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Class X practice studio

CLASS X · 2026-27 · MODEL PAPER

Class X Mathematics Standard (Code 041) Model Paper 2

Worked answers & explanations

Maximum marks: 80

Time: 3 hours

Subject code: 041

Practice, understand, improve.

Instructions

All 38 questions are compulsory, with one selected option for each internal choice.

Section A has 18 MCQs and 2 assertion-reason questions; Section E has 1+1+2-mark case subparts.

Calculators are not allowed; draw labelled figures where needed.

Section A

Question 1

1 mark

Mcq

Choose the correct answer.

(a) If the graph of y = p(x) passes through the point (-1,0), which of the following is a factor of p(x)?
[1 mark]

A. x−1
B. x+1
C. x
D. 1−x

Answer

B. x+1

Step by step

1. **Select and justify the option:** Because p(−1)=0 (point on x-axis), (x−(−1))=(x+1) must be a factor of p(x). Why: A point (k,0) on the graph means k is a zero and (x−k) is a factor.

Topic and concept

Zeros of a polynomial; geometric meaning of zeros as x-intercepts. The zeros of p(x) correspond to the x-coordinates where the graph of y=p(x) crosses the x-axis.

How marks are earned

1. Because p(−1)=0 (point on x-axis), (x−(−1))=(x+1) must be a factor of p(x). (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 2, §2.2 Geometrical Meaning of the Zeroes of a Polynomial: A point (k,0) on the graph means k is a zero and (x−k) is a factor.

Question 2

1 mark

Mcq

Choose the correct answer.

(a) According to the profit model $P(x)=2x^2-10x+12$, which production levels x (in hundreds of units) make the company break even? [1 mark]

A. $x=-2$ and $x=-3$

B. $x=2$ and $x=3$

C. $x=5$ and $x=6$

D. $x=-5$ and $x=-6$

Answer

B. x=2 and x=3

Step by step

1. **Select and justify the option:** Set $P(x)=0 \Rightarrow 2x^2-10x+12=0$, divide by 2 $\Rightarrow x^2-5x+6=0$, which factors to $(x-2)(x-3)=0$, giving $x=2$ or $x=3$. Why: The zeroes of p(x) are the x-coordinates where the graph of y=p(x) meets the x-axis, hence break-even points.

Topic and concept

Zeros of a polynomial; geometric meaning of zeros as x-intercepts. A polynomial of degree n can have at most n real zeros; in particular a quadratic can have at most two zeros.

How marks are earned

1. Set $P(x)=0 \Rightarrow 2x^2-10x+12=0$, divide by 2 $\Rightarrow x^2-5x+6=0$, which factors to $(x-2)(x-3)=0$, giving $x=2$ or $x=3$. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 2, §2.2: The zeroes of p(x) are the x-coordinates where the graph of y=p(x) meets the x-axis, hence break-even points.

Question 3

1 mark

Mcq

Choose the correct answer.

(a) For the linear polynomial $p(x)=2x+3$, what is its unique zero? [1 mark]

A. $-3/2$

B. 1.5

C. −3

D. 3

Answer

A. −3/2

Step by step

1. **Set p(x)=0, solve for x, and verify the keyed option:** Set 2x+3=0. Subtract 3 from both sides to get 2x=−3, then divide by 2 to get x=−3/2 (= −1.5). Check: p(−3/2)=2(−3/2)+3=−3+3=0, so −3/2 is the zero and option A is correct. Why: A zero of a polynomial is a value k with p(k)=0, and for a linear polynomial ax+b this gives exactly one value, the x-coordinate where its graph meets the x-axis. Solving the equation and then substituting back confirms the value satisfies the definition.

Topic and concept

Zeros of a polynomial; geometric meaning of zeros as x-intercepts. A real number k is a zero of p(x) if p(k)=0. For the linear polynomial 2x+3, equating it to zero and solving gives its single zero, which is the point where the graph crosses the x-axis.

How marks are earned

1. Set 2x+3=0, giving x=−3/2 as the zero. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 2, §2.2 Geometrical Meaning of the Zeroes of a Polynomial: The zero of ax+b is the x-coordinate where the graph meets the x-axis.

Question 4

1 mark

Mcq

Choose the correct answer.

(a) Which of the following correctly relates the zeros α, β of ax^2+bx+c to its coefficients? [1 mark]

A. α+β=c/a and αβ=−b/a

B. α+β=−b/a and αβ=c/a

C. α+β=b/a and αβ=c/a

D. α+β=−c/a and αβ=b/a

Answer

B. α+β=−b/a and αβ=c/a

Step by step

1. **Derive the sum and product of zeros by comparing coefficients, then select option B:** If α and β are the zeros of ax^2+bx+c, then $ax^2+bx+c = a(x-\alpha)(x-\beta) = ax^2 - a(\alpha+\beta)x + a\alpha\beta$. Comparing the coefficients of x gives $b = -a(\alpha+\beta)$, so $\alpha+\beta = -b/a$. Comparing the constant terms gives $c = a\alpha\beta$, so $\alpha\beta = c/a$. Check with x^2-5x+6 ($a=1$, $b=-5$, $c=6$): the zeros are 2 and 3, the sum $5 = -(-5)/1$ and the product $6 = 6/1$, which matches option B. Why: A quadratic with zeros α and β must be a constant multiple of $(x-\alpha)(x-\beta)$, so equating coefficients of like powers links the zeros directly to a, b and c. The sum carries a negative sign because it comes from the x-term of $a(x-\alpha)(x-\beta)$, which is $-a(\alpha+\beta)x$. Option A swaps the two results, and C and D drop or misplace the sign or the coefficient, so each fails the x^2-5x+6 check.

Topic and concept

Relationship between zeros and coefficients for quadratic polynomials. For a quadratic ax^2+bx+c with zeros α and β, factorising as $a(x-\alpha)(x-\beta)$ and comparing coefficients shows that the sum of the zeros is $-b/a$ and the product is c/a. These are the relations in NCERT Class 10 Maths Ch. 2, §2.3.

How marks are earned

1. For ax^2+bx+c, $\alpha+\beta=-b/a$ and $\alpha\beta=c/a$. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 2, §2.3 Relationship between Zeroes and Coefficients of a Polynomial: $\alpha+\beta=-b/a$ and $\alpha\beta=c/a$ for a quadratic ax^2+bx+c.

Question 5

1 mark

Mcq

Choose the correct answer.

(a) In a tower height-and-distance problem, why may a horizontal line be drawn from the top of the tower? [1 mark]

A. To form right triangles for trigonometric ratios

B. To measure vertical angles directly

C. To construct a circle radius

D. To create a chord

Answer

A. The horizontal line and vertical tower height form right triangles for trigonometric ratios.

Step by step

1. **Choose the geometrical purpose:** A horizontal line drawn from the tower top is parallel to level ground and helps form right triangles with the vertical height. Why: The right triangles then allow sine, cosine or tangent ratios to connect heights, distances and elevation or depression angles.

Topic and concept

Problems involving two right triangles and two angles of elevation/depression. In heights-and-distances problems, horizontal and vertical lines form right angles. The resulting right triangles support trigonometric ratios for angles of elevation and depression.

How marks are earned

1. Select A and explain that horizontal and vertical lines give right triangles in which trigonometric ratios can be used. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 9, §9.1 Heights and Distances: A horizontal through the top and the vertical tower form right triangles.

Question 6

1 mark

Mcq

Choose the correct answer.

(a) Which condition is necessary and sufficient for a number to be irrational? [1 mark]

A. It cannot be written as p/q for integers p,q

B. Its decimal expansion terminates

C. Its decimal expansion repeats periodically

D. It can be expressed as a root of a prime

Answer

A. It cannot be written as p/q for integers p,q

Step by step

1. **Match the definition of an irrational number to the options and reject the others:** An irrational number cannot be written as p/q with integers p, q and q≠0, so A is the defining condition. A terminating decimal such as 0.75 = 3/4 and a repeating decimal such as 0.333… = 1/3 can both be written as p/q, so they are rational, which rules out B and C. Option D has no meaning as a test for irrationality. Why: Irrationality is defined by the impossibility of writing the number as p/q. Such numbers have decimal expansions that neither terminate nor repeat, so a terminating or repeating decimal cannot be irrational.

