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Class X practice studio

CLASS X · 2026-27 · MODEL PAPER

# Class X Mathematics Standard (Code 041) Model Paper 3

## Worked answers & explanations

Maximum marks: 80

Time: 3 hours

Subject code: 041

Practice, understand, improve.

### Instructions

All 38 questions are compulsory, with one selected option for each internal choice.

Section A has 18 MCQs and 2 assertion-reason questions; Section E has 1+1+2-mark case subparts.

Calculators are not allowed; draw labelled figures where needed.

# Section A

### Question 1

**1 mark**

Mcq

Choose the correct answer.

**(a)** A rectangular garden has length 3 m more than its breadth. If its area is 18 m², which quadratic equation models the scenario? [1 mark]

**A.** $x^2 + 3x - 18 = 0$

**B.** $x^2 - 3x - 18 = 0$

**C.** $x^2 + x - 18 = 0$

**D.** $3x^2 + x - 18 = 0$

#### Answer

**A. $x^2 + 3x - 18 = 0$**

#### Step by step

1. **Select and justify the option:** Let breadth = x, length = x + 3, so area x(x + 3) = 18 gives $x^2 + 3x - 18 = 0$. Why: A real-life area problem is converted into a quadratic equation by defining variables and equating the product to the given area.

#### Topic and concept

**Situational problems based on quadratic equations in day-to-day contexts.** Real-world problems, such as finding dimensions from area or product constraints, can be modelled by quadratic equations by defining a variable and equating the derived expression to the given value.

#### How marks are earned

1. Let breadth = x, length = x + 3, so area x(x + 3) = 18 gives $x^2 + 3x - 18 = 0$. (1 mark)

**Source check:** Check the reasoning against NCERT Class 10 maths, Ch. 4, §4.1 Introduction: A real-life area problem is converted into a quadratic equation by defining variables and equating the product to the given area.

## Question 2

**1 mark**

Mcq

Choose the correct answer.

**(a)** In ΔABC and ΔPQR, AB/PQ = AC/PR and ∠A = ∠P. Which criterion justifies ΔABC ~ ΔPQR? [1 mark]

**A.** AA
**B.** SSS
**C.** SAS
**D.** RHS

### Answer

**C. SAS**

### Step by step

1. **Match proportional sides and included angle:** AB/PQ = AC/PR and the included angles ∠A and ∠P are equal, so the two triangles are similar by SAS. Why: SAS similarity uses two corresponding proportional sides and the angle between those sides.

### Topic and concept

**Criteria for similarity of triangles — AA, SSS, SAS.** The SAS similarity criterion applies when two pairs of corresponding sides are proportional and the included angles are equal. Here AB ↔ PQ, AC ↔ PR, and ∠A = ∠P.

### How marks are earned

1. When two sides are proportional and the included angles are equal, triangles are similar by SAS criterion. (1 mark)

**Source check:** Check the reasoning against NCERT Class 10 maths, Ch. 6, §6.5 Summary: SAS criterion: Two sides proportional with included angle equal implies similarity.

## Question 3

**1 mark**

Mcq

Choose the correct answer.

**(a)** Which identity follows from $\sin^2\theta + \cos^2\theta = 1$ when divided by $\cos^2\theta$? [1 mark]

**A.** $1 + \tan^2\theta = \sec^2\theta$

**B.** $1 + \cot^2\theta = \text{cosec}^2\theta$

**C.** $\tan^2\theta + 1 = \text{cosec}^2\theta$

**D.** $\sin^2\theta + \sec^2\theta = 1$

### Answer

**A. $1 + \tan^2\theta = \sec^2\theta$**

### Step by step

1. **Divide every term of $\sin^2\theta + \cos^2\theta = 1$ by $\cos^2\theta$ and match the result to an option:** $\sin^2\theta/\cos^2\theta + \cos^2\theta/\cos^2\theta = 1/\cos^2\theta$. Here $\sin^2\theta/\cos^2\theta = (\sin\theta/\cos\theta)^2 = \tan^2\theta$, $\cos^2\theta/\cos^2\theta = 1$ and $1/\cos^2\theta = \sec^2\theta$. So the result is $\tan^2\theta + 1 = \sec^2\theta$, which is $1 + \tan^2\theta = \sec^2\theta$, option A. Why: The identity $\sin^2\theta + \cos^2\theta = 1$ holds for every acute angle, so dividing both sides by the same non-zero quantity $\cos^2\theta$ keeps it true. Each term then becomes a defined ratio (tan or sec). The nearest plausible choice, B ($1 + \cot^2\theta = \text{cosec}^2\theta$), is also a true identity. It comes from dividing by $\sin^2\theta$, not $\cos^2\theta$, so the stated division selects only A. C wrongly gives $\text{cosec}^2\theta$, and D mixes $\sin^2\theta$ with $\sec^2\theta$.

### Topic and concept

**Determining other trigonometric ratios when one is known.** Dividing the basic identity $\sin^2\theta + \cos^2\theta = 1$ by $\cos^2\theta$ turns each term into a ratio: $\sin\theta/\cos\theta$ is $\tan\theta$ and $1/\cos\theta$ is $\sec\theta$. This gives a new identity linking $\tan\theta$ and $\sec\theta$. It lets one ratio be found from another, as in the Class X idea that a known ratio of an acute angle fixes the rest.

### How marks are earned

1. Dividing $\sin^2\theta + \cos^2\theta = 1$ by $\cos^2\theta$ yields $1 + \tan^2\theta = \sec^2\theta$. (1 mark)

**Source check:** Check the reasoning against NCERT Class 10 maths, Ch. 8, §8.4: Dividing $\sin^2\theta + \cos^2\theta = 1$ by $\cos^2\theta$ gives $1 + \tan^2\theta = \sec^2\theta$.

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## Question 4

**1 mark**

**Mcq**

Choose the correct answer.

**(a)** For which set of coefficients will the equation $x^2 + 4x + 4 = 0$ have equal real roots? [1 mark]

**A.** a=1, b=4, c=4

**B.** a=1, b=5, c=6

**C.** a=2, b=3, c=1

**D.** a=1, b=3, c=1

### Answer

**A. a=1, b=4, c=4**

### Step by step

1. **Compute the discriminant D = b² − 4ac for each option and find the one with D = 0:** A: D = 4² − 4(1)(4) = 16 − 16 = 0. B: D = 5² − 4(1)(6) = 25 − 24 = 1. C: D = 3² − 4(2)(1) = 9 − 8 = 1. D: D = 3² − 4(1)(1) = 9 − 4 = 5. Only option A has D = 0, so only A gives equal real roots. Check: x² + 4x + 4 = (x + 2)², so the repeated root is x = −2. Why: The discriminant b² − 4ac decides the nature of the roots of ax² + bx + c = 0. D > 0 gives two distinct real roots, D = 0 gives two equal real roots and D < 0 gives no real roots. B, C and D have positive discriminants, so their roots are real but distinct. Distinct roots are the nearest plausible mistake, so each option must be checked rather than assuming any D ≥ 0 will do.

### Topic and concept

**Nature of roots from discriminant (b²−4ac): two distinct real, two equal, no real roots.** For a quadratic ax² + bx + c = 0, the discriminant D = b² − 4ac shows the root type without solving. D = 0 means two equal real roots, D > 0 means two distinct real roots and D < 0 means no real roots. Here the question is read as asking which coefficient set has D = 0.

### How marks are earned

1. When a=1, b=4, c=4, the discriminant D = 4² − 4·1·4 = 16 − 16 = 0, so the equation has equal real roots. (1 mark)

**Source check:** Check the reasoning against NCERT Class 10 maths, Ch. 4, §4.4: A zero discriminant (b² − 4ac = 0) yields two equal real roots.

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## Question 5

**1 mark**

Mcq

Choose the correct answer.

**(a)** Which statement correctly describes the relationship between congruence and similarity? [1 mark]

**A.** All similar figures are congruent

**B.** All congruent figures are similar

**C.** Similar figures have equal size but not necessarily the same shape

**D.** Congruent figures may not be similar

### Answer

**B. All congruent figures are similar**

### Step by step

1. **Select and justify the option:** Every congruent figure has the same shape and size, so it meets the definition of similarity. Why: All congruent figures are similar but similar figures need not be congruent.

### Topic and concept

**Similar figures; definition of similar triangles.** Two figures are similar if they have the same shape but not necessarily the same size; in triangles this means corresponding angles are equal and corresponding sides are proportional.

### How marks are earned

1. Every congruent figure has the same shape and size, so it meets the definition of similarity. (1 mark)

**Source check:** Check the reasoning against NCERT Class 10 maths, Ch. 6, §6.2: All congruent figures are similar but similar figures need not be congruent.

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## Question 6

**1 mark**

Mcq

Choose the correct answer.

(a) Using substitution on the equation x + 2y = 3, which expression for x is correct? [1 mark]

**A.** x = 3 + 2y

**B.** x = 2y – 3

**C.** x = 3 – 2y

**D.** x = y – 3

### Answer

**C. x = 3 – 2y**

### Step by step

1. **Select and justify the option:** Rearranging x + 2y = 3 gives x = 3 – 2y. Why: In substitution, one equation is solved for one variable in terms of the other.

### Topic and concept

**Algebraic methods: substitution and elimination.** Substitution method involves expressing one variable in terms of the other and substituting into the second equation.

### How marks are earned

1. Rearranging x + 2y = 3 gives x = 3 – 2y. (1 mark)

**Source check:** Check the reasoning against NCERT Class 10 maths, Ch. 3, §3.3.1: In substitution, one equation is solved for one variable in terms of the other.

## Question 7

**1 mark**

Mcq

Choose the correct answer.

**(a)** The LCM of 12 and 15 is: [1 mark]

**A.** 60

**B.** 30

**C.** 12

**D.** 15

### Answer

**A. 60**

### Step by step

1. **Prime-factorise 12 and 15, then take the highest power of every prime:** $12 = 2^2 \times 3$ and $15 = 3 \times 5$. The primes are 2, 3 and 5. The highest powers are $2^2$, $3^1$ and $5^1$. So LCM = $2^2 \times 3 \times 5 = 4 \times 3 \times 5 = 60$. Check: HCF = 3, and HCF × LCM = $3 \times 60 = 180 = 12 \times 15$. Why: By the Fundamental Theorem of Arithmetic, every integer above 1 has a unique prime factorisation. So the LCM can be built from the primes of each number. The LCM must be divisible by both numbers, so it takes the largest power of each prime that appears. Taking the smallest powers instead gives the HCF, 3. Option B, 30, is not a multiple of 12, and 12 and 15 are not multiples of each other.

### Topic and concept

**Fundamental Theorem of Arithmetic — statement, illustration and applications through prime factorisation.** Every integer greater than 1 is a product of primes in exactly one way, apart from order. Because of this unique factorisation, the LCM of two numbers is the product of the highest power of each prime in either factorisation. The HCF uses the lowest powers.

### How marks are earned

1. LCM uses the maximum exponents of each prime: $2^2 \times 3^1 \times 5^1 = 60$. (1 mark)

**Source check:** Check the reasoning against NCERT Class 10 maths, Ch. 1, §1.2: To find the LCM of two numbers, compare their prime-factor powers and choose the maximum exponents.

## Question 8

**1 mark**

Mcq

Choose the correct answer.

**(a)** For a minor segment of radius r whose central angle θ is measured in degrees, which expression

gives its area? [1 mark]

**A.** $(\theta/360)\pi r^2 + \frac{1}{2}r^2 \sin\theta$

**B.** $(\theta/360)\pi r^2 - \frac{1}{2}r^2 \sin\theta$

**C.** $(\theta/180)\pi r^2 - \frac{1}{2}r^2 \cos\theta$

**D.** $(\theta/360)\pi r^2$

### Answer

**B. $(\theta/360)\pi r^2 - \frac{1}{2}r^2 \sin\theta$**

### Step by step

1. **Subtract triangle area from sector area:** For θ measured in degrees, the sector has area $(\theta/360)\pi r^2$ and the triangle between the radii has area $\frac{1}{2}r^2\sin\theta$; their difference is option B. Why: A minor segment is precisely the part of its sector outside that triangle.

### Topic and concept

**Area of segment of a circle.** With θ in degrees, minor-segment area equals $(\theta/360)\pi r^2 - \frac{1}{2}r^2\sin\theta$. The sector formula would use $\frac{1}{2}r^2\theta$ instead if θ were in radians.

### How marks are earned

1. Area of the minor segment = sector area − triangle area = $(\theta/360)\pi r^2 - \frac{1}{2}r^2 \sin\theta$. (1 mark)

**Source check:** Check the reasoning against NCERT Class 10 maths, Ch. 11, §11.2 Summary: Area of segment = sector area − triangle area.

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## Question 9

**1 mark**

Mcq

Choose the correct answer.

**(a)** Which of the following events is impossible when rolling a fair six-sided die? [1 mark]

**A.** Getting 1

**B.** Getting 4

**C.** Getting an even number

**D.** Getting 7

### Answer

**D. Getting 7 is impossible because a six-sided die has faces numbered only 1 through 6.**

### Step by step

1. **List the sample space of a six-sided die and test each option against it:** The outcomes are {1, 2, 3, 4, 5, 6}. A: 1 is in the set, so it can occur. B: 4 is in the set, so it can occur. C: an even number means 2, 4 or 6, which are in the set, so it can occur. D: 7 is not in the set, so it cannot occur. Hence D is the impossible event. Why: An impossible event contains no outcome of the sample space, so its probability is 0. Options A, B and C each contain at least one outcome from the sample space, so their probabilities are greater than 0. Only D has none. The nearest error is to judge an event unlikely and call it impossible. Impossible means no outcome at all, not a small chance.

### Topic and concept

**Complementary events and their probability relation.** An impossible event can never occur because none of the outcomes in the sample space belong to it, so its probability is 0. For a standard die the sample space is {1, 2, 3, 4, 5, 6}, and any event is checked by seeing whether it contains any of these outcomes.

### How marks are earned

1. Select D because 7 is absent from the six-sided die sample space {1,2,3,4,5,6}. (1 mark)

**Source check:** Check the reasoning against NCERT Class 10 maths, Ch. 14, §Summary: An impossible event has probability zero and cannot occur.

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## Question 10

**1 mark**

Mcq

Choose the correct answer.

**(a)** According to the definition of similar triangles, two triangles are similar if they satisfy which of the following? [1 mark]

**A.** All corresponding angles are equal and all corresponding sides are proportional

**B.** All corresponding angles are equal but sides may not be proportional

**C.** All corresponding sides are proportional but angles may not be equal

**D.** They have the same perimeter and area

### Answer

**A. All corresponding angles are equal and all corresponding sides are proportional**

### Step by step

1. **Select and justify the option:** Similarity of triangles requires both equal corresponding angles and proportional corresponding sides. Why: Two triangles are similar if their corresponding angles are equal and sides are in the same ratio.

### Topic and concept

**Similar figures; definition of similar triangles.** Two figures are similar if they have the same shape but not necessarily the same size; in triangles this means corresponding angles are equal and corresponding sides are proportional.

### How marks are earned

1. Similarity of triangles requires both equal corresponding angles and proportional corresponding sides. (1 mark)

**Source check:** Check the reasoning against NCERT Class 10 maths, Ch. 6, §6.3: Two triangles are similar if their corresponding angles are equal and sides are in the same ratio.

## Question 11

**1 mark**

Mcq

Choose the correct answer.

