinate geometry for testing right angles.

Question 35

5 marks

Long Answer

Attempt one option.

Option A

Answer both connected parts. Show the required method, explanation and units.

(a) Two fair coins are tossed together. Find the probability of obtaining exactly one head. [2 marks]

Answer

When two fair coins are tossed together, the complete set of equally likely outcomes is {HH, HT, TH, TT}, totalling 4 possible outcomes for the experiment. Among these four outcomes, exactly two (HT and TH) contain one head, giving the probability of exactly one head as 2/4 = 1/2.

Step by step

1. **List the sample space for the two fair coin tosses.:** The possible ordered outcomes are HH, HT, TH and TT, so the sample space is S = {HH, HT, TH, TT}. Hence, the total number of equally likely outcomes is n(S) = 4. Why: Listing every elementary outcome prevents treating HT and TH as the same outcome when the two toss positions are distinct.

2. **Count the outcomes with exactly one head and divide by the total outcomes.:** The favourable outcomes for exactly one head are HT and TH, so n(E) = 2. Therefore, P(exactly one head) = n(E)/n(S) = 2/4 = 1/2. Why: For equally likely outcomes, probability is found by the ratio of favourable outcomes to all possible outcomes.

Topic and concept

Simple real-life problems on finding the probability of an event. To find the probability of getting a head in a coin toss or a particular face in a die roll, list the total possible outcomes and count those that fit the event.

How marks are earned

1. When two fair coins are tossed together, the complete set of equally likely outcomes is {HH, HT, TH, TT}, totalling 4 possible outcomes for the experiment. (1 mark)
2. Among these four outcomes, exactly two (HT and TH) contain one head, giving the probability of exactly one head as 2/4 = 1/2. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 14, §14.1: In a fair coin toss, outcomes are equally likely and combinations like HT and TH each have the same chance.

(b) Two fair dice are rolled. Calculate the probability that the sum of the points is 8. [3 marks]

Answer

When two fair dice are rolled simultaneously, the total sample space consists of 6 × 6 = 36 equally likely ordered pairs of outcomes. The ordered pairs yielding a total sum of 8 are (2,6), (3,5), (4,4), (5,3), and (6,2), making a count of five favourable outcomes. Therefore, the probability of obtaining a sum of eight when rolling two dice is given by the ratio 5/36 of favourable outcomes to the total of 36 possible outcomes.

Step by step

1. **Find the total number of ordered outcomes for rolling two fair dice.:** Each die has 6 possible outcomes. For two dice, the sample space has 6 × 6 = 36 ordered pairs, so n(S) = 36. Why: Using ordered pairs counts results such as (2, 6) and (6, 2) separately, as they are different elementary outcomes for two dice.
2. **List and count the ordered pairs whose sum is 8.:** For sum 8, the valid ordered pairs are (2,6), (3,5), (4,4), (5,3), and (6,2). Thus, the number of favourable outcomes is n(E) = 5. Why: Checking the sum in each listed pair ensures that only outcomes satisfying the event are counted.
3. **Calculate the probability of getting a sum of 8.:** P(sum = 8) = n(E)/n(S) = 5/36. Since 5 and 36 have no common factor other than 1, the probability is 5/36. Why: The required probability is the favourable count divided by the total count of equally likely dice outcomes.

Topic and concept

Simple real-life problems on finding the probability of an event. To find the probability of getting a head in a coin toss or a particular face in a die roll, list the total possible outcomes and count those that fit the event.

How marks are earned

1. When two fair dice are rolled simultaneously, the total sample space consists of 6 × 6 = 36 equally likely ordered pairs of outcomes. (1 mark)
2. The ordered pairs yielding a total sum of 8 are (2,6), (3,5), (4,4), (5,3), and (6,2), making a count of five favourable outcomes. (1 mark)
3. Therefore, the probability of obtaining a sum of eight when rolling two dice is given by the ratio 5/36 of favourable outcomes to the total of 36 possible outcomes. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 14, §14.2: The probability is computed by dividing the number of favourable outcomes by the total number of equally likely outcomes.

OR

Option B

Answer both connected parts. Show the required method, explanation and units.