Topic and concept

Proofs of irrationality of $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$ and similar surds. A number is irrational exactly when it cannot be expressed as p/q for integers p and q with q≠0. Equivalently, its decimal expansion is non-terminating and non-repeating. Terminating and repeating decimals are therefore rational.

How marks are earned

1. An irrational number cannot be expressed as a ratio of two integers and has a non-terminating, non-repeating decimal expansion. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 1, §1.3: A number is irrational if it cannot be written as p/q and its decimal is non-terminating, non-repeating.

Question 7

1 mark

Mcq

Choose the correct answer.

(a) A student uses the diameter d instead of the radius r in the area formula and computes area as πd^2. How many times larger is this result than the correct area πr^2? [1 mark]

A. 4

B. 2

C. 1/2

D. 1/4

Answer

A. 4

Step by step

1. **Select and justify the option:** Since $d = 2r$, $\pi d^2 = \pi(2r)^2 = 4\pi r^2$, so the mistaken area is four times larger. Why: Area of a circle is πr^2; substituting $r=d/2$ shows $\pi d^2 = 4\pi r^2$.

Topic and concept

Area and circumference of a circle. The area of a circle with radius r is given by $A = \pi r^2$.

How marks are earned

1. Since $d = 2r$, $\pi d^2 = \pi(2r)^2 = 4\pi r^2$, so the mistaken area is four times larger. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 11, §11.2 Summary: Area of a circle is πr^2; substituting $r=d/2$ shows $\pi d^2 = 4\pi r^2$.

Question 8

1 mark

Mcq

Choose the correct answer.

(a) Which of the following is not a polynomial in the variable x? [1 mark]

A. $\sqrt{x} + 3$

B. $3x - 2$

C. $x^2 + 1$

D. $x^3 - x$

Answer

A. $\sqrt{x} + 3$

Step by step

1. **Test each expression against the definition of a polynomial and select option A:** A polynomial in x has only whole-number powers of x (0, 1, 2, 3, ...). Writing $\sqrt{x} + 3$ as x^(1/2) + 3 gives a power 1/2, which is not a whole number. The other options are $3x - 2$ (powers 1 and 0), $x^2 + 1$ (powers 2 and 0) and $x^3 - x$ (powers 3 and 1), and all of these are whole numbers. Only $\sqrt{x} + 3$ fails, so the answer is A. Why: A polynomial is a sum of terms of the form (coefficient) × x^n with n a non-negative whole number. A fractional exponent such as 1/2 breaks this rule, so $\sqrt{x} + 3$ is not a polynomial even though it looks simple. Options B, C and D each satisfy the rule, so none of them can be the keyed answer.

Topic and concept

Zeros of a polynomial; geometric meaning of zeros as x-intercepts. A polynomial in x is an expression whose terms are constants times non-negative whole-number powers of x. An expression with a fractional exponent, such as $\sqrt{x}$ = x^(1/2), or with x in a denominator is not a polynomial (NCERT Class 10 Maths Ch. 2, §2.1).

How marks are earned

1. An expression involving a fractional exponent ($\sqrt{x}$+3) is not a polynomial. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 2, §2.1 Introduction: Expressions with non-integer exponents or variables in the denominator are not polynomials.

Question 9

1 mark

Mcq

Choose the correct answer.

(a) Which formula gives the solutions of $ax^2 + bx + c = 0$? [1 mark]

A. $x = (-b \pm \sqrt{(b^2 + 4ac)}) / 2a$

B. $x = (-b \pm \sqrt{(b^2 - 4ac)}) / 2a$

C. $x = (b \pm \sqrt{(b^2 - 4ac)}) / 2a$

D. $x = (-b + \sqrt{(4ac - b^2)}) / a$

Answer

B. $x = (-b \pm \sqrt{(b^2 - 4ac)}) / 2a$

Step by step

1. **Compare each option with the Sridharacharya formula and eliminate the faulty ones:** The roots of $ax^2+bx+c=0$ are $x=(-b \pm \sqrt{(b^2-4ac)})/2a$, which is option B. Option A has $+4ac$ under the root, so the discriminant is wrong. Option C starts with $+b$ instead of $-b$. Option D has $-b + \sqrt{(4ac-b^2)}$ only, drops the $\pm$ and divides by a instead of $2a$. Only B has all four features correct. Why: The formula comes from completing the square on $ax^2+bx+c=0$. This produces $-b$ in the numerator, the discriminant b^2-4ac under the root, $\pm$ for the two roots, and the denominator $2a$. Any change to these parts gives wrong roots.

Topic and concept

Solution by quadratic formula $x = (-b \pm \sqrt{(b^2-4ac)}) / 2a$. The Sridharacharya (quadratic) formula $x=(-b \pm \sqrt{(b^2-4ac)})/2a$ gives the roots of $ax^2+bx+c=0$ ($a \neq 0$) when the discriminant b^2-4ac is non-negative. Its parts are $-b$, the discriminant under the root, $\pm$, and the denominator $2a$.

How marks are earned

1. Option B matches $x = (-b \pm \sqrt{(b^2 - 4ac)}) / 2a$ exactly as given in Sridharacharya's formula. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 4, §4.4 Nature of Roots: The roots of $ax^2 + bx + c = 0$ are given by $x = (-b \pm \sqrt{(b^2 - 4ac)}) / 2a$.

Question 10

1 mark

Mcq

Choose the correct answer.

(a) Which of the following points lies on the y-axis? [1 mark]

A. (0, 5)

B. (5, 0)

C. (−3, 4)

D. (2, −1)

Answer

A. (0, 5)

Step by step

1. **Select and justify the option:** A point on the y-axis has coordinates of the form (0, y), so (0, 5) lies on the y-axis. Why: Coordinates of a point on the y-axis are of the form (0, y).

Topic and concept

Using coordinate axes to locate points. Every point in the plane is identified by an ordered pair (x,y), where x (abscissa) is its distance from the y-axis and y (ordinate) is its distance from the x-axis.

How marks are earned

1. A point on the y-axis has coordinates of the form (0, y), so (0, 5) lies on the y-axis. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 7, §7.1 Introduction: Coordinates of a point on the y-axis are of the form (0, y).

Question 11

1 mark

Mcq

Choose the correct answer.

(a) The graph of a pair of linear equations in two variables always consists of: [1 mark]

A. a single straight line

B. two straight lines

C. a parabola

D. a circle

Answer

B. two straight lines

Step by step

1. **Select and justify the option:** Each equation in two variables is represented by one straight line, so a pair gives two lines. Why: Each linear equation in two variables is represented by a straight line in the plane.

Topic and concept

Graphical representation of pair of linear equations; consistent, inconsistent, dependent conditions. A pair of linear equations in two variables can be represented graphically as two lines in the plane.

How marks are earned

1. Each equation in two variables is represented by one straight line, so a pair gives two lines. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 3, §3.2: Each linear equation in two variables is represented by a straight line in the plane.

Question 12

1 mark

Mcq

Choose the correct answer.

(a) Using the distance formula, find the distance between P(2, 3) and Q(5, 7). [1 mark]

A. 5 units

B. √10 units

C. √13 units

D. √20 units

Answer

A. 5 units

Step by step

1. **Apply the distance formula:** PQ=√[(5−2)²+(7−3)²]=√(9+16)=√25=5 units. Why: The horizontal and vertical coordinate changes are 3 and 4, so Pythagoras gives distance 5.

Topic and concept

Distance formula between two points. For any two points (x_1,y_1) and (x_2,y_2), distance is $\sqrt{[(x_2-x_1)^2+(y_2-y_1)^2]}$. Here both coordinates change, so use both squared differences.

How marks are earned

1. Distance = √[(5−2)² + (7−3)²] = √[3² + 4²] = 5 units. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 7, §7.2 Distance Formula: The distance between $P(x_1,y_1)$ and $Q(x_2,y_2)$ is $\sqrt{[(x_2-x_1)^2+(y_2-y_1)^2]}$.

Question 13

1 mark

Mcq

Choose the correct answer.