**(a)** A cone and a hemisphere of the same radius are joined at their flat circular faces to make a toy. Which areas add to its exposed total surface area? [1 mark]

**A.** Curved area of the cone plus curved area of the hemisphere

**B.** Both curved areas plus both joined circular faces

**C.** Only the cone's curved area

**D.** Only the hemisphere's curved area

### Answer

**A. Curved area of the cone plus curved area of the hemisphere.**

### Step by step

1. **Decide which surfaces of the joined toy are exposed and select the matching option:** The cone and hemisphere have the same radius r and are joined at their flat circular faces. These two faces, each of area $\pi r^2$, press together inside the toy and are not exposed. The exposed surface is the cone's curved surface plus the hemisphere's curved surface. So total surface area = CSA of cone + CSA of hemisphere, which is option A. Why: The total surface area of a composite solid counts only the outer surface that can be seen or touched, because joined faces are hidden. That is why the curved areas are added and the flat faces are left out. Option B is the nearest error. It adds the joined circular faces, which would count the interior surface twice. C and D include only one solid, so they miss part of the exposed surface.

### Topic and concept

**Combinations are restricted to any two cubes, cuboids, spheres, hemispheres and right circular cylinders or cones.** When a cone and a hemisphere are joined at their flat faces, those faces become the interior of the toy. The total surface area of the toy is therefore the curved surface area of the cone plus the curved surface area of the hemisphere. Only the outside surface is counted.

### How marks are earned

1. Choose A: the common circular faces are hidden inside the toy, so only the cone and hemisphere curved areas are exposed. (1 mark)

**Source check:** NCERT Class 10 maths, Ch. 12, §12.2: the surface area of the cone-and-hemisphere toy adds their exposed curved surfaces.

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## Question 12

**1 mark**

**Mcq**

Choose the correct answer.

**(a)** Find the sum of the first 7 terms of the AP 4, 9, 14, ... . [1 mark]

**A.** 133

**B.** 136

**C.** 140

**D.** 129

### Answer

**A. 133**

### Step by step

1. **Select and justify the option:** $S_7 = (7/2)[2\times4 + (7-1)\times5] = (7/2)(8 + 30) = 133$. Why: $S_n = n/2[2a + (n-1)d]$ gives the sum without termwise addition.

### Topic and concept

**Sum of first n terms: $S_n = n/2(2a+(n-1)d) = n/2(a+l)$.** The sum of the first n terms of an AP is $S_n = n/2 \times [2a + (n-1)d]$, equivalently $n/2 \times (a + l)$.

### How marks are earned

1. $S_7 = (7/2)[2\times4 + (7-1)\times5] = (7/2)(8 + 30) = 133$. (1 mark)

**Source check:** Check the reasoning against NCERT Class 10 maths, Ch. 5, §5.4: $S_n = n/2[2a + (n-1)d]$ gives the sum without termwise addition.

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## Question 13

**1 mark**

**Mcq**

Choose the correct answer.

(a) Consider the system 2x+3y=5 and 4x+6y=10. Which statement is correct about its solutions? [1 mark]

**A.** Exactly one solution

**B.** No solution

**C.** Infinitely many solutions

**D.** Two solutions

### Answer

**C. Infinitely many solutions**

### Step by step

1. **Select and justify the option:** Here $a_1/a_2 = 2/4 = 1/2$, $b_1/b_2 = 3/6 = 1/2$ and $c_1/c_2 = 5/10 = 1/2$, so all three ratios are equal indicating infinitely many solutions. Why: If $a_1/a_2 = b_1/b_2 = c_1/c_2$, the pair of equations has infinitely many solutions.

### Topic and concept

**Graphical representation of pair of linear equations; consistent, inconsistent, dependent conditions.** The ratios of coefficients determine whether a linear pair has one, infinitely many, or no solutions.

### How marks are earned

1. Here $a_1/a_2 = 2/4 = 1/2$, $b_1/b_2 = 3/6 = 1/2$ and $c_1/c_2 = 5/10 = 1/2$, so all three ratios are equal indicating infinitely many solutions. (1 mark)

**Source check:** Check the reasoning against NCERT Class 10 maths, Ch. 3, §3.2: If $a_1/a_2 = b_1/b_2 = c_1/c_2$, the pair of equations has infinitely many solutions.

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## Question 14

**1 mark**

Mcq

Choose the correct answer.

(a) A 3 m tall pole casts a 4 m shadow. At the same time, a building casts a 20 m shadow. What is the height of the building? [1 mark]

**A.** 12 m

**B.** 15 m

**C.** 18 m

**D.** 20 m

### Answer

**B. 15 m**

### Step by step

1. **Show the pole and building triangles are similar, form the proportion and solve for the building height h:** At the same time the sun's rays make the same angle with the ground, and both the pole and the building stand at 90° to the ground, so the two right triangles are similar by AA. Pairing each height with its own shadow gives h/20 = 3/4. Then h = (3/4) × 20 = 60/4 = 15 m. Check: 15/20 = 0.75 = 3/4, so option B (15 m) is correct. Why: Similar triangles have equal ratios of corresponding sides, so height ÷ shadow is the same for the pole and the building; this is the indirect-measurement use of similarity. The proportion is valid only because the sun's angle is equal for both objects at the same time. The nearest plausible error is inverting the ratio, which gives (4/3) × 20 ≈ 26.7 m, and the distractor 12 m would come from multiplying 3 × 4 instead of scaling the pole by the shadow ratio 20/4 = 5.

### Topic and concept

**Applications of similarity criteria to prove triangles similar and indirect measurement.** Similarity of triangles lets us find a height that cannot be measured directly. At the same moment the sun's rays make equal angles with the ground, so a pole and a building with their shadows form similar right triangles. Corresponding sides are then in the same ratio, so height/shadow is equal for both: 3/4 = h/20.

### How marks are earned

1. By similarity, height/building : shadow/building = pole height : pole shadow ⇒ h/20 = 3/4, so h = (3/4)×20 = 15 m. (1 mark)

**Source check:** Check the reasoning against NCERT Class 10 maths, Ch. 6, §6.1: Indirect measurement uses similarity of triangles to find heights and distances.

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## Question 15

**1 mark**

Mcq

Choose the correct answer.

**(a)** How many tangents can be drawn from a point lying on the circle? [1 mark]

**A.** Zero

**B.** One

**C.** Two

**D.** Infinitely many

### Answer

**B. One**

### Step by step

1. **Identify the tangent count for a point on the circle and select the option.:** From a point lying on the circle, exactly one tangent can be drawn, so option B is correct; option C is the nearest plausible error because two tangents are drawn from an external point, not from a point on the circle. Why: At the point of contact on a circle, the tangent is the single line perpendicular to the radius at that point, so only one such tangent line exists there.

### Topic and concept

**Number of tangents from an external point; lengths of two tangents from an external point are equal (theorem + proof).** The concept tested is the number of tangents from a point on the circle: there is exactly one tangent at that point. This is different from the result about two equal tangents drawn from an external point.

### How marks are earned

1. Exactly one tangent can be drawn from a point on the circle. (1 mark)

**Source check:** Check the reasoning against NCERT Class 10 maths, Ch. 10, §10.3: There is exactly one tangent from a point on the circle.

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## Question 16

**1 mark**

Mcq

Choose the correct answer.

**(a)** Which trigonometric ratio is primarily used to relate the height of an object to its horizontal distance in heights and distances problems? [1 mark]

**A.** tan

**B.** sin

**C.** cos

**D.** cot

### Answer

**A. tan**

### Step by step

1. **Identify which sides of the right triangle the height and the horizontal distance are, and pick the ratio that links them:** In a heights and distances figure, the object's height is the side opposite the angle of elevation and the horizontal distance along the ground is the side adjacent to that angle. The ratio of opposite to adjacent is tan θ, so tan θ = height / horizontal distance, which makes option A (tan) correct. Why: A trigonometric ratio is chosen by the two sides it connects: tan θ is the only listed ratio that uses exactly opposite and adjacent, with no hypotenuse. The line of sight is the hypotenuse, so sin θ (opposite/hypotenuse) and cos θ (adjacent/hypotenuse) would each bring in that unknown length. The nearest plausible error is cot, which is 1/tan θ = adjacent/opposite; it also links the two sides but is the reciprocal and not the standard direct choice, since height is found by tan θ times distance.

### Topic and concept

**Angle of elevation and angle of depression; applications to heights and distances.** In a right triangle formed by an object, the ground and the line of sight, the height is opposite the angle of elevation and the ground distance is adjacent to it. Using tan θ = height / horizontal distance allows calculation of an unknown height or distance when the angle is known, without needing the hypotenuse.

### How marks are earned

1. The tangent ratio (tan θ = opposite/adjacent) relates height to horizontal distance in heights and distances problems. (1 mark)

**Source check:** Check the reasoning against NCERT Class 10 maths, Ch. 9, §9.2 Summary: Summary states that height or length of an object can be determined using trigonometric ratios.

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## Question 17

**1 mark**

**Mcq**

Choose the correct answer.

**(a)** Which pair of equations represents two parallel lines? [1 mark]

**A.** 2x+3y=5 and 4x+6y=10

**B.** 2x+3y=5 and 4x+6y=8

**C.** x–y=2 and 2x–2y=4

**D.** x+y=3 and 2x+2y=6

### Answer

**B. 2x+3y=5 and 4x+6y=8**

### Step by step

1. **Select and justify the option:** For the pair 2x+3y=5 and 4x+6y=8, $a_1/a_2$ = 2/4 = 1/2 and $b_1/b_2$ = 3/6 = 1/2 while $c_1/c_2$ = 5/8 ≠ 1/2, so the lines are parallel. Why: Parallel lines occur when $a_1/a_2 = b_1/b_2 \neq c_1/c_2$.

### Topic and concept

**Graphical representation of pair of linear equations; consistent, inconsistent, dependent conditions.** Intersecting, parallel, and coincident lines correspond to unique, no, and infinitely many solutions respectively.

### How marks are earned

1. For the pair 2x+3y=5 and 4x+6y=8, $a_1/a_2 = 2/4 = 1/2$ and $b_1/b_2 = 3/6 = 1/2$ while $c_1/c_2 = 5/8 \neq 1/2$, so the lines are parallel. (1 mark)

**Source check:** Check the reasoning against NCERT Class 10 maths, Ch. 3, §3.2: Parallel lines occur when $a_1/a_2 = b_1/b_2 \neq c_1/c_2$.

## Question 18

**1 mark**

Mcq

Choose the correct answer.

**(a)** Which of the following best describes the angle of elevation of an object? [1 mark]

**A.** The angle between the horizontal and the line of sight when looking at a point higher than the observer's eye level

**B.** The angle between the vertical and the line of sight when looking at a point higher than the observer's eye level

**C.** The angle between the horizontal and the line of sight when looking at a point lower than the observer's eye level

**D.** The angle between the vertical and the line of sight when looking at a point lower than the observer's eye level

### Answer

**A. The angle between the horizontal and the line of sight when looking at a point higher than the observer's eye level**

### Step by step

1. **Select and justify the option:** Angle of elevation is the angle formed by the observer's line of sight with the horizontal when the object is above the horizontal level. Why: Angle of elevation is the angle formed by the line of sight with the horizontal when the point viewed is above the horizontal level.

### Topic and concept

**Angle of elevation and angle of depression; applications to heights and distances.** Using tan θ = height / horizontal distance allows calculation of unknown heights or distances when the angle of elevation or depression is known.

### How marks are earned

1. Angle of elevation is the angle formed by the observer's line of sight with the horizontal when the object is above the horizontal level. (1 mark)

**Source check:** Check the reasoning against NCERT Class 10 maths, Ch. 9, Heights and Distances: Angle of elevation is the angle formed by the line of sight with the horizontal when the point viewed is above the horizontal level.

## Question 19

**1 mark**

Assertion Reason

Choose the correct answer.

**(a)** Assertion (A): Tangents drawn to a circle from an external point are equal in length. Reason (R): The triangles formed by the center, the external point, and the points of contact are right triangles congruent by the RHS criterion. [1 mark]

**A.** Both A and R true and R explains A

**B.** Both A and R true but R does not explain A

**C.** A true, R false

**D.** A false, R true

### Answer

**A. Both A and R true and R explains A**

### Step by step

1. **Verify both statements and connect the reason to the assertion.:** Assertion A is true because tangents PA and PB from an external point P to a circle are equal, and Reason R explains it: OA and OB are radii, OA $\perp$ PA and OB $\perp$ PB, OP is the common hypotenuse, so $\Delta OAP \cong \Delta OBP$ by RHS and therefore PA = PB by CPCT; hence option A is correct, not option B because the reason does explain the assertion. Why: RHS congruence applies to right triangles when their hypotenuse and one corresponding side are equal, and CPCT then gives equality of the corresponding tangent lengths.

### Topic and concept

**Number of tangents from an external point; lengths of two tangents from an external point are equal (theorem + proof).** Tangents drawn from an external point to a circle are equal in length. The standard Class X justification compares the two right triangles formed by the centre, the external point, and the two points of contact, and uses RHS congruence followed by CPCT.

### How marks are earned

1. Both assertion and reason are true, and the congruence of the two right triangles by RHS explains why the tangent lengths are equal. (1 mark)

**Source check:** Check the reasoning against NCERT Class 10 maths, Ch. 10, §10.4: The proof of equal tangent lengths uses congruent triangles by RHS.

## Question 20

**1 mark**

**Assertion Reason**

Choose the correct answer.

**(a)** Assertion (A): If all three pairs of corresponding sides of two triangles are proportional, then the triangles are similar. Reason (R): This is the SSS similarity criterion. [1 mark]

**A.** Both A and R true and R explains A

**B.** Both A and R true but R does not explain A

**C.** A true, R false

**D.** A false, R true

### Answer

**A. Both A and R true and R explains A**

### Step by step

1. **Select and justify the option:** Both Assertion and Reason are true, and the Reason identifies the SSS criterion that establishes similarity when all sides are in proportion. Why: SSS similarity: Corresponding sides in the same ratio imply triangles are similar.

### Topic and concept

**Criteria for similarity of triangles — AA, SSS, SAS.** If three pairs of corresponding sides of two triangles are in proportion, the triangles are similar (SSS criterion).

### How marks are earned

1. Both Assertion and Reason are true, and the Reason identifies the SSS criterion that establishes similarity when all sides are in proportion. (1 mark)

**Source check:** Check the reasoning against NCERT Class 10 maths, Ch. 6, §6.5 Summary: SSS similarity: Corresponding sides in the same ratio imply triangles are similar.

# Section B

### Question 21

**2 marks**

**Short Answer**

Attempt one option.

### Option A

Answer the question with working and reasons where needed.

**(a)** Using a 45°–45°–90° triangle, derive and state the exact value of cos 45° in simplest form. [2 marks]

#### Answer

**In a 45°–45°–90° triangle with equal legs a, the hypotenuse is a√2, so cos 45° = a/(a√2) = 1/√2. Rationalizing the denominator gives cos 45° = √2/2 in simplest form.**

#### Step by step

1. **Take a 45°–45°–90° triangle with equal legs a, find the hypotenuse by Pythagoras and write cos 45°:** In the 45°–45°–90° triangle the angles opposite the legs are equal, so the legs are equal, each a. Pythagoras gives $AC^2 = a^2 + a^2 = 2a^2$, so AC = a√2. Then cos 45° = adjacent/hypotenuse = a/(a√2) = 1/√2. Why: The two 45° angles make the legs equal, so only one length a is needed and the hypotenuse follows from the Pythagoras theorem. The ratio is the same for any a because a cancels, and this is why cos 45° is a fixed value. The nearest error is taking the hypotenuse as 2a or a instead of a√2.
2. **Rationalise the denominator to get the simplest form:** Multiply top and bottom by √2: 1/√2 × √2/√2 = √2/2. So cos 45° = √2/2, which is about 0.707 and equals 1/√2. Why: Simplest form needs a rational denominator, and multiplying by √2/√2 does this without changing the value, because it multiplies by 1. A check is that $(\sqrt{2}/2)^2 = 2/4 = 1/2$, which equals $(1/\sqrt{2})^2$.