(a) In a single toss of a fair coin, let E be the event ‘head appears’. Calculate P(E), P(not E), and verify that P(E)+P(not E)=1. [2 marks]

Answer

For a single toss of a fair coin, the probability of the event E, defined as obtaining a head, is P(E) = 1/2, and consequently P(not E) is calculated as 1 − 1/2 = 1/2. By summing these values, P(E) + P(not E) = 1/2 + 1/2 = 1, which verifies the fundamental complementary rule in probability theory.

Step by step

1. **Determine P(E) and the probability of the complementary event.:** For one fair coin toss, the outcomes are {H, T}. Event E = {H}, so P(E) = 1/2. The event not E is {T}, hence P(not E) = 1 − 1/2 = 1/2. Why: The two elementary outcomes of a fair coin toss are equally likely, and the complement of head is tail.
2. **Add the event and complement probabilities to verify the rule.:** P(E) + P(not E) = 1/2 + 1/2 = 2/2 = 1. Hence, P(E) + P(not E) = 1 is verified. Why: An event and its complement together cover all elementary outcomes, whose probabilities add up to 1.

Topic and concept

Elementary events and sum of probabilities. An elementary event is a single outcome, and the sum of probabilities of all elementary events in an experiment is 1.

How marks are earned

1. For a single toss of a fair coin, the probability of the event E, defined as obtaining a head, is P(E) = 1/2, and consequently P(not E) is calculated as 1 − 1/2 = 1/2. (1 mark)
2. By summing these values, P(E) + P(not E) = 1/2 + 1/2 = 1, which verifies the fundamental complementary rule in probability theory. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 14, §14.2: For any event E, P(E) + P(not E) = 1.

(b) A letter is chosen at random from the letters of the word ‘MATHEMATICS’. Find the probability that it is a vowel. [3 marks]

Answer

The word ‘MATHEMATICS’ contains 11 letters in total, with certain letters like A appearing multiple times, each considered a distinct outcome when selecting one letter. The vowels present in the word are A, E, A, and I, which together appear in four distinct positions out of the total eleven letters. Consequently, the probability of choosing a vowel at random from the letters of ‘MATHEMATICS’ is P(vowel) = 4/11, reflecting the ratio of vowel occurrences to the total letters.

Step by step

1. **Count the total number of letter positions in ‘MATHEMATICS’.:** MATHEMATICS has the letters M, A, T, H, E, M, A, T, I, C, S. Counting these gives 11 letters in total, so n(S) = 11. Why: When one letter is chosen at random from the word, each letter position is treated as a possible elementary outcome.
2. **Identify and count the vowel occurrences in the word.:** The vowels in MATHEMATICS are A, E, A and I. Therefore, the number of favourable outcomes is n(E) = 4. Why: Repeated letters must be counted by occurrence, so the two A’s contribute two separate favourable positions.
3. **Compute the probability of selecting a vowel.:** P(vowel) = n(E)/n(S) = 4/11. Thus, the probability that the chosen letter is a vowel is 4/11. Why: For equally likely selections, probability is obtained by dividing the number of favourable letter positions by the total letter positions.

Topic and concept

Simple real-life problems on finding the probability of an event. To find the probability of getting a head in a coin toss or a particular face in a die roll, list the total possible outcomes and count those that fit the event.

How marks are earned

1. The word ‘MATHEMATICS’ contains 11 letters in total, with certain letters like A appearing multiple times, each considered a distinct outcome when selecting one letter. (1 mark)
2. The vowels present in the word are A, E, A, and I, which together appear in four distinct positions out of the total eleven letters. (1 mark)
3. Consequently, the probability of choosing a vowel at random from the letters of ‘MATHEMATICS’ is P(vowel) = 4/11, reflecting the ratio of vowel occurrences to the total letters. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 14, §14.2: Probability is determined by dividing the number of favourable equally likely outcomes by the total number of equally likely outcomes.

Section E

Question 36

4 marks

Case Based

A surveyor wants to determine the height of a tall tree without climbing it. She places a 2 m staff vertically on level ground and measures its shadow as 3 m at a particular time. At that same time, the tree's shadow measures 15 m. Assuming the sun's rays are parallel, she applies the principle of similar triangles to find the tree's height.