(a) A solid is made by joining a cylinder and a hemisphere without overlap. How is its volume found? [1 mark]

A. Add the cylinder volume and the hemisphere volume

B. Multiply the two volumes

C. Subtract the hemisphere volume from the cylinder volume

D. Add their curved surface areas

Answer

A. Add the cylinder volume and the hemisphere volume.

Step by step

1. **Apply the additive rule for volumes of a composite solid and reject the other options:** The solid is a cylinder joined to a hemisphere with no overlap, so its volume is V = V_cylinder + V_hemisphere, which is option A. Multiplying volumes (B) is not meaningful, since volume is added. Subtracting (C) would apply only if a part were hollowed out. Adding curved surface areas (D) gives an area, not a volume. Why: Volume measures the space occupied. Two non-overlapping parts occupy separate regions, so the total space is the sum of the two volumes.

Topic and concept

Combinations are restricted to any two cubes, cuboids, spheres, hemispheres and right circular cylinders or cones. When two solids are combined without overlap, the volume of the new solid equals the sum of the volumes of the individual solids. For a cylinder and a hemisphere, V = V_cylinder + V_hemisphere.

How marks are earned

1. Choose A because the composite solid contains both non-overlapping parts, so its volume is V_cylinder+V_hemisphere. (1 mark)

Source check: NCERT Class 10 maths, Ch. 12, §12.3: the volume of joined non-overlapping solids is the sum of their volumes.

Question 14

1 mark

Mcq

Choose the correct answer.

(a) A non-overlapping composite figure consists of an equilateral triangle of side r and a 60° sector of a circle of radius r. Which expression gives its total area? [1 mark]

A. $r^2(\pi/6 + \sqrt{3}/4)$

B. $r^2(\pi/6 - \sqrt{3}/4)$

C. $r^2(\pi/3 + \sqrt{3}/4)$

D. $r^2(\pi/4 + \sqrt{3}/2)$

Answer

A. Add triangle area $(\sqrt{3}/4)r^2$ and sector area $(\pi/6)r^2$ to obtain $r^2(\pi/6+\sqrt{3}/4)$.

Step by step

1. **Add the two areas:** A 60° sector of radius r has area $(60/360)\pi r^2=\pi r^2/6$. An equilateral triangle of side r has area $(\sqrt{3}/4)r^2$. Their non-overlapping total is $r^2(\pi/6+\sqrt{3}/4)$. Why: The revised prompt explicitly states both the sector radius and triangle side as r, so each formula uses the same length.

Topic and concept

Combination of plane figures (triangle + sector, etc.). For a non-overlapping composite figure, total area is the sum of its component areas. A 60° sector of radius r contributes $\pi r^2/6$; an equilateral triangle of side r contributes $\sqrt{3}r^2/4$.

How marks are earned

1. Select A by adding the 60° sector area $\pi r^2/6$ and equilateral-triangle area $\sqrt{3}r^2/4$ for the specified radius and side r. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 11, §11.1 Areas of Sector and Segment of a Circle: Total area of composite figures is obtained by adding sectors and triangles as required.

Question 15

1 mark

Mcq

Choose the correct answer.

(a) What is the midpoint of the points P(−2, 7) and Q(4, 1)? [1 mark]

A. (1, 4)

B. (2, 3)

C. (−1, 8)

D. (6, 2)

Answer

A. (1, 4)

Step by step

1. **Select and justify the option:** Midpoint = ((−2+4)/2, (7+1)/2) = (1, 4). Why: The midpoint of (x_1,y_1) and (x_2,y_2) is $((x_1+x_2)/2,(y_1+y_2)/2)$.

Topic and concept

Midpoint as a special case of internal division. The midpoint of segment PQ is obtained by setting m=n=1 in the section formula, giving $((x_1+x_2)/2,(y_1+y_2)/2)$.

How marks are earned

1. Midpoint = ((−2+4)/2, (7+1)/2) = (1, 4). (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 7, §7.4 Summary: The midpoint of (x_1,y_1) and (x_2,y_2) is $((x_1+x_2)/2,(y_1+y_2)/2)$.

Question 16

1 mark

Mcq

Choose the correct answer.

(a) If P(E) = 0.3 for some event E, what is P(Ē), where Ē is the complement of E? [1 mark]

A. 0.7

B. 0.3

C. 1.3

D. 0

Answer

A. 0.7

Step by step

1. **Use the complement rule:** Use the complement rule: $P(\bar{E})=1-P(E)=1-0.3=0.7$. Why: E and its complement cover the whole sample space without overlap, so their probabilities add to one.

Topic and concept

Elementary events and sum of probabilities. For any event E, $P(\bar{E})=1-P(E)$, because E and Ē are mutually exclusive and exhaustive.

How marks are earned

1. By the complement rule, $P(\bar{E}) = 1 - P(E) = 1 - 0.3 = 0.7$. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 14, §Summary: $P(E) + P(\bar{E}) = 1$, so $P(\bar{E})=1-P(E)$.

Question 17

1 mark

Mcq

Choose the correct answer.

(a) In an AP, the first term is 7 and the fifth term is 19. What is the common difference? [1 mark]

A. 2

B. 3

C. 4

D. 5

Answer

B. 3

Step by step

1. **Select and justify the option:** Using $a_5 = a + 4d$, we have $19 = 7 + 4d$, giving $d = 3$. Why: The nth term $a_n = a + (n-1)d$ relates the term number to a and d.

Topic and concept

nth term formula: $a_n = a + (n-1)d$. The nth term of an AP with first term a and common difference d is given by $a_n = a + (n - 1)d$.

How marks are earned

1. Using $a_5 = a + 4d$, we have $19 = 7 + 4d$, giving $d = 3$. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 5, §5.3: The nth term $a_n = a + (n-1)d$ relates the term number to a and d.

Question 18

1 mark

Mcq

Choose the correct answer.

(a) Express the circumference of a circle in terms of its diameter d. [1 mark]

A. $2\pi d$

B. πd

C. πd^2

D. $2\pi r$

Answer

B. πd

Step by step

1. **Replace radius with diameter:** Start with $C=2\pi r$ and $d=2r$; substitute to obtain $C=\pi d$. Why: The diameter is twice the radius, and substituting it gives the circumference in the requested variable.

Topic and concept

Area and circumference of a circle. A circle's circumference is $C=2\pi r$. Since $d=2r$, the same length is $C=\pi d$.

How marks are earned

1. Since circumference $C = 2\pi r$ and $d = 2r$, substituting $r = d/2$ gives $C = \pi d$. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 11, §11.2 Summary: Circumference of a circle is 2πr, and since d=2r, C = πd.

Question 19

1 mark

Assertion Reason

Choose the correct answer.

(a) Assertion (A): A line intersecting a circle at two points is a tangent to the circle. Reason (R): A tangent touches the circle at exactly one point. [1 mark]

A. Both A and R true and R explains A

B. Both A and R true but R does not explain A

C. A true, R false

D. A false, R true

Answer

D. A false, R true

Step by step

1. **Select and justify the option:** The assertion is false: a line intersecting a circle in two points is a secant, not a tangent. The reason is true: a tangent touches the circle at exactly one point. Why: A secant meets the circle at two points; a tangent meets it at one.

Topic and concept

Tangent to a circle; tangent at any point is perpendicular to the radius (theorem + proof). A tangent to a circle is a line that touches the circle at exactly one point, unlike a secant which meets it at two points or a non-intersecting line which meets it at none.

How marks are earned

1. The assertion is false: a line intersecting a circle in two points is a secant, not a tangent. The reason is true: a tangent touches the circle at exactly one point. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 10, §10.1: A secant meets the circle at two points; a tangent meets it at one.

Question 20

1 mark

Assertion Reason

Choose the correct answer.

(a) Assertion (A): In a heights and distances problem, the angle of depression from the top of a tower equals the angle of elevation of the tower from the point on the ground. Reason (R): A horizontal line drawn through the top of the tower is parallel to the ground level, making alternate interior angles equal. [1 mark]

A. Both A and R true and R explains A

B. Both A and R true but R does not explain A

C. A true, R false

D. A false, R true

Answer

A. Both A and R true and R explains A

Step by step

1. **Compare the two angles:** A horizontal through the tower top is parallel to level ground. The line of sight crosses them, making the depression and elevation angles equal as alternate interior angles; choose A. Why: The parallel-line angle relationship directly explains the equality in the assertion.