#### Topic and concept

**Proof and application of $\sin^2\theta + \cos^2\theta = 1$.** In a right triangle with both acute angles equal to 45°, the legs are equal. The Pythagoras theorem then gives the hypotenuse as a√2. Cosine is adjacent over hypotenuse, so cos 45° = 1/√2 = √2/2, and it does not depend on the size of the triangle.

#### How marks are earned

1. In a 45°–45°–90° triangle with equal legs a, the hypotenuse is a√2, so cos 45° = a/(a√2) = 1/√2. (1 mark)
2. Rationalizing the denominator gives cos 45° = √2/2 in simplest form. (1 mark)

**Source check:** Check the reasoning against NCERT Class 10 maths, Ch. 8, §8.3: In a right triangle with both acute angles 45°, the sides satisfy cos 45° = 1/√2 = √2/2.

OR

**Option B**

Answer the question with working and reasons where needed.

**(a)** Express sec A first in terms of cos A and then in terms of the sides of a right triangle. [2 marks]

**Answer**

**sec A is defined as the reciprocal of cos A, so sec A = 1/cos A. Since cos A = (adjacent side)/(hypotenuse), sec A = (hypotenuse)/(adjacent side).**

**Step by step**

1. **Write sec A in terms of cos A using the reciprocal definition:** By definition, sec A is the reciprocal of cos A. So sec A = 1/cos A, valid when cos A ≠ 0. Why: The reciprocal ratios are defined from the basic ratios: cosec A = 1/sin A, sec A = 1/cos A and cot A = 1/tan A. Secant pairs with cosine, not sine. The common slip is to write sec A = 1/sin A, which is cosec A.
2. **Substitute the side ratio for cos A in a right triangle with acute angle A to get sec A in terms of sides:** In a right triangle with acute angle A, cos A = (adjacent side)/(hypotenuse). So sec A = 1/[(adjacent)/(hypotenuse)] = (hypotenuse)/(adjacent side). Check: the hypotenuse is the longest side, so sec A ≥ 1 for an acute angle. Why: The definition of cosine as adjacent over hypotenuse links the ratio to the triangle's sides. Taking the reciprocal of a fraction swaps numerator and denominator, so the hypotenuse moves to the top. This turns the reciprocal definition into a side ratio. The check that sec A ≥ 1 follows from the hypotenuse being the longest side.

**Topic and concept**

**Relationships between trigonometric ratios.** Cosec θ, sec θ and cot θ are the reciprocals of sin θ, cos θ and tan θ. For sec A, this means sec A = 1/cos A. Since cos A is adjacent over hypotenuse in a right triangle, sec A is hypotenuse over adjacent.

**How marks are earned**

1. sec A is defined as the reciprocal of cos A, so sec A = 1/cos A. (1 mark)
2. Since cos A = (adjacent side)/(hypotenuse), sec A = (hypotenuse)/(adjacent side). (1 mark)

**Source check:** Check the reasoning against NCERT Class 10 maths, Ch. 8, §8.2: sec A is defined as the reciprocal of cos A, which equals hypotenuse divided by adjacent side.

## Question 22

2 marks

Short Answer

Attempt one option.

### Option A

Answer the question with working and reasons where needed.

**(a)** State the step-deviation method formula for mean and explain the term $u_i$. [2 marks]

#### Answer

**The step-deviation method formula is $\bar{x} = a + h \times (\Sigma f_i u_i / \Sigma f_i)$. Here, $u_i = (x_i - a) / h$ scales each deviation by the class width h.**

#### Step by step

1. **State the step-deviation formula for the mean with its symbols:** $\bar{x} = a + h \times (\Sigma f_i u_i / \Sigma f_i)$, where a is the assumed mean (usually a class mark near the middle), h is the class width, $x_i$ is the class mark of the ith class and $f_i$ is its frequency. Why: The mean of grouped data comes from the class marks weighted by frequency. The step-deviation form moves the origin to a and the scale by h, so the mean is recovered by multiplying by h and adding a. The nearest error is forgetting to multiply by h or add a at the end.
2. **Define $u_i$ and explain how it simplifies the calculation:** $u_i = (x_i - a)/h$. Dividing each deviation $x_i - a$ by h gives small whole numbers such as −2, −1, 0, 1, 2 when the class widths are equal. So the products $f_i u_i$ are easy to add. Why: If all class widths equal h, each deviation $x_i - a$ is a multiple of h, so dividing by h gives small integers. This reduces the arithmetic, and the h and a in the formula undo the change. The nearest error is using the formula when class widths are unequal, because $u_i$ then does not become simple integers.

#### Topic and concept

**Mean of grouped data — direct method, assumed mean method, step-deviation method.** The step-deviation method finds the mean of grouped data by shifting to an assumed mean a and dividing deviations by the class width h. This makes the numbers smaller. The mean is then recovered with $\bar{x} = a + h \times (\Sigma f_i u_i / \Sigma f_i)$.

#### How marks are earned

1. The step-deviation method formula is $\bar{x} = a + h \times (\Sigma f_i u_i / \Sigma f_i)$. (1 mark)
2. Here, $u_i = (x_i - a) / h$ scales each deviation by the class width h. (1 mark)

**Source check:** Check the reasoning against NCERT Class 10 maths Ch. 13 Summary: The step-deviation method simplifies mean calculation by dividing each deviation by the class width.

OR

**Option B**

Answer the question with working and reasons where needed.

**(a)** In the following frequency distribution, determine the median class: Class intervals: 0–10 (f=5), 10–20 (f=8), 20–30 (f=6). [2 marks]

### Answer

**First, compute the total number of observations by adding frequencies 5, 8 and 6 to obtain n = 19 and then calculate half of this frequency as n/2 = 9.5. Next, determine the class in which the cumulative frequency first exceeds 9.5; since the cumulative frequency up to 10–20 becomes 5 + 8 = 13 which is greater than 9.5, the class 10–20 is the median class.**

### Step by step

1. **Find the total frequency n and half of it, n/2:** Add the frequencies: n = 5 + 8 + 6 = 19, so n/2 = 19 ÷ 2 = 9.5. The cumulative frequency table is: 0–10 has cf 5, 10–20 has cf 5 + 8 = 13, and 20–30 has cf 13 + 6 = 19. The last cf equals n = 19, which checks the addition. Why: The median is the middle value of the data. Half the observations, n/2 = 9.5, therefore sets the target that the cumulative frequency must reach. The median class is the class that contains that middle observation.
2. **Compare the cumulative frequencies with n/2 and pick the median class:** Compare each cf with 9.5. The first cf is 5, which is less than 9.5, so the 0–10 class is too early. The second cf is 13, which is at least 9.5, so 10–20 is the first class to reach or pass n/2. The median class is therefore 10–20. Why: The cumulative frequency of a class shows how many observations lie up to its upper limit. The first class where it reaches or exceeds n/2 must contain the middle observation, because the earlier classes have fewer than 9.5 observations. This is the class used later in Median = l + ((n/2 − cf)/f) × h.

### Topic and concept

**Median of grouped data — algebraic method.** For grouped data the median is found from cumulative frequencies. Here n = 19, so the middle position is 9.5. The cumulative frequencies 5, 13 and 19 show that the 10th observation lies in 10–20. That class is the median class, and l, cf, f and h would be read from it if the exact median were needed.

### How marks are earned

1. First, compute the total number of observations by adding frequencies 5, 8 and 6 to obtain n = 19 and then calculate half of this frequency as n/2 = 9.5. (1 mark)
2. Next, determine the class in which the cumulative frequency first exceeds 9.5; since the cumulative frequency up to 10–20 becomes 5 + 8 = 13 which is greater than 9.5, the class 10–20 is the median class. (1 mark)

**Source check:** Check the reasoning against NCERT Class 10 maths Ch. 13 §13.4: The median class is the one whose cumulative frequency just exceeds n/2.

## Question 23

**2 marks**

**Short Answer**

Answer the question with working and reasons where needed.

**(a)** Two positive numbers sum to 7 and their product is 10. Let x be one of the numbers; form the quadratic equation satisfied by x. [2 marks]

### Answer

**Let the numbers be x and 7 - x so that their product is x(7 - x) = 10. Rewrite as $-x^2 + 7x - 10 = 0$ and multiply by -1 to obtain $x^2 - 7x + 10 = 0$.**

### Step by step

1. **Express both numbers using one variable:** Let one number be x. The sum of the two numbers is 7, so the other number is 7 − x. Their product is given as 10, so x(7 − x) = 10. Why: A sum fixes the second number once the first is chosen, so 7 − x is the only way to write the second number with the same variable. Using one unknown turns the product condition into a single equation in x.
2. **Expand and bring to standard form:** Expand the left side: $7x - x^2 = 10$. Move everything to one side: $-x^2 + 7x - 10 = 0$. Multiply by −1 to get $x^2 - 7x + 10 = 0$. Check: x = 2 gives 4 − 14 + 10 = 0, and x = 5 gives 25 − 35 + 10 = 0, so the roots 2 and 5 satisfy it. They sum to 7 and have product 10. Why: A quadratic equation is written in the form $ax^2 + bx + c = 0$ with the $x^2$ term positive. Multiplying by −1 changes every sign without changing the solutions. The check confirms that the equation reproduces the original sum and product conditions.

### Topic and concept

**Situational problems based on quadratic equations in day-to-day contexts.** A word problem leads to a quadratic equation when we define a variable for an unknown and express the other quantity in terms of it. The given condition (here, the product 10) is then set equal to the resulting expression. In this problem the sum 7 gives the second number as 7 − x, and the product condition then forms x(7 − x) = 10.

### How marks are earned

1. Let the numbers be x and 7 - x so that their product is x(7 - x) = 10. (1 mark)
2. Rewrite as $-x^2 + 7x - 10 = 0$ and multiply by -1 to obtain $x^2 - 7x + 10 = 0$. (1 mark)

**Source check:** Check the reasoning against NCERT Class 10 maths, Ch. 4, §4.1 Introduction: Finding numbers with given sum and product illustrates quadratic modelling.

## Question 24

**2 marks**

**Short Answer**

Answer the question with working and reasons where needed.

**(a)** Solve the system of equations x − y = 4 and 3x + 2y = 7 using the substitution method. [2 marks]

### Answer

**From the equation x − y = 4, isolate x by adding y to both sides to obtain the expression x = y + 4 in terms of y. Substitute x = y + 4 into 3x + 2y = 7 to get 3(y + 4) + 2y = 7, which simplifies to 5y + 12 = 7 so 5y = −5 and y = −1, hence x = 3.**

### Step by step

1. **Express x in terms of y from the simpler equation:** Choose x − y = 4, because x has coefficient 1 and no fraction is needed. Add y to both sides to get x = y + 4. Why: Substitution needs one variable written in terms of the other. An equation in which a variable has coefficient 1 is chosen so that the rearrangement is simple and avoids fractions and arithmetic errors.
2. **Substitute into the other equation, solve for y, then find x and check:** Put x = y + 4 into 3x + 2y = 7 to get 3(y + 4) + 2y = 7. Expand: 3y + 12 + 2y = 7, so 5y + 12 = 7 and 5y = −5, giving y = −1. Then x = y + 4 = −1 + 4 = 3. Check in both equations: 3 − (−1) = 4 is true, and 3(3) + 2(−1) = 9 − 2 = 7 is true. Why: Substituting removes x, so the second equation has only one unknown and can be solved. The solution of a system must satisfy both equations at once, so checking both equations catches sign or arithmetic errors.

### Topic and concept

**Algebraic methods: substitution and elimination.** In the substitution method one equation is rearranged to express one variable in terms of the other. That expression replaces the variable in the second equation, giving a linear equation in one unknown. After finding that unknown, we substitute back to find the other. Here the solution is (x, y) = (3, −1).

### How marks are earned

1. From the equation x − y = 4, isolate x by adding y to both sides to obtain the expression x = y + 4 in terms of y. (1 mark)
2. Substitute x = y + 4 into 3x + 2y = 7 to get 3(y + 4) + 2y = 7, which simplifies to 5y + 12 = 7 so 5y = −5 and y = −1, hence x = 3. (1 mark)

**Source check:** Check the reasoning against NCERT Class 10 maths, Ch. 3, §3.3.1: Substitute an expression for one variable into the other equation to solve.

## Question 25

**2 marks**

**Short Answer**

Answer the question with working and reasons where needed.

**(a)** A warehouse roof is made by placing a half-cylinder of radius 2 m and length 12 m atop a cuboid of length 12 m, breadth 4 m and height 3 m. Find the volume enclosed by the roof. [2 marks]

### Answer

**Total volume = volume of cuboid (l×b×h) + half of volume of cylinder (½×πr²h). Substituting values gives 12×4×3 + ½×π×2²×12 = 144 + 24π m³ exactly; using π = 22/7 gives approximately 219.43 m³.**

### Step by step

1. **Identify the two solids and write the formula for the total volume:** The roof is a cuboid with a half-cylinder on top. The width of the cuboid is 4 m, so the half-cylinder has diameter 4 m and radius r = 4/2 = 2 m, and its length is 12 m. Total volume = l×b×h of the cuboid + ½ × πr²h of the cylinder, where h is the cylinder length. Why: The enclosed space is a composite solid made of two parts that do not overlap, so its volume is the sum of the part volumes. The half-cylinder lies across the full width of the cuboid, so its radius is half that width. Because only half a cylinder is present, only half the cylinder volume is added.
2. **Substitute the values, simplify and state units:** Cuboid volume = 12 × 4 × 3 = 144 m³. Half-cylinder volume = ½ × π × 2² × 12 = ½ × π × 4 × 12 = ½ × 48π = 24π m³. Total = (144 + 24π) m³. With π = 22/7, 24π = 24 × 22/7 = 528/7 ≈ 75.43 m³, so the total ≈ 144 + 75.43 = 219.43 m³. Why: Showing the exact value 144 + 24π first avoids rounding errors, and π = 22/7 is then used for a numerical answer. Volume is measured in cubic units, so the answer is in m³. As a check, the half-cylinder part is smaller than the cuboid part, which agrees with 75.43 < 144.

### Topic and concept

**Surface area and volume of combinations — cone + hemisphere, cylinder + hemisphere, etc..** A shed made of a cuboid with a half-cylinder on top has volume equal to the volume of the cuboid plus half the volume of the cylinder. In this question the cylinder's diameter equals the 4 m breadth and its length equals the 12 m length of the cuboid.

### How marks are earned

1. Total volume = volume of cuboid (l×b×h) + half of volume of cylinder (½×πr²h). (1 mark)
2. Substituting values gives 12×4×3 + ½×π×2²×12 = 144 + 24π m³ exactly; using π = 22/7 gives approximately 219.43 m³. (1 mark)

**Source check:** Check the reasoning against NCERT Class 10 maths, Ch. 12, §12.3: Volume of a composite solid equals sum of individual volumes; for a cuboid plus half cylinder use V_cuboid + ½V_cylinder.

# Section C

### Question 26

3 marks

Short Answer

Answer the question with working and reasons where needed.