(a) Refer to the scenario: A surveyor places a 2 m staff vertically on level ground. At a certain instant, the staff's shadow measures 3 m and the shadow of a nearby tree measures 15 m. Assuming the sun's rays are parallel, identify the two triangles that are similar. [1 mark]

Answer

The right triangle formed by the 2 m staff and its 3 m shadow and the right triangle formed by the tree and its 15 m shadow are similar.

Step by step

1. **Identify the two right triangles and the angles that make them similar:** The 2 m staff and its 3 m shadow form one right triangle; the tree and its 15 m shadow form the other. Each has a 90° angle where the vertical object meets level ground. Each also has the same angle at the shadow tip, because the parallel sun rays make the same elevation angle with the ground at the same instant. Two equal pairs of angles give AA similarity, so the staff-shadow triangle is similar to the tree-shadow triangle. Why: If two triangles have equal corresponding angles, their sides are in proportion. The right angle and the common sun-elevation angle are enough to prove similarity without knowing any side ratio first. This is why the staff can stand in for the tree.

Topic and concept

Applications of similarity criteria to prove triangles similar and indirect measurement. Indirect measurement uses similar triangles to find lengths that cannot be measured directly. The vertical staff and the tree each form a right triangle with their shadow, and the parallel sun rays give both the same elevation angle. Two equal angles (AA) make the triangles similar, so their corresponding sides are proportional.

How marks are earned

1. The right triangle formed by the 2 m staff and its 3 m shadow and the right triangle formed by the tree and its 15 m shadow are similar. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 6, §6.3: If two triangles have equal corresponding angles, their sides are in proportion.

(b) Using the similar triangles from the scenario, write the correct proportional relation connecting the staff height (2 m), tree height (h), staff shadow (3 m), and tree shadow (15 m). [1 mark]

Answer

2 : h = 3 : 15, i.e., 2/h = 3/15.

Step by step

1. **Match corresponding sides of the similar triangles and write the ratio:** The staff corresponds to the tree, since both are the vertical sides opposite the sun's elevation angle. The staff's shadow corresponds to the tree's shadow, since both lie on the ground adjacent to that angle. Equating the ratios of corresponding sides gives staff : tree = shadow : shadow, that is 2 : h = 3 : 15, or 2/h = 3/15. The equivalent forms h/2 = 15/3 or 2/3 = h/15 are also correct. Why: In similar triangles, ratios of corresponding sides are equal. The relation is correct only if height is paired with height and shadow with shadow, in the same order on both sides. Inverting only one side breaks the correspondence.

Topic and concept

Applications of similarity criteria to prove triangles similar and indirect measurement. In similar triangles the ratios of corresponding sides are equal. Here staff corresponds to tree and staff shadow to tree shadow, so staff height : tree height = staff shadow : tree shadow. This proportion lets us find the unknown height h.

How marks are earned

1. 2 : h = 3 : 15, i.e., 2/h = 3/15. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 6, §6.3: In similar triangles, ratios of corresponding sides are equal.

Attempt one of the final two-mark options.

Option A

(c) (i) Calculate the height of the tree based on the proportionality you have written. (ii) Which concept from Chapter 6 justifies the use of proportional triangles to find the tree’s height indirectly? [2 marks]

Answer

From 2/h = 3/15, cross-multiply to get 2×15 = 3×h, so 30 = 3h and h = 10 m. The application of similarity of triangles for indirect measurement (Concept CH6-C08) justifies this method.

Step by step

1. **Solve the proportion 2/h = 3/15 for h, with units:** Start from 2/h = 3/15. Cross-multiplying gives 2 × 15 = 3 × h, so 30 = 3h. Dividing both sides by 3 gives h = 30 ÷ 3 = 10 m. Check: 3/15 = 1/5 and 2/10 = 1/5, so the ratios match. The tree is 10 m tall. Why: Solving the proportionality gives the unknown length. Cross-multiplication is valid because the two ratios are equal, and the answer is a length, so it needs the unit metres.

2. **Name the Chapter 6 idea that justifies the method:** The method rests on the AA criterion for similarity of triangles: the staff and tree triangles have equal angles, so their corresponding sides are proportional. This lets the known staff measurements give the unreachable tree height without climbing the tree. This is the application of similarity of triangles to indirect measurement (concept CH6-C08). Why: Similarity lets us find lengths indirectly using proportional triangles. Without the equal-angle condition the sides would not be proportional and the calculation would not be justified.