Topic and concept

Problems involving two right triangles and two angles of elevation/depression. When two horizontal lines are parallel and a line of sight crosses them, the angle of depression at the top equals the angle of elevation at the bottom as alternate interior angles.

How marks are earned

1. Both A and R are true; the reason explains that since the horizontal through the top is parallel to the ground, alternate interior angles (depression and elevation) are equal. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 9, §9.1 Heights and Distances: Horizontal lines are parallel, so alternate interior angles of depression and elevation are equal.

Section B

Question 21

2 marks

Short Answer

Attempt one option.

Option A

Answer the question with working and reasons where needed.

(a) A student standing 100 m away from the base of a vertical tower measures the angle of elevation of its top as 45°. Calculate the height of the tower. [2 marks]

Answer

Using the model tan θ = height / distance, we write tan 45° = h / 100. Since tan 45° = 1, we get h = 100 m, so the tower's height is 100 metres.

Step by step

1. **Set up the right triangle and write the tangent ratio:** Picture a right triangle with the observer at one vertex, the tower's base at the right-angle vertex and the tower's top at the third vertex. The horizontal side from observer to base is the adjacent side, equal to 100 m. The vertical side, the tower height h, is the opposite side. The angle of elevation at the observer is 45°. Since tan θ = opposite/adjacent = height/distance, we write tan 45° = h/100. Why: The line of sight, the ground and the vertical tower form a right triangle, and the ratio that links the opposite side (the unknown height) with the adjacent side (the known distance) is the tangent. Choosing tan avoids needing the hypotenuse, which is not given. This is the use of trigonometric ratios for heights and distances in NCERT Class 10 Maths Ch. 9, §9.2.
2. **Substitute tan 45° = 1 and solve for h with the unit:** Since tan 45° = 1, the equation becomes 1 = h/100. Multiplying both sides by 100 m gives h = 1 × 100 = 100 m, so the tower is 100 metres high. Check: with a 45° angle the two legs of the right triangle are equal, and 100 m = 100 m. Why: The standard value tan 45° = 1 reflects that at 45° the opposite and adjacent sides are equal, which gives a quick check on the answer. Multiplying by the distance carries its unit, so the height comes out in metres.

Topic and concept

Angle of elevation and angle of depression; applications to heights and distances. In a right triangle formed by the observer, the base and the top of an object, tan θ = height / horizontal distance, where θ is the angle of elevation. Knowing the angle and one side lets us find the unknown height (NCERT Class 10 Maths Ch. 9, §9.2).

How marks are earned

1. Using the model tan θ = height / distance, we write tan 45° = h / 100. (1 mark)
2. Since tan 45° = 1, we get h = 100 m, so the tower's height is 100 metres. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 9, §9.2 Summary point 2: Heights and distances can be determined using trigonometric ratios.

OR

Option B

Answer the question with working and reasons where needed.

(a) A vertical tower has a height of 10√3 m. From a point on level ground, the angle of elevation of its top is found to be 60°. Determine the horizontal distance from the point to the base of the tower. [2 marks]

Answer

Applying tan 60° = height / distance gives √3 = (10√3) / d, so d = (10√3)/√3 = 10 m. Therefore, the horizontal distance from the point to the tower's base is 10 metres.

Step by step

1. **Model the situation as a right triangle and choose tan 60° = height/distance:** Draw a right triangle with the tower as the vertical side of 10√3 m, the level ground from the point to the base as the horizontal side d, and the line of sight to the top as the hypotenuse, with the 60° angle of elevation at the point. The height is opposite the 60° angle and d is adjacent to it, so tan 60° = (10√3)/d, which gives √3 = (10√3)/d. Why: The angle of elevation is measured from the horizontal at the observer, so the tower height is the opposite side and the ground distance is the adjacent side. tan θ = opposite/adjacent is the only ratio that joins these two sides without needing the hypotenuse (NCERT Class 10 Maths Ch. 9, §9.2).
2. **Solve for d, check by substitution and state the answer with units:** From √3 = (10√3)/d, multiply both sides by d to get d√3 = 10√3, then divide both sides by √3 to get d = (10√3)/√3 = 10. Check: tan 60° = 10√3/10 = √3, which matches the known value. So the horizontal distance from the point to the base of the tower is 10 metres. Why: Dividing by the common factor √3 isolates d, and the unit is metres because the height was given in metres. Substituting d = 10 back reproduces tan 60° = √3, which confirms that the ratio was set up the right way up and the arithmetic is right.

Topic and concept

Angle of elevation and angle of depression; applications to heights and distances. When a tower, the level ground and the line of sight form a right triangle, the angle of elevation at the observer's eye gives tan θ = height / horizontal distance. Here tan 60° = √3 and the height is 10√3 m, so the unknown distance follows from one equation (NCERT Class 10 Maths Ch. 9, §9.2).

How marks are earned

1. Applying tan 60° = height / distance gives √3 = (10√3) / d, so d = (10√3)/√3 = 10 m. (1 mark)

2. Therefore, the horizontal distance from the point to the tower's base is 10 metres. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 9, §9.2 Summary point 2: Heights and distances can be determined using trigonometric ratios.

Question 22

2 marks

Short Answer

Attempt one option.

Option A

Answer the question with working and reasons where needed.

(a) Express the equation $4x - 5 + x^2 = 0$ in the standard form of a quadratic equation and identify the coefficients a, b, and c. [2 marks]

Answer

By arranging the given equation $4x - 5 + x^2 = 0$ in descending powers of x, it takes the standard form $x^2 + 4x - 5 = 0$. On comparing the standard form $x^2 + 4x - 5 = 0$ with $ax^2 + bx + c = 0$, we identify the coefficients as a = 1, b = 4, and c = -5.

Step by step

1. **Rearrange the terms in descending powers of x:** The given equation is $4x - 5 + x^2 = 0$. The highest power is x^2 (degree 2), so it is quadratic. Reordering the terms from the highest power to the lowest, keeping each term's sign, gives $x^2 + 4x - 5 = 0$. Why: The standard form $ax^2 + bx + c = 0$ lists the x^2 term first, then the x term, then the constant. Reordering terms does not change the equation, because addition is commutative, but it lines up each term with its place in the standard form. The equation is quadratic because its highest power is 2.
2. **Compare with $ax^2 + bx + c = 0$ to read off a, b and c:** Comparing $x^2 + 4x - 5 = 0$ with $ax^2 + bx + c = 0$ term by term: the coefficient of x^2 is a = 1, the coefficient of x is b = 4, and the constant term is c = −5. Check: a = 1 is not zero, so the equation is a genuine quadratic. Why: A coefficient includes its sign, so the term −5 gives c = −5 and not 5. The condition $a \neq 0$ is needed, because if a were 0 the x^2 term would vanish and the equation would be linear, not quadratic (NCERT Class 10 Maths Ch. 4, §4.2).

Topic and concept

Standard form $ax^2 + bx + c = 0$ ($a \neq 0$); real roots and their existence. A quadratic equation is a second-degree polynomial set equal to zero, written $ax^2 + bx + c = 0$ with $a \neq 0$. To identify a, b and c, arrange the terms in descending powers and compare with this form, keeping the signs (NCERT Class 10 Maths Ch. 4, §4.2).

How marks are earned

1. By arranging the given equation $4x - 5 + x^2 = 0$ in descending powers of x, it takes the standard form $x^2 + 4x - 5 = 0$. (1 mark)

2. On comparing the standard form $x^2 + 4x - 5 = 0$ with $ax^2 + bx + c = 0$, we identify the coefficients as $a = 1$, $b = 4$, and $c = -5$. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 4, §4.2 Quadratic Equations: Writing the terms of a quadratic polynomial in descending order yields its standard form $ax^2 + bx + c = 0$.

OR

Option B

Answer the question with working and reasons where needed.

(a) Solve the quadratic equation $x^2 - x - 12 = 0$ by factorisation. [2 marks]

Answer

Factorize the left-hand side as $(x - 4)(x + 3) = 0$ by finding numbers whose product is -12 and sum is -1. Set each factor to zero: $x - 4 = 0$ or $x + 3 = 0$, yielding roots $x = 4$ and $x = -3$.