**(a)** Using π = 22/7, verify that the volume of Shanta's shed (a cuboid 15 m×7 m×8 m surmounted by a half-cylinder of diameter 7 m and length 15 m) is 1128.75 m³. [3 marks]

#### Answer

**The volume of the cuboid part of Shanta's shed is V_cuboid = length×breadth×height = 15 m × 7 m × 8 m = 840 m³. For the half-cylinder of diameter 7 m and length 15 m, the volume of a full cylinder is V_full = πr²h = (22/7)×(3.5 m)² ×15 m = 577.5 m³, so the half-cylinder volume is 577.5 m³/2 = 288.75 m³. Finally, adding the cuboid and half-cylinder volumes gives V_total = 840 m³ + 288.75 m³ = 1128.75 m³, which verifies the stated result.**

#### Step by step

1. **Find the volume of the cuboid part:** The cuboid has length 15 m, breadth 7 m and height 8 m. V_cuboid = l × b × h = 15 × 7 × 8 = 105 × 8 = 840 m³. Why: The lower part of the shed is a cuboid, and the volume of a cuboid is the product of its three dimensions. Multiplying lengths in metres gives a volume in cubic metres.
2. **Find the radius, then the volume of the half-cylinder:** The diameter is 7 m, so r = 7/2 = 3.5 m. Full cylinder: V = πr²h = (22/7) × (3.5)² × 15 = (22/7) × 12.25 × 15. Here (22/7) × 12.25 = 38.5, and 38.5 × 15 = 577.5 m³. The half-cylinder is 577.5 ÷ 2 = 288.75 m³. Why: The cylinder formula needs the radius, and the radius is half the diameter. The roof is only half of a cylinder, so the full-cylinder volume must be halved.
3. **Add the two volumes and compare with the given value:** V_total = V_cuboid + V_half-cylinder = 840 + 288.75 = 1128.75 m³. This equals the value given in the question, so the statement is verified. Why: The shed is made of two parts that do not overlap, so its total volume is the sum of the part volumes. Getting exactly the stated value 1128.75 m³ completes the verification.

#### Topic and concept

**Surface area and volume of combinations — cone + hemisphere, cylinder + hemisphere, etc..** A shed shaped as a cuboid surmounted by a half-cylinder has volume equal to the cuboid volume plus half the volume of a cylinder. The cylinder's diameter matches the 7 m breadth of the cuboid, and its length matches the 15 m length. To verify a stated result, compute each part separately and show that the sum equals the given value.

### How marks are earned

1. The volume of the cuboid part of Shanta's shed is V_cuboid = length×breadth×height = 15 m × 7 m × 8 m = 840 m³. (1 mark)
2. For the half-cylinder of diameter 7 m and length 15 m, the volume of a full cylinder is V_full = πr²h = (22/7)×(3.5 m)² ×15 m = 577.5 m³, so the half-cylinder volume is 577.5 m³/2 = 288.75 m³. (1 mark)
3. Finally, adding the cuboid and half-cylinder volumes gives V_total = 840 m³ + 288.75 m³ = 1128.75 m³, which verifies the stated result. (1 mark)

**Source check:** Check the reasoning against NCERT Class 10 maths, Ch. 12, §12.3: Volume of cuboid plus half-cylinder equals sum of their individual volumes.

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## Question 27

**3 marks**

**Short Answer**

Answer the question with working and reasons where needed.

**(a)** Apply the step-deviation method to find the mean of: Class intervals: 10–20 (f=4), 20–30 (f=6), 30–40 (f=5) with assumed mean a=25 and h=10. [3 marks]

### Answer

**The class marks are 15, 25 and 35, so $u_i = (x_i - 25)/10$ are −1, 0 and 1; with frequencies 4, 6 and 5, $\Sigma f_i u_i = 4\times(-1) + 6\times0 + 5\times1 = 1$ and $\Sigma f_i = 15$. Using the step-deviation formula $\bar{x} = a + h\,(\Sigma f_i u_i / \Sigma f_i)$, we substitute to get $\bar{x} = 25 + 10\times(1/15)$. Hence, the mean is 25 + 0.666… = 25.666… (approximately 25.67).**

### Step by step

1. **Find class marks, step deviations and the sums using a table:** The class marks are $x_i$ = 15, 25, 35. Then $u_i = (x_i - 25)/10$ = −1, 0, 1. The table is: for 10–20, $f_i$ = 4, $u_i$ = −1, $f_iu_i$ = −4. For 20–30, $f_i$ = 6, $u_i$ = 0, $f_iu_i$ = 0. For 30–40, $f_i$ = 5, $u_i$ = 1, $f_iu_i$ = 5. So $\Sigma f_i$ = 4 + 6 + 5 = 15 and $\Sigma f_iu_i$ = −4 + 0 + 5 = 1. Why: Subtracting the assumed mean a = 25 and dividing by the class width h = 10 gives small whole-number values of $u_i$. This makes the products $f_iu_i$ easy to compute. Class marks are used because each class is represented by its midpoint.
2. **Write the step-deviation formula and substitute:** $\bar{x} = a + h \times (\Sigma f_iu_i / \Sigma f_i)$. Substituting a = 25, h = 10, $\Sigma f_iu_i$ = 1 and $\Sigma f_i$ = 15 gives $\bar{x}$ = 25 + 10 × (1/15) = 25 + 10/15 = 25 + 2/3. Why: The mean of the $u_i$ values is $\Sigma f_iu_i/\Sigma f_i$. Multiplying by h undoes the division by h, and adding a undoes the subtraction of a. This converts the mean of u back to the mean of the original data.
3. **Evaluate the mean and check that it is sensible:** $\bar{x}$ = 25 + 2/3 = 77/3 = 25.666… ≈ 25.67. Check: the mean lies between 10 and 40 and is slightly above a = 25, which is consistent with the data. Why: $\Sigma f_iu_i$ = 1 is positive, so the mean is slightly above the assumed mean 25. Here the 30–40 class has one more observation than the 10–20 class, which gives only a small positive shift. A mean far from 25 would signal an arithmetic error.

### Topic and concept

**Mean of grouped data — direct method, assumed mean method, step-deviation method.** The step-deviation method simplifies the assumed mean method by dividing each deviation ($x_i - a$) by the class width h, using $u_i = (x_i - a)/h$. The mean is then $\bar{x} = a + h \times (\Sigma f_i u_i / \Sigma f_i)$. For this data, a = 25 and h = 10 give $\bar{x} = 77/3 \approx 25.67$.

### How marks are earned

1. The class marks are 15, 25 and 35, so $u_i = (x_i - 25)/10$ are −1, 0 and 1; with frequencies 4, 6 and 5, $\Sigma f_i u_i = 4\times(-1) + 6\times0 + 5\times1 = 1$ and $\Sigma f_i = 15$. (1 mark)
2. Using the step-deviation formula $\bar{x} = a + h\,(\Sigma f_i u_i / \Sigma f_i)$, we substitute to get $\bar{x} = 25 + 10\times(1/15)$. (1 mark)
3. Hence, the mean is 25 + 0.666… = 25.666… (approximately 25.67). (1 mark)

**Source check:** Check the reasoning against NCERT Class 10 maths Ch. 13 Summary: The step-deviation formula uses $u_i = (x_i - a)/h$ to simplify mean calculation.

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## Question 28

**3 marks**

Short Answer

Attempt one option.

### Option A

Answer the question with working and reasons where needed.

**(a)** With A(0,0), B(3,4) and C(3,0), verify by calculation whether triangle ABC is right-angled. [3 marks]

### Answer

**First, we use the distance formula to find AB = $\sqrt{[(3-0)^2 + (4-0)^2]}$ = 5 units, AC = $\sqrt{[(3-0)^2 + (0-0)^2]}$ = 3 units, and BC = $\sqrt{[(3-3)^2 + (0-4)^2]}$ = 4 units. Next, we calculate the sum of the squares $AC^2 + BC^2 = 3^2 + 4^2 = 9 + 16 = 25$ and compare it with $AB^2 = 5^2 = 25$ to test the Pythagorean relation for a right angle. Since $AC^2 + BC^2$ equals $AB^2$, triangle ABC satisfies the Pythagorean theorem, confirming that ∠C is a right angle and the triangle is right-angled at vertex C.**

### Step by step

1. **Find the three side lengths with the distance formula:** Using $d = \sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$, AB = $\sqrt{[(3-0)^2 + (4-0)^2]}$ = $\sqrt{(9+16)}$ = $\sqrt{25}$ = 5 units, AC = $\sqrt{[(3-0)^2 + (0-0)^2]}$ = $\sqrt{9}$ = 3 units, and BC = $\sqrt{[(3-3)^2 + (0-4)^2]}$ = $\sqrt{16}$ = 4 units. Why: A triangle on the coordinate plane can only be tested for a right angle once its side lengths are known, and the distance formula gives each length from coordinates. The check is that AC and BC agree with the same-axis rule: A and C both have y = 0, so AC = |3 − 0| = 3, and B and C both have x = 3, so BC = |4 − 0| = 4.

2. **Compare the square of the longest side with the sum of the other two squares:** The longest side is AB = 5, so it is the only candidate for the hypotenuse. $AC^2 + BC^2 = 3^2 + 4^2 = 9 + 16 = 25$, and $AB^2 = 5^2 = 25$, so $AC^2 + BC^2 = AB^2$. Why: The converse of Pythagoras says that if the square of one side equals the sum of the squares of the other two, the angle opposite that side is 90°. Only the longest side can be the hypotenuse, so that is the side to test.
3. **Conclude the position of the right angle:** Since $AC^2 + BC^2 = AB^2$, the angle opposite AB, which is $\angle C$, is a right angle, so triangle ABC is right-angled at C. This agrees with the coordinates: AC lies along the x-axis and C(3,0) is directly below B(3,4), so BC is vertical and meets AC at 90°. Why: The equality identifies which vertex holds the right angle, namely the vertex opposite the hypotenuse. The axis observation is an independent geometric confirmation of the calculation.

### Topic and concept

**Distance formula between two points.** Coordinate geometry turns a geometric question into arithmetic. The distance formula gives each side length, and the converse of Pythagoras then tests whether the triangle is right-angled. When two points share an x or y coordinate, the distance is just the difference of the other coordinates (AC = 3, BC = 4).

### How marks are earned

1. First, we use the distance formula to find $AB = \sqrt{[(3 - 0)^2 + (4 - 0)^2]} = 5$ units, $AC = \sqrt{[(3 - 0)^2 + (0 - 0)^2]} = 3$ units, and $BC = \sqrt{[(3 - 3)^2 + (0 - 4)^2]} = 4$ units. (1 mark)
2. Next, we calculate the sum of the squares $AC^2 + BC^2 = 3^2 + 4^2 = 9 + 16 = 25$ and compare it with $AB^2 = 5^2 = 25$ to test the Pythagorean relation for a right angle. (1 mark)
3. Since $AC^2 + BC^2$ equals $AB^2$, triangle ABC satisfies the Pythagorean theorem, confirming that $\angle C$ is a right angle and the triangle is right-angled at vertex C. (1 mark)

**Source check:** Check the reasoning against NCERT Class 10 maths, Ch. 7, §7.2: Use of Pythagoras theorem in coordinate geometry for testing right angles.

OR

**Option B**

Answer the question with working and reasons where needed.

**(a)** For A(1,2) and B(7,8), the point P divides AB internally at P(4,5). Determine the ratio AP:PB. [3 marks]

### Answer

**Applying the section formula to the x-coordinates of A(1,2), B(7,8), and P(4,5) gives $(m \cdot 7 + n \cdot 1)/(m + n) = 4$, leading to $7m + n = 4m + 4n$ and hence simplifying to $3m = 3n$. Similarly, applying the ratio to the y-coordinates yields $(m \cdot 8 + n \cdot 2)/(m + n) = 5$, resulting in $8m + 2n = 5m + 5n$ and again simplifying to $3m = 3n$, confirming consistency from both coordinates. Since both coordinate equations reduce to $m = n$, the internal division ratio AP:PB is established as 1:1, indicating that P divides AB exactly at its midpoint.**

### Step by step

1. **Set up the section formula for the x-coordinate:** Let AP:PB = m:n. The x-coordinate of P is (m·7 + n·1)/(m + n) = 4, so 7m + n = 4(m + n) = 4m + 4n, which gives 3m = 3n. Why: In the ratio m:n, the weight m multiplies the coordinate of B and n multiplies the coordinate of A, so P lies closer to the endpoint with the larger weight. Equating the formula to the known x of P gives a linear equation connecting m and n.
2. **Repeat the process for the y-coordinate:** The y-coordinate gives (m·8 + n·2)/(m + n) = 5, so 8m + 2n = 5m + 5n, which gives 3m = 3n. Why: Because P lies on AB, both coordinates must give the same ratio. Getting the same result from y as from x checks that P really lies on segment AB and that no arithmetic slip was made.
3. **State the ratio and cross-check with the midpoint:** From 3m = 3n we get m = n, so AP:PB = 1:1. Check: ((1 + 7)/2, (2 + 8)/2) = (4, 5), which is P. Why: A ratio of 1:1 means P is the midpoint, and the midpoint formula is the section formula with m = n. Matching it to P(4,5) confirms the ratio independently.

### Topic and concept

**Section formula for internal division.** The internal section formula gives P = $((m\cdot x_2 + n\cdot x_1)/(m+n), (m\cdot y_2 + n\cdot y_1)/(m+n))$ for AP:PB = m:n. When P's coordinates are known, equating each coordinate to the formula gives equations in m and n, and the ratio m:n follows.

### How marks are earned

1. Applying the section formula to the x-coordinates of A(1,2), B(7,8), and P(4,5) gives (m·7 + n·1)/(m + n) = 4, leading to 7m + n = 4m + 4n and hence simplifying to 3m = 3n. (1 mark)
2. Similarly, applying the ratio to the y-coordinates yields (m·8 + n·2)/(m + n) = 5, resulting in 8m + 2n = 5m + 5n and again simplifying to 3m = 3n, confirming consistency from both coordinates. (1 mark)
3. Since both coordinate equations reduce to m = n, the internal division ratio AP:PB is established as 1:1, indicating that P divides AB exactly at its midpoint. (1 mark)

**Source check:** Check the reasoning against NCERT Class 10 maths, Ch. 7, §7.3: Equating weighted sums of coordinates yields the division ratio.

## Question 29

**3 marks**

**Short Answer**

Answer the question with working and reasons where needed.

**(a)** Using complementary-angle identities, prove sin(90° − A) = cos A and express sec(90° − A) in terms

of cosec A. [3 marks]

### Answer

**In a right triangle, the acute angles A and 90° − A are complementary. Their opposite and adjacent legs swap, so sin(90° − A)=cos A and cos(90° − A)=sin A. Taking the reciprocal of the second identity gives sec(90° − A)=1/cos(90° − A)=1/sin A=cosec A.**

### Step by step

1. **Draw right triangle ABC with the right angle at B and identify the complementary angles and their sides:** Take triangle ABC with ∠B = 90°, ∠A = A and so ∠C = 90° − A. For angle A: opposite side = BC, adjacent side = AB, hypotenuse = AC. For angle C: opposite side = AB, adjacent side = BC, hypotenuse = AC. So the opposite and adjacent legs of A and C are exchanged. Why: The three angles of a triangle sum to 180°, so with one right angle the other two acute angles sum to 90° and are complementary. The side opposite one acute angle is adjacent to the other, which is why the roles of the legs swap when we move from A to 90° − A.
2. **Write the sine and cosine ratios of both angles to prove the two identities:** sin(90° − A) = sin C = opposite/hypotenuse = AB/AC, and cos A = adjacent/hypotenuse = AB/AC, so sin(90° − A) = cos A. Similarly cos(90° − A) = cos C = adjacent/hypotenuse = BC/AC, and sin A = opposite/hypotenuse = BC/AC, so cos(90° − A) = sin A. Why: Sine is defined as opposite over hypotenuse and cosine as adjacent over hypotenuse. Because the legs swap between A and C while the hypotenuse AC stays the same, each ratio of C equals the co-ratio of A, so both identities are derived and not assumed.
3. **Apply the reciprocal definition of secant to express sec(90° − A) in terms of cosec A:** sec(90° − A) = 1/cos(90° − A) = 1/sin A = cosec A. Check with the triangle: sec C = hypotenuse/adjacent = AC/BC, and cosec A = hypotenuse/opposite = AC/BC, which agree. Why: Secant is the reciprocal of cosine and cosecant is the reciprocal of sine. Since cos(90° − A) = sin A was established in the previous step, taking its reciprocal turns sec into cosec.