Topic and concept

Applications of similarity criteria to prove triangles similar and indirect measurement. Similar triangles have equal corresponding angles and proportional corresponding sides. Because the sun's rays are parallel, the staff and the tree form similar right triangles with their shadows. The known ratio 2 : 3 therefore gives the tree's height from its 15 m shadow.

How marks are earned

1. From 2/h = 3/15, cross-multiply to get 2×15 = 3×h, so 30 = 3h and h = 10 m. (1 mark)
2. The application of similarity of triangles for indirect measurement (Concept CH6-C08) justifies this method. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 6, §6.3: Solving the proportionality gives the unknown length. Check the reasoning against NCERT Class 10 maths, Ch. 6, §6.5: Similarity lets us find lengths indirectly using proportional triangles.

OR

Option B

(c) Another tree casts a 9 m shadow at the same instant. Calculate its height and state why the staff ratio applies. [2 marks]

Answer

Parallel sunlight makes the staff and tree triangles similar, so height/shadow = 2/3 for both. For a 9 m shadow, height = (2/3) × 9 m = 6 m.

Step by step

1. **Explain why the staff ratio applies and write the proportion:** The sun's rays are parallel, so at the same instant they make the same angle with the ground. The staff (2 m, shadow 3 m) and the tree each form a right triangle with their shadow, and these triangles are similar by AA, since both have a right angle and the same sun angle. So height/shadow = 2/3 for both, and for the new tree h/9 = 2/3. Why: In similar triangles corresponding sides are in the same ratio. Height and shadow are corresponding sides, so their ratio is the same for the staff and the tree.
2. **Solve the proportion and check the unit:** h = (2/3) × 9 m = 18/3 m = 6 m. Check: 6/9 = 2/3, which matches the staff ratio 2/3. The height of the tree is 6 m. Why: Cross-multiplying 2/3 = h/9 gives 3h = 18, so h = 6. The shadow was in metres, so the height is in metres, and the check shows the tree has the same height-to-shadow ratio as the staff.

Topic and concept

Applications of similarity criteria to prove triangles similar and indirect measurement. Similar triangles allow indirect measurement. Parallel sunlight at one instant gives the same angle of elevation for all vertical objects, so the ratio of height to shadow length is constant. One known object (the staff) then gives the height of another from its shadow.

How marks are earned

1. Parallel sunlight makes the staff and tree triangles similar, so height/shadow = 2/3 for both. (1 mark)
2. For a 9 m shadow, height = (2/3) × 9 m = 6 m. (1 mark)

Source check: Check the passage facts and NCERT Class 10 maths, Ch. 6, §6.3 against both steps.

Question 37

4 marks

Case Based

In a classroom activity, students are given two composite numbers, 180 and 210. They factorise each number completely into primes and record: 180 = 2^2 × 3^2 × 5; 210 = 2 × 3 × 5 × 7. They will then use these factorisations to determine the highest common factor (HCF) and the least common multiple (LCM).

(a) Refer to the activity described in the case. Which prime numbers appear in the factorisations of both 180 and 210? [1 mark]

Answer

The primes common to both 180 and 210 are 2, 3, and 5.

Step by step

1. **Compare the prime factors shown for 180 and 210 and take the common primes.:** Given 180 = 2^2 × 3^2 × 5, its prime factors are 2, 3 and 5. Given 210 = 2 × 3 × 5 × 7, its prime factors are 2, 3, 5 and 7. The primes appearing in both lists are 2, 3 and 5. Why: Prime factorisation lets us identify common prime factors by matching the primes that occur in both numbers.

Topic and concept

Fundamental Theorem of Arithmetic — statement, illustration and applications through prime factorisation. To find the HCF and LCM of two numbers, compare their prime-factor powers: choose the minimum exponents for HCF and the maximum for LCM.

How marks are earned

1. The primes common to both 180 and 210 are 2, 3, and 5. (1 mark)

Source check: Check the reasoning against NCERT Class 10 mathematics, Ch. 1, §1.2: Use prime factorisation to list all prime factors of a given number.

(b) According to the case, which exponents of the common primes determine the HCF of 180 and 210? [1 mark]

Answer

For each common prime, choose the smaller exponent: for 2 choose 2^1, for 3 choose 3^1, and for 5 choose 5^1.