Step by step

1. **Split the middle term and factorise x^2-x-12:** We need two numbers with product $ac = -12$ and sum $b = -1$. These are -4 and $+3$, since $(-4)(3) = -12$ and $-4+3 = -1$. So $x^2-x-12 = x^2-4x+3x-12 = x(x-4)+3(x-4) = (x-4)(x+3)$. Why: Splitting the middle term lets the quadratic be grouped into two linear factors with a common bracket. The numbers must multiply to ac and add to b so that the split terms combine back to $-x$.
2. **Equate each factor to zero and verify the roots:** $(x-4)(x+3)=0$ means $x-4=0$ or $x+3=0$, so $x=4$ or $x=-3$. Check: for $x=4$, $16-4-12=0$, and for $x=-3$, $9+3-12=0$. So the roots are $x=4$ and $x=-3$. Why: A product of two numbers is zero only if at least one of them is zero, so each linear factor set to zero gives a root. Substituting the roots into the original equation confirms they satisfy it.

Topic and concept

Solution by factorisation. If ax^2+bx+c can be written as a product of two linear factors, each factor set to zero gives a root. Factorisation is done by splitting the middle term into two terms whose product is ac and sum is b.

How marks are earned

1. Factorize the left-hand side as $(x - 4)(x + 3) = 0$ by finding numbers whose product is -12 and sum is -1. (1 mark)
2. Set each factor to zero: $x - 4 = 0$ or $x + 3 = 0$, yielding roots $x = 4$ and $x = -3$. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 4, §4.3 Solution of a Quadratic Equation by Factorisation: If a quadratic can be written as two linear factors, each factor set to zero gives a root.

Question 23

2 marks

Short Answer

Answer the question with working and reasons where needed.

(a) A cylinder and a hemisphere are joined without overlap to form one solid. State how to find the volume of the new solid and explain why the joined circular faces are not subtracted. [2 marks]

Answer

The total volume is V_cylinder+V_hemisphere. The circular faces form a boundary of zero thickness, so joining the solids does not remove any volume.

Step by step

1. **Add the volumes of the two parts:** The new solid contains the full cylinder and the full hemisphere without overlap, so its volume is V = V_cylinder + V_hemisphere. Why: Volume measures occupied three-dimensional space; non-overlapping occupied regions add when the two solids are joined.
2. **Account for the joined faces:** The two matching circular faces touch at the joint, but each face is a two-dimensional boundary with zero thickness. No part is cut away, so no volume is subtracted. Why: Hiding a surface changes exposed surface area, but a surface by itself has no volume. The composite volume is still the sum of both solids.

Topic and concept

Combinations are restricted to any two cubes, cuboids, spheres, hemispheres and right circular cylinders or cones. For non-overlapping joined solids, add the volumes of the component solids. A shared boundary may disappear from exposed surface area, but it has zero thickness and does not change total volume.

How marks are earned

1. State that the volume of the non-overlapping composite is V_cylinder+V_hemisphere. (1 mark)
2. Explain that the joined circular faces have no thickness and do not remove volume from either solid. (1 mark)

Source check: NCERT Class 10 maths, Ch. 12, §12.3 states that the volume of joined non-overlapping solids is the sum of the volumes of their parts.

Question 24

2 marks

Short Answer

Answer the question with working and reasons where needed.

(a) Define a tangent and a secant to a circle, and state one key difference between them. [2 marks]

Answer

A tangent to a circle is a straight line that touches the circle at exactly one point. A secant to a circle is a line that intersects the circle at two distinct points, which is the key difference.

Step by step

1. **Define a tangent to a circle:** A tangent to a circle is a straight line that touches the circle at exactly one point. If the line is drawn through that point of contact, it meets the circle there and nowhere else. Extra property: the tangent is perpendicular to the radius drawn to the point of contact. Why: The definition rests on the number of common points between a line and a circle. One common point means the line only just touches the circle without cutting into it, which is what separates a tangent from the other two cases (NCERT Class 10 Maths Ch. 10, §10.1). The perpendicularity property is a further feature of a tangent and is not part of the definition.
2. **Define a secant and state the key difference:** A secant to a circle is a line that intersects the circle at two distinct points. The key difference is therefore the number of points of intersection: a tangent meets the circle at exactly 1 point, while a secant meets it at 2 points. A line that meets the circle at no point is a third case, which is neither of the two. Why: A line and a circle can have 0, 1 or 2 common points, and these three cases are what classify the line as non-intersecting, a tangent or a secant. Stating the difference as 1 point versus 2 points gives a precise comparison that can be checked on any figure.

Topic and concept

Tangent to a circle; tangent at any point is perpendicular to the radius (theorem + proof). A line in the plane of a circle can meet it in no point, in exactly one point (a tangent) or in two distinct points (a secant). The tangent touches the circle at one point, while the secant cuts through it (NCERT Class 10 Maths Ch. 10, §10.1).

How marks are earned

1. A tangent to a circle is a straight line that touches the circle at exactly one point. (1 mark)
2. A secant to a circle is a line that intersects the circle at two distinct points, which is the key difference. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 10, §10.1 Introduction: A tangent touches the circle at one point while a secant meets it at two points.

Question 25

2 marks

Short Answer

Answer the question with working and reasons where needed.

(a) Express 360 and 540 as products of primes and determine their HCF and LCM using the

Fundamental Theorem of Arithmetic. [2 marks]

Answer

Prime factorisation of 360 = 2^3 × 3^2 × 5 and of 540 = 2^2 × 3^3 × 5. HCF = 2^min(3,2) × 3^min(2,3) × 5^min(1,1) = 2^2 × 3^2 × 5 = 180; LCM = 2^max(3,2) × 3^max(2,3) × 5^max(1,1) = 2^3 × 3^3 × 5 = 1080.

Step by step

1. **Find the prime factorisation of each number by repeated division.:** For 360, repeated division gives 360 = 2 × 180 = 2 × 2 × 90 = 2 × 2 × 2 × 45 = 2^3 × 3 × 15 = 2^3 × 3^2 × 5, and for 540 it gives 540 = 2 × 270 = 2 × 2 × 135 = 2^2 × 3 × 45 = 2^2 × 3^2 × 15 = 2^2 × 3^3 × 5. Why: The Fundamental Theorem of Arithmetic ensures that each integer greater than 1 has a unique prime factorisation apart from the order of factors, so these prime powers can be used reliably for comparison.
2. **Use the prime-power exponents to calculate the HCF and LCM.:** HCF = 2^min(3,2) × 3^min(2,3) × 5^min(1,1) = 2^2 × 3^2 × 5 = 4 × 9 × 5 = 180; LCM = 2^max(3,2) × 3^max(2,3) × 5^max(1,1) = 2^3 × 3^3 × 5 = 8 × 27 × 5 = 1080, and the check is 180 × 1080 = 194400 = 360 × 540. Why: The HCF takes only the common prime factors with the smaller exponent so that it divides both numbers, while the LCM takes every prime factor needed with the larger exponent so that both numbers divide it.

Topic and concept

Fundamental Theorem of Arithmetic — statement, illustration and applications through prime factorisation. For 360 and 540, the Fundamental Theorem of Arithmetic lets us write each number as a unique product of prime powers; comparing the exponents of the same primes then gives the HCF from the least powers and the LCM from the greatest powers.

How marks are earned

1. Prime factorisation of 360 = 2^3 × 3^2 × 5 and of 540 = 2^2 × 3^3 × 5. (1 mark)
2. HCF = 2^min(3,2) × 3^min(2,3) × 5^min(1,1) = 2^2 × 3^2 × 5 = 180; LCM = 2^max(3,2) × 3^max(2,3) × 5^max(1,1) = 2^3 × 3^3 × 5 = 1080. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 1, §1.2 The Fundamental Theorem of Arithmetic: Composite numbers are factorised uniquely into primes via repeated division starting from smallest prime.

Section C

Question 26

3 marks

Short Answer

Answer the question with working and reasons where needed.