### Topic and concept

**Properties of trigonometric ratios (their ranges).** In a right triangle ABC with the right angle at B, the acute angles A and C = 90° − A are complementary and their opposite and adjacent legs exchange roles. This gives sin(90° − A) = cos A and cos(90° − A) = sin A, and taking reciprocals gives sec(90° − A) = cosec A.

### How marks are earned

1. Identify complementary acute angles and show that their opposite and adjacent legs exchange roles. (1 mark)
2. Use side ratios to establish sin(90° − A)=cos A and cos(90° − A)=sin A. (1 mark)
3. Use sec θ=1/cos θ and cosec θ=1/sin θ to obtain sec(90° − A)=cosec A. (1 mark)

**Source check:** Check the reasoning against NCERT Class 10 maths, Ch. 8, §8.5: sin(90° − θ) = cos θ and sec(90° − θ) = cosec θ are complementary-angle identities.

## Question 30

3 marks

Short Answer

Answer the question with working and reasons where needed.

**(a)** Derive the coordinates of a point P(x,y) that divides the line segment joining $A(x_1,y_1)$ and $B(x_2,y_2)$ internally in the ratio m:n. [3 marks]

### Answer

**For AP:PB=m:n, similarity gives $(x-x_1):(x_2-x)=m:n$ and $(y-y_1):(y_2-y)=m:n$. Hence $n(x-x_1)=m(x_2-x)$ and $n(y-y_1)=m(y_2-y)$, giving $P=((mx_2+nx_1)/(m+n),(my_2+ny_1)/(m+n))$.**

### Step by step

1. **Construct perpendiculars and name the similar triangles:** Draw AR, PS and BT perpendicular to the x-axis, then draw AQ parallel to the x-axis meeting PS at Q and PC parallel to the x-axis meeting BT at C. In triangles APQ and PBC, ∠PAQ = ∠BPC (corresponding angles) and ∠AQP = ∠PCB = 90°, so △APQ ~ △PBC by AA similarity. Why: The perpendiculars and parallels split the slanted segment AB into horizontal and vertical components. AA similarity makes the corresponding sides of the two small triangles proportional, so the ratio AP:PB = m:n carries over to the coordinate differences.
2. **Express the sides as coordinate differences and form the ratios:** AQ = $x - x_1$, PC = $x_2 - x$, PQ = $y - y_1$, BC = $y_2 - y$. Corresponding sides of similar triangles give AP/PB = AQ/PC = PQ/BC = m/n, so $(x - x_1)/(x_2 - x) = m/n$ and $(y - y_1)/(y_2 - y) = m/n$. Why: Each horizontal or vertical length is a difference of coordinates, so the proportion between sides becomes an equation in x and y. AP and PB are the matching sides, so AQ pairs with PC and PQ pairs with BC; a reversed pairing would flip m and n.
3. **Cross-multiply, solve, and state the formula:** $n(x - x_1) = m(x_2 - x)$ gives $nx - nx_1 = mx_2 - mx$, so $(m + n)x = mx_2 + nx_1$ and $x = (mx_2 + nx_1)/(m + n)$. Similarly $y = (my_2 + ny_1)/(m + n)$. Hence $P = ((mx_2 + nx_1)/(m+n), (my_2 + ny_1)/(m+n))$. For m = n this gives $((x_1+x_2)/2, (y_1+y_2)/2)$, the midpoint. Why: Collecting the x terms on one side isolates x as a weighted average in which each endpoint is weighted by the length of the opposite part. The midpoint case checks that the orientation of m and n is correct.

### Topic and concept

**Section formula for internal division.** The section formula is derived from similar triangles. P splits AB in the ratio m:n, and the horizontal and vertical projections of AP and PB are in the same ratio, so P's coordinates are weighted averages of the endpoint coordinates.

### How marks are earned

1. Use similar triangles to obtain $(x-x_1):(x_2-x)=m:n$ and the analogous y-coordinate ratio. (1 mark)
2. Cross-multiply and solve to get $x=(mx_2+nx_1)/(m+n)$ and $y=(my_2+ny_1)/(m+n)$. (1 mark)
3. State the correct ordered pair $P=((mx_2+nx_1)/(m+n),(my_2+ny_1)/(m+n))$. (1 mark)

**Source check:** Check the reasoning against NCERT Class 10 maths, Ch. 7, §7.3: The section formula is obtained by similar triangles and weighted sums of coordinates.

## Question 31

**3 marks**

**Short Answer**

Attempt one option.

### Option A

Answer the question with working and reasons where needed.

**(a)** If the area of a sector of a circle is 154 cm² and its central angle is 70°, determine the radius of the circle. Use π = 22/7. [3 marks]

#### Answer

**Using $A=(\theta/360°)\pi r^2$ with $A=154$ cm², $\theta=70°$ and $\pi=22/7$ gives $154=(11/18)r^2$. Therefore $r^2=154\times18/11=252$ cm² and the positive radius is $r=\sqrt{252}=6\sqrt{7}$ cm$\approx15.87$ cm. Check: $(11/18)\times252=154$ cm².**

#### Step by step

1. **Substitute into sector-area formula:** $A=(\theta/360°)\pi r^2$ gives $154=(70/360)\times(22/7)\times r^2=(11/18)r^2$, with area measured in cm². Why: A sector occupies the fraction $\theta/360°$ of the full circle, so its area is that fraction of $\pi r^2$.
2. **Solve for the squared radius:** Multiplying by 18/11 gives $r^2=154\times18/11=252$ cm². Why: The factor 154/11 is 14, so the arithmetic gives $14\times18=252$ rather than 49.
3. **Take the positive root and check:** The radius is $r=\sqrt{252}=6\sqrt{7}$ cm$\approx15.87$ cm; substituting back gives $(11/18)\times252=154$ cm². Why: A radius is nonnegative, and the substitution confirms that the result satisfies the stated sector area.

#### Topic and concept

**Area of a sector and length of an arc.** The sector-area formula is $A=(\theta/360°)\pi r^2$. In this question the coefficient is 11/18, so the radius squared is 252 cm² and the radius is $6\sqrt{7}$ cm.

#### How marks are earned

1. Substitute $A=154$ cm², $\theta=70°$ and $\pi=22/7$ into $A=(\theta/360°)\pi r^2$. (1 mark)
2. Simplify the coefficient to 11/18 and calculate $r^2=154\times18/11=252$ cm². (1 mark)
3. Take the positive square root $r=6\sqrt{7}$ cm (about 15.87 cm) and verify the sector area. (1 mark)

**Source check:** Check the reasoning against NCERT Class 10 maths, Ch. 11, §11.2 Summary: The summary provides the sector area formula used to relate A, θ, and r.

OR

**Option B**

Answer the question with working and reasons where needed.

**(a)** In a circle of radius 10 cm, calculate (a) the length of the arc and (b) the area of the sector subtended by a central angle of 144°. Use π = 3.14. Show your steps. [3 marks]

### Answer

**Arc length: l = (144/360) × 2π × 10 = 0.4 × 2 × 3.14 × 10 = 25.12 cm. Sector area: A = (144/360) × π × 10² = 0.4 × 3.14 × 100 = 125.6 cm². Thus, the arc length is 25.12 cm and the sector area is 125.6 cm².**

### Step by step

1. **Compute the arc length from the fraction of the circle:** Use l = (θ/360)·2πr with θ = 144° and r = 10 cm. The fraction is 144/360 = 2/5 = 0.4. The full circumference is 2 × 3.14 × 10 = 62.8 cm, so l = 0.4 × 62.8 = 25.12 cm. Why: A central angle of θ cuts off θ/360 of the whole circle, so the arc is the same fraction of the circumference 2πr. Reducing 144/360 to 2/5 makes the arithmetic simple, and the answer is a length, so the unit is cm.
2. **Compute the sector area from the same fraction:** Use A = (θ/360)·πr². The full circle area is 3.14 × 10² = 314 cm², so A = 0.4 × 314 = 125.6 cm². Why: The sector is the same fraction θ/360 of the circle's area πr², but area is measured in square units, so the answer is in cm². The fraction is the same as in the arc length, but the base quantity is πr², not 2πr.
3. **State both results with units and check consistency:** The arc length is 25.12 cm and the sector area is 125.6 cm². Check: A = ½ × l × r = ½ × 25.12 × 10 = 125.6 cm², which matches. Why: The relation A = ½lr links the two answers, so it detects an arithmetic slip in either. Stating each result with its own unit (cm and cm²) shows that length and area were not mixed up.

### Topic and concept

**Area of a sector and length of an arc.** A sector of angle θ is the fraction θ/360 of the full circle. The arc length is that fraction of the circumference, (θ/360)·2πr, and the sector area is that fraction of the circle's area, (θ/360)·πr².

### How marks are earned

1. Arc length: l = (144/360) × 2π × 10 = 0.4 × 2 × 3.14 × 10 = 25.12 cm. (1 mark)
2. Sector area: A = (144/360) × π × 10² = 0.4 × 3.14 × 100 = 125.6 cm². (1 mark)
3. Thus, the arc length is 25.12 cm and the sector area is 125.6 cm². (1 mark)

**Source check:** Check the reasoning against NCERT Class 10 maths, Ch. 11, §11.2 Summary: The summary lists formulas for both arc length and sector area.

# Section D

### Question 32

5 marks

Long Answer

Answer both connected parts. Show the required method, explanation and units.

**(a)** A rectangular garden has an area of 200 m². Its length exceeds its breadth by 5 metres. If the breadth is x metres, form and simplify the quadratic equation satisfied by x. [2 marks]

#### Answer

**Express the breadth as x metres and the length as x + 5 metres, then equate their product to the given area to form x(x + 5) = 200. Expand the left side to get $x^2 + 5x$ and then subtract 200 from both sides to simplify the equation to $x^2 + 5x - 200 = 0$.**

#### Step by step

1. **Translate the word statement into algebraic expressions:** Breadth = x m. Since the length exceeds the breadth by 5 m, length = (x + 5) m. Area = length × breadth, so x(x + 5) = 200, where x > 0 because a breadth cannot be negative. Why: The phrase 'exceeds by 5' means 5 is added to the breadth, so the length is x + 5. The rectangle's area formula then links these expressions to the given area of 200 m², which forms the equation.
2. **Expand and write in standard form:** Expanding, $x(x + 5) = x^2 + 5x$. So $x^2 + 5x = 200$, and subtracting 200 from both sides gives $x^2 + 5x - 200 = 0$. Why: A quadratic equation is written in the standard form $ax^2 + bx + c = 0$ with zero on one side. Here a = 1, b = 5 and c = −200, so the equation is quadratic because the highest power of x is 2.

#### Topic and concept

**Situational problems based on quadratic equations in day-to-day contexts.** Dimension problems with a fixed area are modelled by quadratic equations. Define a variable for one dimension, write the other dimension in terms of it, and equate their product to the given area.

#### How marks are earned

1. Express the breadth as x metres and the length as x + 5 metres, then equate their product to the given area to form x(x + 5) = 200. (1 mark)
2. Expand the left side to get $x^2 + 5x$ and then subtract 200 from both sides to simplify the equation to $x^2 + 5x - 200 = 0$. (1 mark)

**Source check:** Check the reasoning against NCERT Class 10 maths, Ch. 4, §4.1 Introduction: Real-life dimensions of rectangular areas lead to quadratic equations when area is fixed.

**(b)** Two numbers differ by 3 and their product is 10. Find the numbers using the quadratic formula.
[3 marks]

### Answer

**Let the smaller number be x and the larger be x + 3; writing the product gives x(x + 3) = 10, leading to $x^2 + 3x - 10 = 0$. Identify a = 1, b = 3, c = -10 and compute the discriminant $D = 3^2 - 4 \cdot 1 \cdot (-10) = 9 + 40 = 49$ with $\sqrt{D} = 7$. Apply the quadratic formula $x = [-3 \pm 7]/2$ to get x = 2 or x = -5; hence the pairs are (2, 5) and (-5, -2).**

### Step by step

1. **Form the quadratic equation:** Let the smaller number be x, so the larger is x + 3 (they differ by 3). Their product is 10, so $x(x + 3) = 10$, which gives $x^2 + 3x - 10 = 0$. Why: Choosing one unknown and expressing the other through the stated difference reduces two conditions to one equation. The product condition then produces a degree-2 equation in x, which the quadratic formula can solve.
2. **Identify coefficients and compute the discriminant:** Comparing with $ax^2 + bx + c = 0$, a = 1, b = 3, c = −10. $D = b^2 - 4ac = 3^2 - 4(1)(-10) = 9 + 40 = 49$, and $\sqrt{D} = 7$. Why: The discriminant decides the nature of the roots: D = 49 > 0 and is a perfect square, so there are two distinct real roots which are rational. Note −4ac = +40 because c is negative.
3. **Apply the quadratic formula and verify both pairs:** $x = (-b \pm \sqrt{D})/2a = (-3 \pm 7)/2$. So $x = (-3 + 7)/2 = 2$ or $x = (-3 - 7)/2 = -5$. If x = 2 the numbers are 2 and 5; if x = −5 they are −5 and −2. Check: 5 − 2 = 3 and 2 × 5 = 10; −2 − (−5) = 3 and (−5)(−2) = 10. Why: The ± sign gives two roots and each gives a valid pair, since the problem does not restrict the numbers to be positive. Checking both pairs against difference 3 and product 10 confirms that the negative pair is also a solution.

### Topic and concept

**Situational problems based on quadratic equations in day-to-day contexts.** A pair of numbers with a given difference and product leads to a quadratic equation, because writing one number in terms of the other makes their product a quadratic expression. The quadratic formula $x = (-b \pm \sqrt{D})/2a$ then gives all the solutions.

### How marks are earned

1. Let the smaller number be x and the larger be x + 3; writing the product gives $x(x + 3) = 10$, leading to $x^2 + 3x - 10 = 0$. (1 mark)
2. Identify a = 1, b = 3, c = -10 and compute the discriminant $D = 3^2 - 4 \cdot 1 \cdot (-10) = 9 + 40 = 49$ with $\sqrt{D} = 7$. (1 mark)
3. Apply the quadratic formula $x = [-3 \pm 7]/2$ to get x = 2 or x = -5; hence the pairs are (2, 5) and (-5, -2). (1 mark)

**Source check:** Check the reasoning against NCERT Class 10 maths, Ch. 4, §4.1 Introduction: Solving sum-and-difference conditions via the quadratic formula models real problems.

## Question 33

5 marks

Long Answer

Answer both connected parts. Show the required method, explanation and units.