Step by step

1. **Compare the powers of the primes that occur in both given factorisations.:** From 180 = $2^2 \times 3^2 \times 5$ and 210 = $2^1 \times 3^1 \times 5^1 \times 7$, the common primes are 2, 3 and 5. Taking the smaller exponent for each common prime gives 2^min(2,1) = 2^1, 3^min(2,1) = 3^1, and 5^min(1,1) = 5^1. Why: The HCF uses only common prime factors, and for each such factor the lower power is the greatest power that can divide both numbers.

Topic and concept

Fundamental Theorem of Arithmetic — statement, illustration and applications through prime factorisation. Every integer greater than one is either prime or can be factored into primes in exactly one way, up to ordering of factors.

How marks are earned

1. For each common prime, choose the smaller exponent: for 2 choose 2^1, for 3 choose 3^1, and for 5 choose 5^1. (1 mark)

Source check: Check the reasoning against NCERT Class 10 mathematics, Ch. 1, §1.2: Choose the minimum exponent of each common prime factor for HCF.

Attempt one of the final two-mark options.

Option A

(c) (i) Using the information in the case, calculate the HCF of 180 and 210. (ii) Using the case data, find the LCM of 180 and 210. [2 marks]

Answer

HCF = 2^1 × 3^1 × 5^1 = 2 × 3 × 5 = 30. Choose the highest exponents: 2^2, 3^2, 5^1, 7^1; LCM = 2^2 × 3^2 × 5 × 7 = 4 × 9 × 5 × 7 = 1260.

Step by step

1. **Compare the exponents of each prime in 180 and 210, and take the minimum of the common primes for the HCF.:** For 180 = $2^2 \times 3^2 \times 5$ and 210 = $2^1 \times 3^1 \times 5^1 \times 7^1$, the exponents are: 2 → (2,1), min 1; 3 → (2,1), min 1; 5 → (1,1), min 1; 7 → (0,1), min 0. HCF = $2^1 \times 3^1 \times 5^1 = 2 \times 3 \times 5 = 30$. Why: The HCF is the largest number dividing both, so it can use each prime only as many times as both numbers have it. That is the minimum exponent. 7 divides only 210, so its minimum is 0 and it is left out.

2. **Take the maximum exponent of every prime for the LCM and multiply.:** Maximum exponents: 2 → 2, 3 → 2, 5 → 1, 7 → 1. LCM = $2^2 \times 3^2 \times 5 \times 7 = 4 \times 9 \times 5 \times 7 = 36 \times 35 = 1260$. Check: HCF × LCM = 30 × 1260 = 37800 = 180 × 210. Why: The LCM must be a multiple of both numbers, so it needs each prime as many times as the larger exponent. 7 appears because 210 needs it, even though 180 does not. The product check confirms both results.

Topic and concept

Fundamental Theorem of Arithmetic — statement, illustration and applications through prime factorisation. By unique prime factorisation, 180 and 210 each have exactly one prime-power form. Writing both with the same primes lets us compare exponents: the minimum exponent of each prime gives the HCF and the maximum gives the LCM. A prime missing from one number has exponent 0 there.

How marks are earned

1. HCF = 2^1 × 3^1 × 5^1 = 2 × 3 × 5 = 30. (1 mark)
2. Choose the highest exponents: 2^2, 3^2, 5^1, 7^1; LCM = 2^2 × 3^2 × 5 × 7 = 4 × 9 × 5 × 7 = 1260. (1 mark)

Source check: Check the reasoning against NCERT Class 10 mathematics, Ch. 1, §1.2: Multiply the selected prime powers to find the HCF. Check the reasoning against NCERT Class 10 mathematics, Ch. 1, §1.2: Select the maximum exponent of each prime factor for LCM and multiply.

OR

Option B

(c) Verify the relation HCF(180,210) × LCM(180,210) = 180 × 210 using the values found from the prime factors. [2 marks]

Answer

Using HCF=30 and LCM=1260, their product is 30×1260=37800. The original numbers give 180×210=37800, so HCF×LCM equals the product of the numbers.