(a) Calculate the mean of the following grouped data by the direct method: Class intervals: 0–10 (f=5), 10–20 (f=8), 20–30 (f=7). [3 marks]

Answer

Compute the class marks by averaging the class boundaries: $x_1 = (0+10)/2 = 5$, $x_2 = (10+20)/2 = 15$ and $x_3 = (20+30)/2 = 25$, and note the corresponding frequencies 5, 8 and 7 respectively. Calculate the sum of the products of frequencies and class marks using $\Sigma f_i x_i = 5\times5 + 8\times15 + 7\times25 = 25 + 120 + 175 = 320$, and find the total frequency $\Sigma f_i = 5 + 8 + 7 = 20$. Apply the direct method formula for the mean $\bar{x} = \Sigma f_i x_i / \Sigma f_i$ to obtain $\bar{x} = 320 / 20 = 16$, which is the mean of the given grouped data.

Step by step

1. **Find each class mark and pair it with its frequency in a table:** Class mark x_i = (lower limit + upper limit)/2. For 0–10, $x_1 = (0+10)/2 = 5$ with $f_1 = 5$. For 10–20, $x_2 = (10+20)/2 = 15$ with $f_2 = 8$. For 20–30, $x_3 = (20+30)/2 = 25$ with $f_3 = 7$. So the (x_i, f_i) pairs are (5, 5), (15, 8) and (25, 7). Why: The individual observations inside a class are not known, so the direct method assumes that every observation in a class sits at the midpoint of that class. The class mark therefore represents the whole class and supplies the x_i needed in the mean formula.
2. **Compute each $f_i x_i$ and add to get $\Sigma f_i x_i$ and Σf_i:** The products are $f_1x_1 = 5\times5 = 25$, $f_2x_2 = 8\times15 = 120$ and $f_3x_3 = 7\times25 = 175$. Adding gives $\Sigma f_i x_i = 25 + 120 + 175 = 320$. The total frequency is $\Sigma f_i = 5 + 8 + 7 = 20$. Why: Each product $f_i x_i$ is the total contribution of that class to the overall sum, as if the class mark were repeated f_i times. Σf_i counts all the observations, and this is the number by which the total must be divided to get an average.
3. **Apply the direct-method formula and check the result:** $\bar{x} = \Sigma f_i x_i / \Sigma f_i = 320/20 = 16$. Check: 16 lies between the smallest class mark 5 and the largest class mark 25. It lies between 15 and 25 rather than at 15 because the 20–30 class has 7 observations, which pulls the mean a little above 15. Why: The mean of grouped data is the sum of $f_i x_i$ divided by the total frequency (NCERT Class 10 Maths Ch. 13, §13.2). An average must lie between the extreme class marks, so 16 is reasonable, and 320/20 divides exactly to 16.

Topic and concept

Mean of grouped data — direct method, assumed mean method, step-deviation method. For grouped data the direct method replaces each class by its class mark x_i, multiplies it by the frequency f_i, adds these products and divides by Σf_i. Here the class marks 5, 15 and 25 with frequencies 5, 8 and 7 give $\Sigma f_i x_i = 320$ and $\Sigma f_i = 20$, so the mean is 16 (NCERT Class 10 Maths Ch. 13, §13.2).

How marks are earned

1. Compute the class marks by averaging the class boundaries: $x_1 = (0+10)/2 = 5$, $x_2 = (10+20)/2 = 15$ and $x_3 = (20+30)/2 = 25$, and note the corresponding frequencies 5, 8 and 7 respectively. (1 mark)
2. Calculate the sum of the products of frequencies and class marks using $\Sigma f_i x_i = 5\times5 + 8\times15 + 7\times25 = 25 + 120 + 175 = 320$, and find the total frequency $\Sigma f_i = 5 + 8 + 7 = 20$. (1 mark)
3. Apply the direct method formula for the mean $\bar{x} = \Sigma f_i x_i / \Sigma f_i$ to obtain $\bar{x} = 320 / 20 = 16$, which is the mean of the given grouped data. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths Ch. 13 §13.2: The direct method calculates the mean as $\Sigma f_i x_i$ divided by Σf_i where x_i are class marks.

Question 27

3 marks

Short Answer

Answer the question with working and reasons where needed.

(a) Determine the mode for the following frequency distribution: Class intervals: 20–30 (f=4), 30–40 (f=9), 40–50 (f=6). [3 marks]

Answer

Identify the modal class as the class with the highest frequency, which is 30–40 having $f_1 = 9$, with the preceding class frequency $f_0 = 4$ and the succeeding class frequency $f_2 = 6$. Use the mode formula Mode = $l + [(f_1 - f_0)/(2f_1 - f_0 - f_2)] \times h$, substituting $l = 30$, $f_0 = 4$, $f_1 = 9$, $f_2 = 6$ and $h = 10$ into the expression. Perform the calculation Mode = $30 + [(9 - 4)/(2\times9 - 4 - 6)] \times 10 = 30 + (5/8) \times 10 = 30 + 6.25 = 36.25$, giving the modal value.

Step by step

1. **Identify the modal class and read off f_0, f_1 and f_2:** The highest frequency is 9, so the modal class is 30–40 and $f_1 = 9$. The class before it is 20–30, so $f_0 = 4$. The class after it is 40–50, so $f_2 = 6$. Why: The mode is the value around which the observations are most concentrated, so it is looked for in the class with the highest frequency. The neighbouring frequencies f_0 and f_2 are needed because they show how the data is pulled toward either side of the modal class.

2. **Write the mode formula and substitute l, h and the three frequencies:** Mode = l + [(f_1 − f_0)/(2f_1 − f_0 − f_2)] × h. Here l = 30 is the lower limit of the modal class and h = 40 − 30 = 10 is the class size. Substituting gives Mode = 30 + [(9 − 4)/(2×9 − 4 − 6)] × 10. Why: The formula places the mode inside the modal class by comparing how much f_1 exceeds f_0 with the sum of both differences in the denominator. Each symbol has a fixed meaning (l lower limit, h class size), so identifying it before substituting prevents misplaced values.
3. **Evaluate the expression and check that the mode lies in the modal class:** Denominator: 2×9 − 4 − 6 = 18 − 10 = 8. Numerator: 9 − 4 = 5. Mode = 30 + (5/8) × 10 = 30 + 6.25 = 36.25. Check: 36.25 lies between 30 and 40, so it is inside the modal class. Why: The mode estimate must lie within the modal class, so a result outside 30–40 would signal an error. The fraction 5/8 is exact, so the mode is 30 + 6.25 = 36.25.

Topic and concept

Mode of grouped data — modal class; mode formula. For grouped data the mode is estimated inside the modal class, which is the class with the highest frequency, using Mode = l + [(f_1 − f_0)/(2f_1 − f_0 − f_2)] × h. Here the modal class is 30–40 with f_0 = 4, f_1 = 9 and f_2 = 6, and the formula gives 36.25 (NCERT Class 10 Maths Ch. 13, §13.3).

How marks are earned

1. Identify the modal class as the class with the highest frequency, which is 30–40 having f_1 = 9, with the preceding class frequency f_0 = 4 and the succeeding class frequency f_2 = 6. (1 mark)
2. Use the mode formula Mode = l + [(f_1 – f_0)/(2f_1 – f_0 – f_2)] × h, substituting l = 30, f_0 = 4, f_1 = 9, f_2 = 6 and h = 10 into the expression. (1 mark)
3. Perform the calculation Mode = 30 + [(9 – 4)/(2×9 – 4 – 6)] × 10 = 30 + (5/8) × 10 = 30 + 6.25 = 36.25, giving the modal value. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths Ch. 13 Summary: The mode is estimated within the modal class using Mode = l + ((f_1–f_0)/(2f_1–f_0–f_2))×h.

Question 28

3 marks

Short Answer

Attempt one option.

Option A

Answer the question with working and reasons where needed.

(a) Show by contradiction that √5 is irrational, citing the role of unique prime factorisation. [3 marks]

Answer

Assume √5 = p/q in lowest terms, where p and q are integers with no common factors, which implies p^2 = 5q^2 upon squaring. Since p^2 is divisible by 5, the unique prime factorisation principle implies p must be divisible by 5, so we write p = 5k for some integer k. Substituting p = 5k into p^2 = 5q^2 gives q^2 = 5k^2, showing q is also divisible by 5, contradicting the assumption that p and q are coprime.