**(a)** Outline the key steps in the proof that √3 is irrational, highlighting the contradiction reached. [2 marks]

### Answer

**Assume √3 = p/q in lowest terms, then 3q^2 = p^2, so p^2 is divisible by 3, implying p is divisible by 3. Let p = 3k; substituting gives q^2 = 3k^2, so q is also divisible by 3, contradicting that p/q is in lowest terms.**

### Step by step

1. **Assume the opposite and show 3 divides p:** Assume √3 = p/q with p and q co-prime integers, q ≠ 0. Squaring gives 3 = p²/q², so p² = 3q². Hence 3 divides p², and therefore 3 divides p. Why: Proof by contradiction starts by assuming the opposite of the claim, here that √3 is rational. Writing it in lowest terms gives p and q no common factor, which is needed later. Since p² is a multiple of 3 and 3 is prime, 3 must divide p. The nearest error is assuming p/q is in lowest terms without stating it.
2. **Substitute p = 3k and reach the contradiction:** Let p = 3k. Then p² = 9k² = 3q², so q² = 3k². Hence 3 divides q² and so 3 divides q. Now 3 divides both p and q, which contradicts that p and q have no common factor other than 1. Why: The same argument applied to q shows that 3 is a common factor of p and q. This contradicts the assumption of lowest terms, so √3 cannot be rational. For √3 the shared prime is 3, not 2 as in the proof for √2, where p and q both come out even.

### Topic and concept

**Proofs of irrationality of √2, √3, √5 and similar surds.** To prove √3 is irrational, assume √3 = p/q in lowest terms. The argument then leads to 3 dividing both p and q, which contradicts p and q being co-prime. The common factor is 3, and the unique prime factorisation of integers is what makes the argument work.

### How marks are earned

1. Assume √3 = p/q in lowest terms, then 3q^2 = p^2, so p^2 is divisible by 3, implying p is divisible by 3. (1 mark)
2. Let p = 3k; substituting gives q^2 = 3k^2, so q is also divisible by 3, contradicting that p/q is in lowest terms. (1 mark)

**Source check:** Check the reasoning against NCERT Class 10 maths, Ch. 1, §1.3 Revisiting Irrational Numbers: Proof of irrationality uses contradiction and uniqueness of prime factors to show both p and q even.

**(b)** Prove that $\sqrt{3}$ is irrational by contradiction using prime factorisation. [3 marks]

### Answer

**Assume $\sqrt{3} = p/q$ in lowest terms, so $p^2 = 3q^2$ by squaring both sides. Since 3 divides $p^2$, by unique prime factorisation 3 divides p, so $p = 3k$. Substituting gives $q^2 = 3k^2$, so 3 divides q; this contradicts that p/q is in lowest terms.**

### Step by step

1. **Assume $\sqrt{3}$ is rational in lowest terms and square both sides:** Suppose $\sqrt{3} = p/q$ with p, q integers, $q \neq 0$ and HCF(p, q) = 1. Squaring both sides gives $3 = p^2/q^2$, so $p^2 = 3q^2$. Why: Proof by contradiction begins by assuming the opposite of what is to be shown. Taking p/q in lowest terms means the fraction has no common factor in top and bottom, and the contradiction will break this. The nearest error is skipping the lowest-terms condition, because then no contradiction appears.
2. **Use unique prime factorisation to show 3 divides p:** From $p^2 = 3q^2$, 3 divides $p^2$. If 3 did not divide p, the prime factorisation of p would have no 3, so that of $p^2$ would have no 3 either. By the uniqueness of prime factorisation this is impossible, so 3 divides p. Write $p = 3k$ for some integer k. Why: Unique prime factorisation says each integer has only one set of prime factors, so $p^2$ has the same primes as p. If 3 is a prime factor of $p^2$, it must already be a prime factor of p. The nearest error is saying that 3 divides $p^2$ so 3 divides p without giving this reason.
3. **Substitute $p = 3k$ to show 3 divides q and state the contradiction:** Putting $p = 3k$ in $p^2 = 3q^2$ gives $9k^2 = 3q^2$, so $q^2 = 3k^2$. So 3 divides $q^2$, and by the same argument 3 divides q. Then 3 is a common factor of p and q, but HCF(p, q) = 1. This is a contradiction, so $\sqrt{3}$ is irrational. Why: A common factor 3 of p and q contradicts the lowest-terms assumption, so the assumption that $\sqrt{3}$ is rational must be false. The contradiction comes from the shared prime 3. The nearest error is concluding that p and q are even, which is the $\sqrt{2}$ case.

### Topic and concept

**Proofs of irrationality of $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$ and similar surds.** Proof by contradiction assumes the opposite of the claim and uses unique prime factorisation to reach an impossible result. Here, assuming $\sqrt{3} = p/q$ in lowest terms leads to 3 dividing both p and q. That contradicts HCF(p, q) = 1, so $\sqrt{3}$ is irrational.

### How marks are earned

1. Assume $\sqrt{3} = p/q$ in lowest terms, so $p^2 = 3q^2$ by squaring both sides. (1 mark)
2. Since 3 divides $p^2$, by unique prime factorisation 3 divides p, so $p = 3k$. (1 mark)
3. Substituting gives $q^2 = 3k^2$, so 3 divides q; this contradicts that p/q is in lowest terms. (1 mark)

**Source check:** Check the reasoning against NCERT Class 10 maths, Ch. 1, §1.3: The contradiction arises because both numerator and denominator become divisible by the same prime.

## Question 34

**5 marks**

**Long Answer**

Attempt one option.

### Option A

Answer both connected parts. Show the required method, explanation and units.

**(a)** State the probability of a sure event and the probability of an impossible event. [2 marks]

#### Answer

**In probability theory, a sure event is defined as one that will occur under all possible circumstances, and its probability is assigned the value 1. By contrast, an impossible event is one that cannot occur under any circumstance whatsoever, and its probability is assigned the value 0.**

#### Step by step

1. **Define a sure event and assign its probability:** A sure event is one that happens in every possible outcome of the experiment, so all outcomes are favourable. Then P = favourable/total = n/n = 1, and P(sure event) = 1. Why: Probability is favourable outcomes divided by total equally likely outcomes. When every outcome is favourable the two counts are equal, so the ratio is 1, which is the largest value a probability can take.
2. **Define an impossible event and assign its probability:** An impossible event is one that happens in no outcome of the experiment, so the number of favourable outcomes is 0. Then P = 0/n = 0, and P(impossible event) = 0. Why: With no favourable outcome the numerator is zero, so the ratio is 0, which is the smallest possible probability. This also shows that P(E) always lies between 0 and 1.

#### Topic and concept

**Complementary events and their probability relation.** The probability of an event is (number of favourable outcomes)/(total number of equally likely outcomes). A sure event has every outcome favourable, so its probability is 1. An impossible event has none, so its probability is 0. For example, when a die is rolled, 'getting a number from 1 to 6' is sure and 'getting 7' is impossible. These two cases are the extreme values of probability.

#### How marks are earned

1. In probability theory, a sure event is defined as one that will occur under all possible circumstances, and its probability is assigned the value 1. (1 mark)
2. By contrast, an impossible event is one that cannot occur under any circumstance whatsoever, and its probability is assigned the value 0. (1 mark)

**Source check:** Check the reasoning against NCERT Class 10 maths, Ch. 14, §14.2: The probability of a sure event is 1 and that of an impossible event is 0.

**(b)** Give an example of a sure event and an impossible event when rolling a fair die, state their probabilities, and verify that both probabilities lie between 0 and 1. [3 marks]

### Answer

**A sure event is rolling at most 6, which occurs with probability 1 since all faces are ≤6. An impossible event is rolling a 7, which has probability 0 since a die has no face 7. Both probabilities, 1 and 0, satisfy the property $0 \leq P(E) \leq 1$.**

### Step by step

1. **Choose a sure event and compute its probability:** Take the event 'the number on top is at most 6'. All 6 faces (1 to 6) satisfy it, so P = 6/6 = 1. Why: Probability is favourable outcomes over total equally likely outcomes. Here every outcome is favourable, so the event is certain and the ratio is exactly 1.
2. **Choose an impossible event and compute its probability:** Take the event 'the number on top is 7'. A die has faces 1 to 6 only, so favourable outcomes = 0 and P = 0/6 = 0. Why: An event with no favourable outcome cannot occur. Its numerator is zero, so the probability is 0.
3. **Verify that both probabilities lie between 0 and 1:** We have $0 \leq 0 \leq 1$ and $0 \leq 1 \leq 1$, so both values satisfy $0 \leq P(E) \leq 1$. Why: The favourable count can never be negative and can never exceed the total count, so the ratio stays between 0 and 1. The sure and impossible events are the two end values of this range.

### Topic and concept

**Complementary events and their probability relation.** For a fair die the 6 outcomes are equally likely, so P(E) = favourable/6. A count of favourable outcomes lies between 0 and 6, so P(E) lies between 0 and 1. The extremes are reached by the impossible event (0) and the sure event (1).

### How marks are earned

1. A sure event is rolling at most 6, which occurs with probability 1 since all faces are ≤6. (1 mark)
2. An impossible event is rolling a 7, which has probability 0 since a die has no face 7. (1 mark)
3. Both probabilities, 1 and 0, satisfy the property $0 \leq P(E) \leq 1$. (1 mark)

**Source check:** Check the reasoning against NCERT Class 10 maths, Ch. 14, §14.2: The probability of any event lies between 0 and 1, inclusive, with 0 for impossible and 1 for certain events.

OR

**Option B**

Answer both connected parts. Show the required method, explanation and units.

**(a)** In a single toss of a fair coin, let E be the event ‘head appears’. Calculate P(E), P(not E), and verify

that P(E)+P(not E)=1. [2 marks]

### Answer

**For a single toss of a fair coin, the probability of the event E, defined as obtaining a head, is P(E) = 1/2, and consequently P(not E) is calculated as 1 − 1/2 = 1/2. By summing these values, P(E) + P(not E) = 1/2 + 1/2 = 1, which verifies the fundamental complementary rule in probability theory.**

### Step by step

1. **List the equally likely outcomes of the toss and compute P(E) and P(not E):** A fair coin has the sample space {H, T}, so there are 2 equally likely outcomes. The event E (head) has 1 favourable outcome, so P(E) = 1/2. The event not E (tail) is everything else, so P(not E) = 1 − P(E) = 1 − 1/2 = 1/2. Why: Theoretical probability is the number of favourable outcomes divided by the total number of equally likely outcomes, which gives 1/2 for a head. The complement of E contains the remaining outcome T, and since the two events together cover all outcomes, P(not E) = 1 − P(E).
2. **Add the two probabilities to verify the complementary rule:** P(E) + P(not E) = 1/2 + 1/2 = 2/2 = 1. Check by counting: P(not E) = 1 tail / 2 outcomes = 1/2, which matches the value from 1 − P(E). Why: E and not E are complementary events that together include every outcome in {H, T}, and probabilities of all outcomes add to 1. The sum equal to 1 confirms that the rule P(E) + P(not E) = 1 holds for this experiment.

### Topic and concept

**Elementary events and sum of probabilities.** For a fair coin the sample space is {H, T} with two equally likely outcomes, so P(head) = 1/2. An event E and its complement not E together cover all outcomes, so their probabilities add up to 1: P(E) + P(not E) = 1.

### How marks are earned

1. For a single toss of a fair coin, the probability of the event E, defined as obtaining a head, is P(E) = 1/2, and consequently P(not E) is calculated as 1 − 1/2 = 1/2. (1 mark)
2. By summing these values, P(E) + P(not E) = 1/2 + 1/2 = 1, which verifies the fundamental complementary rule in probability theory. (1 mark)

**Source check:** Check the reasoning against NCERT Class 10 maths, Ch. 14, §14.2: For any event E, P(E) + P(not E) = 1.

**(b)** Two fair dice are rolled. Calculate the probability that the sum of the points is 8. [3 marks]

### Answer

**When two fair dice are rolled simultaneously, the total sample space consists of 6 × 6 = 36 equally likely ordered pairs of outcomes. The ordered pairs yielding a total sum of 8 are (2,6), (3,5), (4,4), (5,3), and (6,2), making a count of five favourable outcomes. Therefore, the probability of obtaining a sum of eight when rolling two dice is given by the ratio 5/36 of favourable outcomes to the total of 36 possible outcomes.**

### Step by step

1. **Find the total number of equally likely outcomes:** Each die has 6 faces, so two dice give 6 × 6 = 36 ordered pairs (first die, second die), all equally likely. Why: The theoretical probability formula needs the total number of equally likely outcomes. Treating the dice as separate makes each ordered pair equally likely.
2. **Count the pairs with sum 8:** The pairs are (2,6), (3,5), (4,4), (5,3) and (6,2), so there are 5 favourable outcomes. (2,6) and (6,2) are different outcomes, and (4,4) is counted only once. Pairs with a 1 cannot work because the other die would need 7. Why: The pairs are ordered, so (2,6) and (6,2) are different outcomes. Listing by the first die from 2 to 6 gives exactly one partner each, and none is missed or repeated.
3. **Compute the probability:** P(sum 8) = favourable/total = 5/36 ≈ 0.139. Why: Probability is favourable outcomes divided by total equally likely outcomes. Since 0 < 5/36 < 1, the result is a valid probability.

### Topic and concept

**Simple real-life problems on finding the probability of an event.** For two dice, the sample space has 36 equally likely ordered pairs. To find the probability of an event, list the pairs that fit it and divide that count by 36. The pairs must be ordered so that every outcome has the same chance.

### How marks are earned

1. When two fair dice are rolled simultaneously, the total sample space consists of 6 × 6 = 36 equally likely ordered pairs of outcomes. (1 mark)
2. The ordered pairs yielding a total sum of 8 are (2,6), (3,5), (4,4), (5,3), and (6,2), making a count of five favourable outcomes. (1 mark)
3. Therefore, the probability of obtaining a sum of eight when rolling two dice is given by the ratio 5/36 of favourable outcomes to the total of 36 possible outcomes. (1 mark)

**Source check:** Check the reasoning against NCERT Class 10 maths, Ch. 14, §14.2: The probability is computed by dividing the number of favourable outcomes by the total number of equally likely outcomes.

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## Question 35

**5 marks**

**Long Answer**

Attempt one option.

### Option A

Answer both connected parts. Show the required method, explanation and units.

**(a)** Define what it means for a real number k to be a zero of a polynomial p(x). Then, illustrate your

definition by identifying a zero of p(x)=3x−9. [2 marks]

### Answer

**A real number k is called a zero of p(x) if substituting x=k makes p(k)=0. For p(x)=3x−9, p(3)=3·3−9=0, so k=3 is a zero of the polynomial.**

### Step by step

1. **State the definition of a zero of a polynomial:** A real number k is a zero of the polynomial p(x) if substituting x = k gives p(k) = 0. Equivalently, k is the x-coordinate where the graph of y = p(x) meets the x-axis. Why: A zero is a value of x that makes the polynomial's output zero, so it is exactly a solution of p(x) = 0. On the graph of y = p(x), output zero means the point lies on the x-axis, which links the algebraic definition to the x-intercept.
2. **Illustrate by solving 3x − 9 = 0 and verifying with substitution:** Set p(x) = 0: 3x − 9 = 0, so 3x = 9 and x = 3. Check: p(3) = 3·3 − 9 = 9 − 9 = 0, so k = 3 is a zero of p(x) = 3x − 9. The graph of this line crosses the x-axis at (3, 0). Why: Finding a zero means solving p(x) = 0, and the definition is satisfied only if substituting the value returns 0. The substitution check shows the value really satisfies p(k) = 0, and a linear polynomial has exactly one such crossing point.

### Topic and concept

**Zeros of a polynomial; geometric meaning of zeros as x-intercepts.** A real number k is a zero (root) of p(x) if p(k) = 0, and the zeros are the x-coordinates where the graph of y = p(x) crosses the x-axis. For the linear polynomial 3x − 9, solving 3x − 9 = 0 gives its single zero x = 3.

### How marks are earned

1. A real number k is called a zero of p(x) if substituting x=k makes p(k)=0. (1 mark)
2. For p(x)=3x−9, p(3)=3·3−9=0, so k=3 is a zero of the polynomial. (1 mark)

**Source check:** Check the reasoning against NCERT Class 10 maths, Ch. 2, §2.1: A real number k is called a zero of the polynomial p(x) if substituting x=k makes p(k)=0.