Step by step

1. **Multiply the HCF and LCM obtained from the prime-factor method.:** Using HCF = 30 and LCM = 1260, HCF × LCM = 30 × 1260 = 3 × 1260 × 10 = 3780 × 10 = 37800. Why: Calculating the left-hand side of the required relation first gives the exact value to be compared with the product of the original numbers.
2. **Multiply the original two numbers and compare the two products.:** 180 × 210 = 180 × (200 + 10) = 36000 + 1800 = 37800. Since 30 × 1260 = 37800 and 180 × 210 = 37800, HCF(180,210) × LCM(180,210) = 180 × 210. Why: Both sides have been evaluated independently and give the same number, so the stated relation is verified for 180 and 210.

Topic and concept

Fundamental Theorem of Arithmetic — statement, illustration and applications through prime factorisation. For two positive integers, their HCF times LCM equals their product; verify both products using the prime-factor results.

How marks are earned

1. Using HCF=30 and LCM=1260, their product is 30×1260=37800. (1 mark)
2. The original numbers give 180×210=37800, so HCF×LCM equals the product of the numbers. (1 mark)

Source check: Check the passage facts and NCERT Class 10 mathematics, Ch. 1, §1.2 against both steps.

Question 38

4 marks

Case Based

A rectangular garden has length exceeding its breadth by 4 m. The garden's area is 45 m². If the breadth is x metres, then the length is (x + 4) metres, leading to the equation x(x + 4) = 45, which simplifies to $x^2 + 4x - 45 = 0$.

(a) Write the equation x(x + 4) = 45 in the standard form $ax^2 + bx + c = 0$ as presented in the passage. [1 mark]

Answer

Rewriting x(x + 4) = 45 yields $x^2 + 4x - 45 = 0$.

Step by step

1. **Expand the product and move all terms to one side:** Expand the left side: $x(x + 4) = x^2 + 4x$, so the equation becomes $x^2 + 4x = 45$. Subtract 45 from both sides to get $x^2 + 4x - 45 = 0$. Here a = 1, b = 4 and c = −45. Why: Any quadratic equation can be written in the form $ax^2 + bx + c = 0$ with $a \neq 0$. Terms are written in descending powers of x and the constant is moved across with its sign changed so the right side is zero.

Topic and concept

Standard form $ax^2 + bx + c = 0$ ($a \neq 0$); real roots and their existence. A quadratic equation is a second-degree polynomial set equal to zero, $ax^2 + bx + c = 0$ with $a \neq 0$. Expanding x(x + 4) gives a term in x^2, so the garden equation is quadratic. Moving 45 to the left puts it in standard form.

How marks are earned

1. Rewriting x(x + 4) = 45 yields $x^2 + 4x - 45 = 0$. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 4, §4.2: Any quadratic equation can be written in the form $ax^2 + bx + c = 0$ with $a \neq 0$.

(b) From the standard form $x^2 + 4x - 45 = 0$ given in the passage, state the values of a, b and c. [1 mark]

Answer

Comparing $x^2 + 4x - 45 = 0$ with $ax^2 + bx + c = 0$ gives a = 1, b = 4 and c = −45.

Step by step

1. **Compare each term of the given quadratic with the standard form.:** Standard form: $ax^2 + bx + c = 0$. Given equation: $x^2 + 4x - 45 = 0 = 1x^2 + 4x + (-45)$. Therefore, a = 1, b = 4 and c = −45. Why: In the standard quadratic form, a is the coefficient of x^2, b is the coefficient of x, and c is the constant term; matching corresponding terms identifies them directly.

Topic and concept

Compare the given equation term by term with the standard quadratic form $ax^2 + bx + c = 0$, where $a \neq 0$.. In $ax^2 + bx + c = 0$, a is the coefficient of x^2, b is the coefficient of x, and c is the constant term. Write x^2 as $1x^2$; in $x^2 + 4x - 45 = 0$, this gives a = 1, b = 4, c = −45.

How marks are earned

1. Comparing $x^2 + 4x - 45 = 0$ with $ax^2 + bx + c = 0$ gives a = 1, b = 4 and c = −45. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 4, §4.2: In $ax^2 + bx + c = 0$, the coefficients a, b and constant term c are identified directly by comparison.

Attempt one of the final two-mark options.

Option A

(c) (i) Factorise the quadratic $x^2 + 4x - 45$ from the passage into two linear factors. (ii) Compute the discriminant of $x^2 + 4x - 45 = 0$ and state what it tells you about the roots. [2 marks]

Answer

$x^2 + 4x - 45$ factors as $(x + 9)(x - 5)$. $D = 4^2 - 4 \cdot 1 \cdot (-45) = 16 + 180 = 196 > 0$, so the equation has two distinct real roots.