Step by step

1. **Assume the opposite: √5 is rational and write it in lowest terms, then square.:** Suppose √5 = p/q with integers p, q (q ≠ 0) and HCF(p, q) = 1. Squaring both sides gives 5 = p²/q², so p² = 5q². Why: Proof by contradiction starts by assuming the opposite of the claim; writing the fraction in lowest terms gives a precise condition (no common factor) that we can later show is violated.
2. **Use unique prime factorisation to deduce that 5 divides p, and write p = 5k.:** From p² = 5q², 5 divides p². If 5 did not divide p, the prime factorisation of p would have no 5, so p² (which has the same primes as p, each repeated) would have no 5 either. By uniqueness of prime factorisation this is impossible, so 5 divides p. Write p = 5k for some integer k. Why: The Fundamental Theorem of Arithmetic says each number has exactly one prime factorisation, so a prime that divides p² must already appear in the factorisation of p. This is why 'p² divisible by 5' forces 'p divisible by 5'.
3. **Substitute p = 5k to show 5 also divides q, then state the contradiction and conclusion.:** Put p = 5k in p² = 5q²: 25k² = 5q², so q² = 5k². Hence 5 divides q² and, by the same argument, 5 divides q. So 5 is a common factor of p and q, contradicting HCF(p, q) = 1. Therefore √5 is irrational. Why: The contradiction arises because the lowest-terms assumption says p and q share no prime, yet both are divisible by 5. Since every step was valid, the only faulty item is the initial assumption that √5 is rational.

Topic and concept

Proofs of irrationality of √2, √3, √5 and similar surds. To prove a number like √5 is irrational, assume it equals p/q in lowest terms and use unique prime factorisation to show that the prime 5 must divide both p and q. That contradicts 'lowest terms', so the assumption fails and √5 is irrational (NCERT Class 10 Maths, Ch. 1, §1.3).

How marks are earned

1. Assume √5 = p/q in lowest terms, where p and q are integers with no common factors, which implies p^2 = 5q^2 upon squaring. (1 mark)
2. Since p^2 is divisible by 5, the unique prime factorisation principle implies p must be divisible by 5, so we write p = 5k for some integer k. (1 mark)
3. Substituting p = 5k into p^2 = 5q^2 gives q^2 = 5k^2, showing q is also divisible by 5, contradicting the assumption that p and q are coprime. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 1, §1.3: The contradiction follows because assuming rationality forces both numerator and denominator to share prime 5.

OR

Option B

Answer the question with working and reasons where needed.

(a) Prove that √3 is irrational by contradiction using prime factorisation. [3 marks]

Answer

Assume √3 = p/q in lowest terms, so p² = 3q² by squaring both sides. Since 3 divides p², by unique prime factorisation 3 divides p, so p = 3k. Substituting gives q² = 3k², so 3 divides q; this contradicts that p/q is in lowest terms.

Step by step

1. **Assume the opposite: √3 = p/q in lowest terms, and square it:** Suppose √3 is rational, so √3 = p/q where p and q are integers, q ≠ 0, and p and q have no common factor other than 1. Squaring both sides gives 3 = p²/q², so p² = 3q². Why: Proof by contradiction starts by assuming the opposite of what is to be proved, here that √3 is rational. Writing p/q in lowest terms matters because the contradiction will be a common factor, which is impossible only if the fraction is fully reduced.
2. **Use prime factorisation to show that 3 divides p, and write p = 3k:** From p² = 3q², p² is a multiple of 3, so 3 divides p². If 3 did not divide p, the prime factorisation of p would contain no 3, and p² would then contain the same primes as p, so no 3 either. So 3 divides p, and we write p = 3k for some integer k. Why: By unique prime factorisation the primes in p² are exactly the primes in p, each repeated. So if the prime 3 divides p², it must already appear in p. This step turns a statement about p² into one about p.
3. **Substitute p = 3k, show that 3 divides q, and state the contradiction:** Substituting p = 3k in p² = 3q² gives 9k² = 3q², so q² = 3k². Then 3 divides q², and by the same prime factorisation reasoning 3 divides q. So 3 is a common factor of p and q, which contradicts that p/q is in lowest terms. Why: The contradiction shows that the starting assumption was false, because p and q cannot both be divisible by 3 when they have no common factor. Hence √3 cannot be written as p/q and is irrational (NCERT Class 10 Maths Ch. 1, §1.3).

Topic and concept

Proofs of irrationality of √2, √3, √5 and similar surds. To prove that a number such as √3 is irrational, assume it equals p/q in lowest terms and use unique prime factorisation to show that a prime divides both p and q. For √3 the prime is 3. This contradicts lowest terms, so the assumption is false (NCERT Class 10 Maths Ch. 1, §1.3).

How marks are earned

1. Assume √3 = p/q in lowest terms, so p² = 3q² by squaring both sides. (1 mark)
2. Since 3 divides p², by unique prime factorisation 3 divides p, so p = 3k. (1 mark)
3. Substituting gives q² = 3k², so 3 divides q; this contradicts that p/q is in lowest terms. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 1, §1.3: The contradiction arises because both numerator and denominator become divisible by the same prime.

Question 29

3 marks

Short Answer

Answer the question with working and reasons where needed.

(a) For A(1,2) and B(7,8), the point P divides AB internally at P(4,5). Determine the ratio AP:PB. [3 marks]

Answer

Applying the section formula to the x-coordinates of A(1,2), B(7,8), and P(4,5) gives (m·7 + n·1)/(m + n) = 4, leading to 7m + n = 4m + 4n and hence simplifying to 3m = 3n. Similarly, applying the ratio to the y-coordinates yields (m·8 + n·2)/(m + n) = 5, resulting in 8m + 2n = 5m + 5n and again simplifying to 3m = 3n, confirming consistency from both coordinates. Since both coordinate equations reduce to m = n, the internal division ratio AP:PB is established as 1:1, indicating that P divides AB exactly at its midpoint.

Step by step

1. **Let AP:PB = m:n and equate the x-coordinate from the internal section formula to 4.:** Section formula: $x = (m \cdot x_2 + n \cdot x_1)/(m+n)$. With A(1,2), B(7,8): $(7m + n \cdot 1)/(m+n) = 4$, so $7m + n = 4m + 4n$, giving $3m = 3n$. Why: The internal section formula gives the point dividing AB as a weighted average of the endpoints, with weight m on B and n on A for the ratio m:n. Setting it equal to the known x of P turns the ratio into an equation.
2. **Repeat with the y-coordinate to check consistency.:** $y = (m \cdot y_2 + n \cdot y_1)/(m+n)$: $(8m + 2n)/(m+n) = 5$, so $8m + 2n = 5m + 5n$, giving $3m = 3n$. Why: P is a single point, so the same ratio must satisfy both coordinate equations. Getting $3m = 3n$ again confirms that P(4,5) really lies on AB with this ratio.
3. **Conclude the ratio and verify with the midpoint formula.:** $3m = 3n$ gives $m = n$, so AP:PB = 1:1. Check: midpoint of AB = ((1+7)/2, (2+8)/2) = (4,5) = P. Why: Equal weights m = n mean P is equally far from both endpoints, which is exactly the midpoint, so the midpoint formula gives an independent check of the answer.

Topic and concept

Section formula for internal division. If P divides AB internally in the ratio m:n, then $P = ((m \cdot x_2 + n \cdot x_1)/(m+n), (m \cdot y_2 + n \cdot y_1)/(m+n))$. Here the known P lets us solve for m:n from either coordinate (NCERT Class 10 Maths, Ch. 7, §7.3–7.4).

How marks are earned

1. Applying the section formula to the x-coordinates of A(1,2), B(7,8), and P(4,5) gives (m·7 + n·1)/(m + n) = 4, leading to 7m + n = 4m + 4n and hence simplifying to 3m = 3n. (1 mark)
2. Similarly, applying the ratio to the y-coordinates yields (m·8 + n·2)/(m + n) = 5, resulting in 8m + 2n = 5m + 5n and again simplifying to 3m = 3n, confirming consistency from both coordinates. (1 mark)

3. Since both coordinate equations reduce to m = n, the internal division ratio AP:PB is established as 1:1, indicating that P divides AB exactly at its midpoint. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 7, §7.3: Equating weighted sums of coordinates yields the division ratio.