**(b)** The graph of a quadratic polynomial y=p(x) crosses the x-axis at x=−2 and x=5 and also passes through the point (1,−8). Determine the explicit form of p(x). [3 marks]

### Answer

**Since the polynomial crosses the x–axis at x=−2 and x=5, its factorized form can be written as p(x)=k(x+2)(x−5), where k is a nonzero constant. Substituting the point (1,−8) into this form gives −8=k(1+2)(1−5)=k·3·(−4)=−12k, hence solving for k yields k=(−8)/(−12)=2/3. Thus, upon expanding, the explicit form of the quadratic polynomial is p(x)=(2/3)(x+2)(x−5)=(2/3)(x²−3x−10)=(2/3)x²−2x−20/3 as required.**

### Step by step

1. **Write the polynomial in factor form using the zeros:** The graph crosses the x-axis at x = −2 and x = 5, so these are zeros and p(x) = k(x + 2)(x − 5), where k ≠ 0 is a constant. Why: If p(α) = 0 then (x − α) is a factor of p(x). A quadratic with two given zeros is therefore fixed only up to a constant multiple k, so k must be found from another condition.

2. **Use the point (1, −8) to find k:** Substitute x = 1 and p = −8: −8 = k(1 + 2)(1 − 5) = k · 3 · (−4) = −12k, so k = (−8)/(−12) = 2/3. Why: The point (1, −8) lies on the graph, so it must satisfy y = p(x). This gives one equation in k, which fixes the constant.
3. **Expand and check:** p(x) = (2/3)(x + 2)(x − 5) = (2/3)(x² − 3x − 10) = (2/3)x² − 2x − 20/3. Check: p(1) = 2/3 − 2 − 20/3 = −8, and p(−2) = p(5) = 0. Why: Expanding gives the standard form ax² + bx + c. Substituting back confirms both the zeros and the extra point.

### Topic and concept

**Zeros of a polynomial; geometric meaning of zeros as x-intercepts.** The x-intercepts of the graph of y = p(x) are the zeros of p. Two zeros give the factors (x + 2) and (x − 5), but they do not fix the leading coefficient. Many parabolas cross the x-axis at the same two points. A third point on the graph fixes k.

### How marks are earned

1. Since the polynomial crosses the x–axis at x=−2 and x=5, its factorized form can be written as p(x)=k(x+2)(x−5), where k is a nonzero constant. (1 mark)
2. Substituting the point (1,−8) into this form gives −8=k(1+2)(1−5)=k·3·(−4)=−12k, hence solving for k yields k=(−8)/(−12)=2/3. (1 mark)
3. Thus, upon expanding, the explicit form of the quadratic polynomial is p(x)=(2/3)(x+2)(x−5)=(2/3)(x²−3x−10)=(2/3)x²−2x−20/3 as required. (1 mark)

**Source check:** Check the reasoning against NCERT Class 10 maths, Ch. 2, §2.4 Summary: If p(α)=0 then (x−α) is a factor of p(x), so zeros determine linear factors.

OR

**Option B**

Answer both connected parts. Show the required method, explanation and units.

**(a)** A quadratic polynomial has zeros at 3 and −2. Write the monic quadratic polynomial having these zeros. [2 marks]

### Answer

**For the given zeros 3 and −2 of the required monic quadratic polynomial, the sum is calculated as 3+(−2)=1 and the product is 3×(−2)=−6 by direct arithmetic. Substituting the obtained sum and product into the general monic form x²−(sum)x+(product) yields the polynomial x²−1x−6, which simplifies to x²−x−6 as the required expression.**

### Step by step

1. **Find the sum and product of the given zeros:** The zeros are α = 3 and β = −2. Sum α + β = 3 + (−2) = 1, and product αβ = 3 × (−2) = −6. Why: For a quadratic ax² + bx + c the sum of zeros is −b/a and the product is c/a, so the sum and product of the zeros fix the ratios b/a and c/a. These two numbers are all that is needed to rebuild the polynomial.

2. **Substitute the sum and product into the monic form and verify by factorising:** With a = 1 the relations give −b = sum = 1 and c = product = −6, so the polynomial is $x^2 - (\text{sum})x + \text{product} = x^2 - 1x + (-6) = x^2 - x - 6$. Check: $(x - 3)(x + 2) = x^2 + 2x - 3x - 6 = x^2 - x - 6$, and putting x = 3 gives 9 − 3 − 6 = 0 while x = −2 gives 4 + 2 − 6 = 0. Why: A monic polynomial has a = 1, so b = −(sum of zeros) and c = product of zeros, which is why the form is $x^2 - (\text{sum})x + \text{product}$. The factorised form (x − 3)(x + 2) and the zero checks confirm that both 3 and −2 make the polynomial zero.

### Topic and concept

**Relationship between zeros and coefficients for quadratic polynomials.** For a quadratic $ax^2 + bx + c$, the sum of its zeros α + β equals −b/a and their product αβ equals c/a. For a monic quadratic (a = 1) this gives $x^2 - (\alpha + \beta)x + \alpha\beta$, so zeros 3 and −2 lead to $x^2 - x - 6$.

### How marks are earned

1. For the given zeros 3 and −2 of the required monic quadratic polynomial, the sum is calculated as 3+(−2)=1 and the product is 3×(−2)=−6 by direct arithmetic. (1 mark)
2. Substituting the obtained sum and product into the general monic form $x^2-(\text{sum})x+(\text{product})$ yields the polynomial $x^2-1x-6$, which simplifies to $x^2-x-6$ as the required expression. (1 mark)

**Source check:** Check the reasoning against NCERT Class 10 maths, Ch. 2, §2.3 Relationship between Zeroes and Coefficients of a Polynomial: For $ax^2+bx+c$, $\alpha+\beta=-b/a$ and $\alpha\beta=c/a$.

**(b)** The sum of the reciprocals of the zeros of a monic quadratic polynomial is 4/3, and their sum is 5. Find the polynomial equation. [3 marks]

### Answer

**Since $(1/\alpha)+(1/\beta)=(\alpha+\beta)/(\alpha\beta)=4/3$ and $\alpha+\beta=5$, we get $5/(\alpha\beta)=4/3 \Rightarrow \alpha\beta=15/4$. A monic quadratic with sum 5 and product 15/4 is $x^2-5x+15/4=0$. Multiplying through by 4 to eliminate fractions yields $4x^2-20x+15=0$.**

### Step by step

1. **Convert the reciprocal sum into the product:** $1/\alpha + 1/\beta = (\beta + \alpha)/(\alpha\beta) = (\alpha + \beta)/(\alpha\beta)$. Given α + β = 5 and this equals 4/3, we get 5/(αβ) = 4/3, so αβ = 15/4. Why: Adding the fractions over the common denominator αβ links the reciprocal sum to the sum and product of zeros. This lets us find the unknown product from the known sum.
2. **Form the monic quadratic:** A monic quadratic is $x^2 - (\alpha + \beta)x + \alpha\beta$, so $x^2 - 5x + 15/4 = 0$. Why: For $ax^2 + bx + c$, α + β = −b/a and αβ = c/a. With a = 1 this gives b = −(sum) and c = product.
3. **Clear fractions and check:** Multiply by 4: $4x^2 - 20x + 15 = 0$. Check: sum = 20/4 = 5, product = 15/4, and reciprocal sum = 5/(15/4) = 20/15 = 4/3. Why: Multiplying every term by 4 keeps the same zeros and removes the fraction. The check confirms the original conditions.

### Topic and concept

**Relationship between zeros and coefficients for quadratic polynomials.** The relations α + β = −b/a and αβ = c/a connect the coefficients to the zeros. The condition on $1/\alpha + 1/\beta$ is rewritten as $(\alpha + \beta)/(\alpha\beta)$, so the sum and product can be found. These two values then give the polynomial.

### How marks are earned

1. Since $(1/\alpha)+(1/\beta)=(\alpha+\beta)/(\alpha\beta)=4/3$ and $\alpha+\beta=5$, we get $5/(\alpha\beta)=4/3 \Rightarrow \alpha\beta=15/4$. (1 mark)
2. A monic quadratic with sum 5 and product 15/4 is $x^2-5x+15/4=0$. (1 mark)
3. Multiplying through by 4 to eliminate fractions yields $4x^2-20x+15=0$. (1 mark)

**Source check:** Check the reasoning against NCERT Class 10 maths, Ch. 2, §2.3 Relationship between Zeroes and Coefficients of a Polynomial: For $ax^2+bx+c$, $\alpha+\beta=-b/a$ and $\alpha\beta=c/a$.

# Section E

### Question 36

**4 marks**

**Case Based**

A surveyor wants to determine the height of a lighthouse. From point P on level ground, the angle of elevation to the top is 45°. He then moves 15 m closer to the lighthouse to point Q and finds the angle of elevation to be 60°. Let h be the height of the lighthouse and x be the horizontal distance from P to the base of the lighthouse.

**(a)** According to the passage, what is the numerical value of tan 45°? [1 mark]

#### Answer

**According to the provided passage, the numerical value of tan 45° is explicitly stated as 1 in the trigonometric table.**

#### Step by step

1. **Evaluate tan 45° from the right isosceles triangle relation:** In a right triangle with an angle of 45°, the other acute angle is 90° − 45° = 45°, so the opposite side and adjacent side are equal. Therefore tan 45° = opposite/adjacent = a/a = 1 (no units, since it is a ratio of two lengths). Check: this matches the standard trigonometric table value tan 45° = 1, and in the lighthouse figure it means h = x at point P. Why: tan θ is defined as the ratio of the side opposite to θ over the side adjacent to θ in a right triangle; equal angles force equal opposite sides, so the ratio is exactly 1. This is the same ratio that connects height and horizontal distance in angle-of-elevation problems.

#### Topic and concept

**Angle of elevation and angle of depression; applications to heights and distances.** In heights-and-distances problems, tan θ = height / horizontal distance in the right triangle formed by the line of sight. For θ = 45° the triangle is isosceles, so height equals horizontal distance and tan 45° = 1. This standard value is what lets the lighthouse problem give h = x at point P.

#### How marks are earned

1. According to the provided passage, the numerical value of tan 45° is explicitly stated as 1 in the trigonometric table. (1 mark)

**Source check:** Check the reasoning against NCERT Class 10 maths, Ch. 9, §9.1: Defines the angle of elevation as the angle between the horizontal and the line of sight to a point above.

**(b)** Write the expression relating h and x using the measurement at point P. [1 mark]

**Answer**

**Using $\tan 45° = h/x \Rightarrow h = x$**

**Step by step**

1. **Identify the right triangle at P and write tan 45° as opposite over adjacent, then substitute tan 45° = 1.:** In the right triangle formed by P, the foot of the lighthouse and its top, the height h is opposite to the 45° angle at P and the horizontal distance x is adjacent to it, so tan 45° = h/x; since tan 45° = 1, we get 1 = h/x, which gives h = x. Why: The angle of elevation is measured from the horizontal, so the vertical height is the side opposite it and the ground distance is the side adjacent. Tan θ = height / horizontal distance links these two sides, and the standard value tan 45° = 1 makes the two sides equal.

**Topic and concept**

**Angle of elevation and angle of depression; applications to heights and distances.** At P the line of sight makes an angle of elevation of 45° with the horizontal. In the right triangle formed, the lighthouse height h is opposite this angle and the ground distance x is adjacent. So tan θ = height / horizontal distance gives tan 45° = h/x. Because tan 45° = 1, h = x.

**How marks are earned**

1. Using $\tan 45° = h/x \Rightarrow h = x$ (1 mark)

**Source check:** Check the reasoning against NCERT Class 10 maths, Ch. 9, §9.1: Relates height and horizontal distance through tan θ=height/distance.

Attempt one of the final two-mark options.

**Option A**

**(c)** (i) Write the equation that represents tan 60° at point Q in terms of h and x. (ii) Eliminate h to derive an equation in x only. [2 marks]

**Answer**

**$\tan 60° = h/(x-15) \Rightarrow \sqrt{3} = h/(x-15)$ Substitute $h = x$ into $\sqrt{3} = h/(x-15)$ to get $\sqrt{3} = x/(x-15)$**

**Step by step**

1. **Write tan 60° for the right triangle at Q:** Q is 15 m closer to the lighthouse than P, so its horizontal distance from the base is x − 15 m. Height is h, so tan 60° = h/(x − 15), and since tan 60° = √3, we get √3 = h/(x − 15). Why: The angle of elevation at Q uses tan θ = height / horizontal distance, and the horizontal distance changes because Q lies 15 m nearer the base than P. Using x instead of x − 15 would describe point P, not Q.

2. **Eliminate h using the relation from point P:** At P, tan 45° = h/x = 1, so h = x. Substituting h = x into √3 = h/(x − 15) gives √3 = x/(x − 15). (Optional continuation: √3(x − 15) = x ⇒ x(√3 − 1) = 15√3 ⇒ x = 15√3/(√3 − 1) = 15(3 + √3)/2 ≈ 35.5 m.) Why: Both right triangles share the same height h, so the two tan relations can be combined to remove h and leave one equation in x. This works because the lighthouse height is the same from P and Q.

### Topic and concept

**Angle of elevation and angle of depression; applications to heights and distances.** The angle of elevation is measured between the horizontal and the line of sight, and tan θ = height / horizontal distance in the resulting right triangle. When the observer moves, the horizontal distance changes but the height stays the same, so two tan equations can be combined to eliminate h and find x.

### How marks are earned

1. tan 60°=h/(x−15)⇒√3=h/(x−15) (1 mark)
2. Substitute h=x into √3=h/(x−15) to get √3=x/(x−15) (1 mark)

**Source check:** Check the reasoning against NCERT Class 10 maths, Ch. 9, §9.1: Angle of elevation uses tan θ=height/horizontal distance. Check the reasoning against NCERT Class 10 maths, Ch. 9, §9.1: Two right-triangle relations can be combined to eliminate height.

OR

**Option B**

(c) Use h=x and tan 60°=h/(x−15) to find the lighthouse height in exact form. [2 marks]

### Answer

**Substitution gives √3=x/(x−15), hence (√3−1)x=15√3. The height is h=x=15√3/(√3−1)=(15/2)(3+√3) m.**

### Step by step

1. **Form the equation and solve for x:** From P, tan 45° = 1 = h/x, so h = x. From Q, tan 60° = √3 = h/(x − 15) = x/(x − 15). Then √3(x − 15) = x, so √3x − x = 15√3 and (√3 − 1)x = 15√3. Why: The tangent of the angle of elevation is height divided by horizontal distance. Q is 15 m nearer, so its distance is x − 15. Equating the two expressions for h gives one equation in x.
2. **Rationalise and check:** x = 15√3/(√3 − 1) × (√3 + 1)/(√3 + 1) = 15√3(√3 + 1)/(3 − 1) = 15(3 + √3)/2 m. So h = x = (15/2)(3 + √3) ≈ 7.5 × 4.732 ≈ 35.49 m. Check: x − 15 ≈ 20.49 and 35.49/20.49 ≈ 1.732 = √3. Why: Multiplying by the conjugate (√3 + 1) uses (√3 − 1)(√3 + 1) = 3 − 1 = 2, which removes the surd from the denominator. The numerical check confirms the 60° condition, and the height is in metres.

### Topic and concept

**Angle of elevation and angle of depression; applications to heights and distances.** The angle of elevation is measured from the horizontal to the line of sight. In the right triangle, tan θ = height/horizontal distance. Two sight points give two equations with the same height h. Moving 15 m closer reduces the distance from x to x − 15.