Step by step

1. **Factorise $x^2 + 4x - 45$ by splitting the middle term:** Here a = 1, b = 4, c = −45, so ac = −45. We need two numbers with product −45 and sum 4: these are 9 and −5. Split the middle term: $x^2 + 9x - 5x - 45 = x(x + 9) - 5(x + 9) = (x + 9)(x - 5)$. Setting each factor to zero gives roots x = −9 and x = 5. Check: $(x + 9)(x - 5) = x^2 - 5x + 9x - 45 = x^2 + 4x - 45$. Why: If $ax^2 + bx + c$ can be written as a product of two linear factors, each factor set to zero gives a root. Splitting the middle term finds those factors systematically by using the product and sum of the two numbers.
2. **Compute the discriminant and interpret its sign:** With a = 1, b = 4, c = −45: $D = b^2 - 4ac = 4^2 - 4(1)(-45) = 16 + 180 = 196$. Since 196 > 0 (and $196 = 14^2$), the equation has two distinct real roots, which agree with −9 and 5 found by factorising. Why: The discriminant $D = b^2 - 4ac$ decides the nature of the roots: D > 0 gives two distinct real roots, D = 0 two equal real roots, and D < 0 no real roots. The double negative in −4ac with c negative makes the 4ac term add to 16.

Topic and concept

Solution by factorisation. A quadratic that factorises into two linear factors has roots given by setting each factor to zero. The discriminant D = b² − 4ac tells the nature of the roots without solving: D > 0 means two distinct real roots. In the garden problem, the roots are −9 and 5, and the breadth x = 5 m is the valid one.

How marks are earned

1. $x^2 + 4x - 45$ factors as $(x + 9)(x - 5)$. (1 mark)
2. $D = 4^2 - 4 \cdot 1 \cdot (-45) = 16 + 180 = 196 > 0$, so the equation has two distinct real roots. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 4, §4.3: A quadratic $ax^2 + bx + c$ can be expressed as a product of two linear factors whose zeros are the roots. Check the reasoning against NCERT Class 10 maths, Ch. 4, §4.4: The discriminant $D = b^2 - 4ac$ determines if a quadratic has two distinct real, equal, or no real roots.

OR

Option B

(c) Find the physically possible breadth and length of the garden from $x^2+4x-45=0$. [2 marks]

Answer

Factorisation gives (x+9)(x−5)=0, so x=−9 or x=5; breadth cannot be negative. The breadth is 5 m and the length is x+4=9 m, whose area is 45 m².

Step by step

1. **Solve $x^2 + 4x - 45 = 0$ by splitting the middle term, and reject the root that is not physically possible.:** Find two numbers with product −45 and sum 4: they are 9 and −5. So $x^2 + 9x - 5x - 45 = x(x + 9) - 5(x + 9) = (x + 9)(x - 5) = 0$, giving x = −9 or x = 5. Breadth cannot be negative, so x = −9 is rejected and x = 5. Why: A quadratic that factors into two linear factors has roots where each factor is zero. A breadth is a length, so it must be positive. The root x = −9 solves the equation but cannot describe a garden.
2. **Find the length and verify it against the given area.:** Breadth = 5 m, so length = x + 4 = 5 + 4 = 9 m. Check: area = 5 m × 9 m = 45 m², which matches the given area. Why: The case defines the length as the breadth plus 4 m, so the length follows from the accepted root. The area check confirms that this root satisfies the original equation x(x + 4) = 45.

Topic and concept

Solution by factorisation. A quadratic $ax^2 + bx + c = 0$ can be solved by writing it as a product of two linear factors and setting each to zero. Both roots satisfy the equation, but the context decides which is valid. A breadth must be positive, so only x = 5 is accepted.

How marks are earned

1. Factorisation gives (x+9)(x−5)=0, so x=−9 or x=5; breadth cannot be negative. (1 mark)
2. The breadth is 5 m and the length is x+4=9 m, whose area is 45 m². (1 mark)

Source check: Check the passage facts and NCERT Class 10 maths, Ch. 4, §4.2 against both steps.