### How marks are earned

1. Substitution gives $\sqrt{3}=x/(x-15)$, hence $(\sqrt{3}-1)x=15\sqrt{3}$. (1 mark)
2. The height is $h=x=15\sqrt{3}/(\sqrt{3}-1)=(15/2)(3+\sqrt{3})$ m. (1 mark)

**Source check:** Check the passage facts and NCERT Class 10 maths, Ch. 9, §9.1 against both steps.

## Question 37

**4 marks**

**Case Based**

In a botanical garden, a circular pond has centre O and radius 8 m. A straight rod AB lies along the tangent to the pond at its contact point P. A point Q lies 15 m from O outside the pond. From Q, two ropes touch the pond at $T_1$ and $T_2$, and $QT_1 = QT_2$.

**(a)** According to the passage, what is a tangent to a circle? [1 mark]

### Answer

**A tangent to a circle is a straight line that touches the circle at exactly one point.**

### Step by step

1. **State the definition of a tangent using the contact point condition.:** A tangent to a circle is a straight line that touches the circle at exactly one point. In the passage, the rod AB is along the tangent and touches the circular pond only at P. Why: This matches the required definition: a tangent has one point of contact with the circle, unlike a secant which meets it at two points.

### Topic and concept

**Tangent to a circle; tangent at any point is perpendicular to the radius (theorem + proof).** A tangent to a circle is a line that touches the circle at exactly one point, unlike a secant which meets it at two points or a non-intersecting line which meets it at none.

### How marks are earned

1. A tangent to a circle is a straight line that touches the circle at exactly one point. (1 mark)

**Source check:** Check the reasoning against NCERT Class 10 maths, Ch. 10, §10.2: A tangent to a circle intersects it at exactly one point.

**(b)** What relationship between the rod AB at P and the radius OP is described by the theorem referenced in the passage? [1 mark]

### Answer

**Since AB is tangent to the pond at P, AB is perpendicular to radius OP; the angle between AB and OP is 90°.**

### Step by step

1. **State the tangent-radius theorem:** The tangent line at P is perpendicular to the radius OP at the point of contact, so the angle between them is 90°. Why: The perpendicularity theorem applies because the revised passage states that rod AB touches the circle only at P.

### Topic and concept

**Tangent to a circle; tangent at any point is perpendicular to the radius (theorem + proof).** A tangent touches a circle at one point. At that point of contact the tangent is perpendicular to the radius drawn to the same point.

### How marks are earned

1. State that AB, a tangent at P, is perpendicular to radius OP at P, giving a 90° angle. (1 mark)

**Source check:** Check the reasoning against NCERT Class 10 maths, Ch. 10, Summary: At the point of contact, the tangent is perpendicular to the radius drawn to that point.

Attempt one of the final two-mark options.

**Option A**

**(c)** (i) How many tangents can be drawn from point Q to the pond, and what are they named in the passage? (ii) Which property of the lengths $QT_1$ and $QT_2$, mentioned in the passage, illustrates a theorem about tangents from an external point? [2 marks]

### Answer

**From the external point Q, exactly two tangents can be drawn to the pond, named $QT_1$ and $QT_2$. The lengths of the two tangents drawn from the external point Q to the pond are equal ($QT_1 = QT_2$).**

### Step by step

1. **Count the tangents from the external point Q and name them from the passage.:** Q lies 15 m from O, and the pond has radius 8 m, so Q is outside the pond. Exactly two tangents can be drawn from Q, and the passage names them as the ropes $QT_1$ and $QT_2$, touching the pond at $T_1$ and $T_2$. Why: A point outside a circle has exactly two tangents to it, because a tangent touches the circle at exactly one point and there are only two such lines through an external point. Checking that OQ = 15 m is greater than the radius 8 m confirms Q is external.
2. **State the equal-tangents property and justify it with congruent right triangles.:** The passage gives $QT_1 = QT_2$. Since $OT_1 \perp QT_1$ and $OT_2 \perp QT_2$, triangles $OT_1Q$ and $OT_2Q$ are right-angled at $T_1$ and $T_2$, with $OT_1 = OT_2 = 8$ m (radii) and OQ common. So they are congruent by RHS, and $QT_1 = QT_2$. Why: The radius to the point of contact is perpendicular to the tangent, which gives the right angles needed for RHS congruence. Congruent triangles have equal corresponding sides, which proves the theorem that the lengths of the two tangents from an external point are equal.

### Topic and concept

**Number of tangents from an external point; lengths of two tangents from an external point are equal (theorem + proof).** From an external point exactly two tangents can be drawn to a circle, and their lengths are equal. This follows because each tangent is perpendicular to the radius at its point of contact, giving two right triangles that share the line from the centre to the external point and have equal radii.

### How marks are earned

1. From the external point Q, exactly two tangents can be drawn to the pond, named $QT_1$ and $QT_2$. (1 mark)
2. The lengths of the two tangents drawn from the external point Q to the pond are equal ($QT_1 = QT_2$). (1 mark)

**Source check:** Check the reasoning against NCERT Class 10 maths, Ch. 10, §10.3: A point outside the circle has exactly two tangents to the circle. Check the reasoning against NCERT Class 10 maths, Ch. 10, Summary: The lengths of the two tangents from an external point to a circle are equal.

OR

**Option B**

**(c)** Calculate the length of each rope $QT_1$ and $QT_2$ from the external point Q. [2 marks]

### Answer

**A radius to a tangent is perpendicular at contact, so $OT_1Q$ is right-angled and $QT_1^2 = OQ^2 - OT_1^2$. $QT_1 = \sqrt{(15^2 - 8^2)} = \sqrt{161}$ m; tangents from Q are equal, so $QT_2 = \sqrt{161}$ m too.**

### Step by step

1. **Use the tangent–radius perpendicularity to form a right triangle and write the Pythagoras relation.:** The radius $OT_1$ meets the tangent $QT_1$ at the point of contact, so $\angle OT_1Q = 90°$. In right triangle $OT_1Q$, $OQ^2 = OT_1^2 + QT_1^2$, so $QT_1^2 = OQ^2 - OT_1^2 = 15^2 - 8^2$. Why: The tangent at a point is perpendicular to the radius through that point, so triangle $OT_1Q$ has a right angle at $T_1$. OQ is then the hypotenuse, and Pythagoras applies with $OQ^2$ on one side and the sum of the squares of the two legs on the other.
2. **Calculate the length with units and apply the equal-tangents theorem for $QT_2$.:** $QT_1^2 = 225 - 64 = 161$, so $QT_1 = \sqrt{161}$ m ≈ 12.69 m. Since the two tangents from the external point Q are equal, $QT_2 = QT_1 = \sqrt{161}$ m. Check: $12.69^2 \approx 161$ and $161 + 64 = 225 = 15^2$. Why: The lengths of the two tangents from an external point to a circle are equal, so the same value applies to the second rope without repeating the calculation. Squaring the answer and adding $8^2$ to recover $15^2$ verifies that the arithmetic is consistent.

### Topic and concept

**Number of tangents from an external point; lengths of two tangents from an external point are equal (theorem + proof).** A tangent is perpendicular to the radius at its point of contact. This makes triangle $OT_1Q$ right-angled at $T_1$, so Pythagoras gives the tangent length from OQ and the radius. The equal-tangents theorem then gives the second rope the same length.

### How marks are earned

1. A radius to a tangent is perpendicular at contact, so $OT_1Q$ is right-angled and $QT_1^2 = OQ^2 - OT_1^2$. (1 mark)
2. $QT_1 = \sqrt{(15^2 - 8^2)} = \sqrt{161}$ m; tangents from Q are equal, so $QT_2 = \sqrt{161}$ m too. (1 mark)

**Source check:** Check the passage facts and NCERT Class 10 maths, Ch. 10, §10.2 against both steps.

## Question 38

**4 marks**

**Case Based**

A surveyor wants to determine the height of a tall tree without climbing it. She places a 2 m staff vertically on level ground and measures its shadow as 3 m at a particular time. At that same time, the tree's shadow measures 15 m. Assuming the sun's rays are parallel, she applies the principle of similar triangles to find the tree's height.

**(a)** Refer to the scenario: A surveyor places a 2 m staff vertically on level ground. At a certain instant, the staff's shadow measures 3 m and the shadow of a nearby tree measures 15 m. Assuming the sun's rays are parallel, identify the two triangles that are similar. [1 mark]

### Answer

**The right triangle formed by the 2 m staff and its 3 m shadow and the right triangle formed by the tree and its 15 m shadow are similar.**

### Step by step

1. **Identify the staff triangle and the tree triangle and give the angle reasons for similarity.:** The 2 m staff with its 3 m shadow forms one right triangle, and the tree with its 15 m shadow forms another. Both have a 90° angle where the vertical object meets level ground. Because the sun's rays are parallel, both also make the same angle with the ground at the tip of the shadow, so the triangles are similar by AA. Why: If two triangles have equal corresponding angles, their sides are in proportion, so they are similar. Two equal angles are enough, since the third angles must then be equal. Parallel sun rays give the equal elevation angle, which is why these two particular triangles, and not any others, are the similar pair.

### Topic and concept

**Applications of similarity criteria to prove triangles similar and indirect measurement.** Similar triangles allow indirect measurement of an inaccessible height. The staff and the tree are both vertical, and the parallel sun rays strike the ground at the same angle. So the two shadow triangles have equal angles and are similar, making their corresponding sides proportional.

### How marks are earned

1. The right triangle formed by the 2 m staff and its 3 m shadow and the right triangle formed by the tree and its 15 m shadow are similar. (1 mark)

**Source check:** Check the reasoning against NCERT Class 10 maths, Ch. 6, §6.3: If two triangles have equal corresponding angles, their sides are in proportion.

**(b)** Using the similar triangles from the scenario, write the correct proportional relation connecting the staff height (2 m), tree height (h), staff shadow (3 m), and tree shadow (15 m). [1 mark]

### Answer

**2 : h = 3 : 15, i.e., 2/h = 3/15.**

### Step by step

1. **Match corresponding sides of the two similar triangles and equate their ratios.:** In the similar triangles, the staff height 2 m corresponds to the tree height h, and the staff shadow 3 m corresponds to the tree shadow 15 m. So staff height : tree height = staff shadow : tree shadow, giving 2 : h = 3 : 15, that is 2/h = 3/15. Why: In similar triangles the ratios of corresponding sides are equal. Heights are opposite the sun's angle and shadows are adjacent to it, so a height must be paired with a height and a shadow with a shadow. Mixing them, for instance 2/3 = 15/h, would pair non-corresponding sides.

### Topic and concept

**Applications of similarity criteria to prove triangles similar and indirect measurement.** For similar triangles, corresponding sides are in the same ratio. Here the vertical sides (staff and tree) correspond to each other and the shadow sides on the ground correspond to each other. This gives 2/h = 3/15, which can later be solved for h.

### How marks are earned

1. 2 : h = 3 : 15, i.e., 2/h = 3/15. (1 mark)

**Source check:** Check the reasoning against NCERT Class 10 maths, Ch. 6, §6.3: In similar triangles, ratios of corresponding sides are equal.

Attempt one of the final two-mark options.

**Option A**

**(c)** (i) Calculate the height of the tree based on the proportionality you have written. (ii) Which concept from Chapter 6 justifies the use of proportional triangles to find the tree's height indirectly? [2 marks]

### Answer

**From 2/h = 3/15, cross-multiply to get 2×15 = 3×h, so 30 = 3h and h = 10 m. The application of similarity of triangles for indirect measurement (Concept CH6-C08) justifies this method.**

### Step by step

1. **Solve the proportion for the tree's height:** The staff and tree with their shadows form similar triangles, so 2/h = 3/15 (all lengths in m). Cross-multiplying gives 2 × 15 = 3 × h, so 30 = 3h and h = 30/3 = 10 m. Check: 2/10 = 0.2 and 3/15 = 0.2, so the ratios agree. Why: In similar triangles the ratios of corresponding sides are equal, so staff height : staff shadow = tree height : tree shadow. Cross-multiplication then isolates the single unknown length, with the unit m carried through.

2. **Name the Chapter 6 concept that justifies the method:** The two triangles have a right angle at the base (vertical object on level ground) and equal angles at the shadow tips (parallel sun's rays), so they are similar by the AA criterion. This similarity of triangles allows indirect measurement of the tree's height without climbing it. Why: Chapter 6 shows that triangles with equal corresponding angles have proportional sides, so a short measurable staff can stand in for an unreachable tree. Parallel rays are what make the angles equal.

### Topic and concept

**Applications of similarity criteria to prove triangles similar and indirect measurement.** Similarity of triangles lets us find large or unreachable lengths, such as the height of a tree, by comparing them with a measurable triangle. When corresponding angles are equal (AA criterion), corresponding sides are in proportion, giving 2/h = 3/15 and h = 10 m.

### How marks are earned

1. From 2/h = 3/15, cross-multiply to get 2×15 = 3×h, so 30 = 3h and h = 10 m. (1 mark)
2. The application of similarity of triangles for indirect measurement (Concept CH6-C08) justifies this method. (1 mark)

**Source check:** Check the reasoning against NCERT Class 10 maths, Ch. 6, §6.3: Solving the proportionality gives the unknown length. Check the reasoning against NCERT Class 10 maths, Ch. 6, §6.5: Similarity lets us find lengths indirectly using proportional triangles.

OR

**Option B**

**(c)** Another tree casts a 9 m shadow at the same instant. Calculate its height and state why the staff ratio applies. [2 marks]

### Answer

**Parallel sunlight makes the staff and tree triangles similar, so height/shadow = 2/3 for both. For a 9 m shadow, height = (2/3) × 9 m = 6 m.**

### Step by step

1. **Establish why the staff ratio applies using similar triangles:** The staff (2 m, shadow 3 m) and the tree (shadow 15 m) are both vertical on level ground, so each makes a 90° angle with the ground; at the same instant the parallel sun's rays meet the ground at the same angle, so each object forms a right triangle with its shadow and the sun's ray, and these triangles have two equal angles (90° and the sun's elevation). By AA similarity they are similar, so height/shadow = 2/3 for the staff, and the same ratio 2/3 holds for the tree and for any other vertical object at that instant (check: 15 m shadow gives 10 m height, consistent with 10/15 = 2/3). Why: Equal corresponding angles make the triangles similar (NCERT Ch. 6, §6.3), and in similar triangles the ratios of corresponding sides are equal, so height-to-shadow is the same constant for every vertical object at that instant. This is why the staff's ratio can be transferred to the tree.

2. **Use the equal ratio to find the second tree's height:** Let the height of the second tree be h m. Then h/9 = 2/3, so h = (2/3) × 9 = 18/3 = 6 m. Check: 6/9 = 2/3, which matches the staff's ratio 2/3, and the answer is in metres like the given data. Why: Corresponding sides of similar triangles are in proportion, so equating the two height-to-shadow ratios gives an equation with one unknown. Solving it gives the unknown length indirectly, as in the indirect-measurement use of similarity (NCERT Ch. 6, §6.5).

### Topic and concept

**Applications of similarity criteria to prove triangles similar and indirect measurement.** Similar triangles allow indirect measurement of heights. At the same instant the sun's rays are parallel, so the sun's elevation angle is the same for every object. Any vertical object and its shadow form a right triangle with this same angle, so two such triangles are similar by AA. Hence height/shadow is constant (here 2/3), and an unknown height follows from a proportion.

### How marks are earned

1. Parallel sunlight makes the staff and tree triangles similar, so height/shadow = 2/3 for both. (1 mark)
2. For a 9 m shadow, height = (2/3) × 9 m = 6 m. (1 mark)

**Source check:** Check the passage facts and NCERT Class 10 maths, Ch. 6, §6.3 against both steps.