Question 30

3 marks

Short Answer

Answer the question with working and reasons where needed.

(a) A 3 m tall lamppost casts a 4 m shadow. At the same time, a tree casts a 10 m shadow. Find the height of the tree, showing all steps and units. [3 marks]

Answer

Modeling the lamppost and its shadow as a right triangle similar to the triangle formed by the tree and its shadow, we set up the proportion height/shadow = 3/4 = h/10. By cross-multiplying the proportion 3/4 = h/10, we obtain h = (3 × 10)/4, which yields h = 7.5 when calculated. Hence, completing all calculations and including units, the height of the tree is determined to be 7.5 m.

Step by step

1. **Justify similarity and set up the proportion of height to shadow.:** At the same time the sun's rays make the same angle with the ground, and both the lamppost and the tree are vertical (90°). So the two right triangles are similar by AA. Hence height/shadow is equal: 3 m/4 m = h/10 m. Why: Similar triangles have corresponding sides in the same ratio. The shared sun angle and the right angles give AA similarity, which is what allows the unknown height to be found indirectly.
2. **Cross-multiply and compute h.:** 3/4 = h/10, so h = (3 × 10)/4 = 30/4 = 7.5. Why: Multiplying both sides by 10 isolates h. The ratio 3/4 = 0.75 means height is 0.75 times the shadow, so a 10 m shadow gives 0.75 × 10 = 7.5, which checks the result.
3. **State the answer with units and a sense check.:** The height of the tree is 7.5 m. Check: tree shadow (10 m) is longer than the lamppost shadow (4 m), so the tree should be taller than 3 m; 7.5 m fits. Why: Both quantities in the ratio are lengths in metres, so the units cancel in the ratio and the answer takes the unit of the height, metres.

Topic and concept

Applications of similarity criteria to prove triangles similar and indirect measurement. Similar triangles let us find an unreachable length by comparing it with a measurable one: corresponding sides are proportional. Here the sun's rays form similar right triangles for the lamppost and the tree (NCERT Class 10 Maths, Ch. 6, §6.1).

How marks are earned

1. Modeling the lamppost and its shadow as a right triangle similar to the triangle formed by the tree and its shadow, we set up the proportion height/shadow = 3/4 = h/10. (1 mark)

2. By cross-multiplying the proportion 3/4 = h/10, we obtain h = (3 × 10)/4, which yields h = 7.5 when calculated. (1 mark)
3. Hence, completing all calculations and including units, the height of the tree is determined to be 7.5 m. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 6, §6.1: Indirect measurement of heights uses the principle of similar triangles.

Question 31

3 marks

Short Answer

Attempt one option.

Option A

Answer the question with working and reasons where needed.

(a) A man 1.7 m tall stands 10 m away from a tower and measures the angle of elevation to its top as 30°. Find the height of the tower. [3 marks]

Answer

Let h represent the total height of the tower, so the vertical drop from the tower's top to the man's eye level is (h−1.7) metres, which is opposite to the angle of elevation. Use the relation tan 30°=(h−1.7)/10 and substitute tan 30°=1/√3 to formulate the equation 1/√3=(h−1.7)/10, which gives h−1.7=10/√3 after cross-multiplying. Adding the observer's eye level height of 1.7 m to 10/√3 and rationalizing the denominator yields h=1.7+(10√3)/3≈7.47 metres as the tower's height.

Step by step

1. **Describe the figure and define the height above eye level.:** Let the tower height be h m. The man's eye is 1.7 m above the ground, so the part of the tower above eye level is (h − 1.7) m. This is the side opposite the 30° angle in a right triangle, and the horizontal distance 10 m is the adjacent side. Why: The angle of elevation is measured from the horizontal line through the eye, so only the height above eye level is opposite to it. Using h directly would ignore the 1.7 m offset.
2. **Choose tan, substitute, and solve for h − 1.7.:** tan 30° = opposite/adjacent = (h − 1.7)/10. With tan 30° = 1/√3: 1/√3 = (h − 1.7)/10, so h − 1.7 = 10/√3. Why: Tan is chosen because it links the two sides known or sought, opposite and adjacent, without needing the hypotenuse.
3. **Add back the eye height and evaluate with units.:** 10/√3 = 10√3/3 ≈ 10 × 1.73/3 ≈ 5.77 m, so h = 1.7 + 5.77 ≈ 7.47 m. Why: The tower's total height is the part above eye level plus the 1.7 m below it. Rationalising gives a numerical value to compare: about 7.47 m is reasonable as it is a little more than the man's height plus 5.77 m.

Topic and concept

Angle of elevation and angle of depression; applications to heights and distances. In a right triangle, tan θ = opposite/adjacent. For an angle of elevation, the opposite side is the height above eye level and the adjacent side is the horizontal distance (NCERT Class 10 Maths, Ch. 9, §9.2).

How marks are earned

1. Let h represent the total height of the tower, so the vertical drop from the tower's top to the man's eye level is (h−1.7) metres, which is opposite to the angle of elevation. (1 mark)
2. Use the relation tan 30°=(h−1.7)/10 and substitute tan 30°=1/√3 to formulate the equation 1/√3=(h−1.7)/10, which gives h−1.7=10/√3 after cross-multiplying. (1 mark)
3. Adding the observer's eye level height of 1.7 m to 10/√3 and rationalizing the denominator yields h=1.7+(10√3)/3≈7.47 metres as the tower's height. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 9, §9.2 Summary point 2: Heights and distances can be determined using trigonometric ratios.

OR

Option B

Answer the question with working and reasons where needed.

(a) A tree is observed from a point 20 m away at an angle of elevation of 30°. Determine the height of the tree. [3 marks]

Answer

By applying the definition of tangent for the observed angle of elevation, we write tan 30°=height of the tree/20 m as the foundational trigonometric relation. Since tan 30° equals 1/√3, we set 1/√3=h/20 and solve algebraically to find that h=20/√3 metres represents the tree's height. Rationalizing the denominator of the expression 20/√3 yields the exact value h=(20√3)/3 metres, which is the final height of the tree.

Step by step

1. **Set up the right triangle and choose tan for the known and unknown sides:** Assume the observation is made at ground level, so the tree is vertical and forms a right triangle with the ground. The horizontal distance from the observer to the tree is 20 m (adjacent side), the tree height h is the opposite side, and the angle of elevation is 30°. So tan 30° = h/20. Why: The angle of elevation is measured at the observer between the horizontal and the line of sight. The ratio that links the opposite side (height) and the adjacent side (distance) is tan θ = opposite/adjacent, so tan is the right choice. The ground-level assumption means the observer's eye height is ignored.
2. **Substitute tan 30° = 1/√3 and solve for h:** tan 30° = 1/√3, so 1/√3 = h/20. Multiplying both sides by 20 gives h = 20/√3 m. Why: The standard value of tan 30° is 1/√3 from the table of trigonometric ratios. Multiplying both sides by 20 isolates h, keeping the units in metres because 20 m is a length.

3. **Rationalise the denominator and check the value with units:** h = 20/√3 = (20×√3)/(√3×√3) = 20√3/3 m. Using √3 ≈ 1.732, h ≈ 20×1.732/3 = 34.64/3 ≈ 11.55 m. Check: tan 30° ≈ 0.577 and 11.55/20 ≈ 0.577. Why: Rationalising removes the surd from the denominator and gives the exact form. The decimal check confirms the answer reproduces tan 30°. The height is less than the distance 20 m, as expected for an angle below 45° (NCERT Class 10 Maths Ch. 9, §9.2).

Topic and concept

Angle of elevation and angle of depression; applications to heights and distances. In a right triangle formed by an observer, the tree and the ground, tan θ = height/horizontal distance (NCERT Class 10 Maths Ch. 9, §9.2). With θ = 30° and distance 20 m, tan 30° = 1/√3 gives the height 20√3/3 m, about 11.55 m.

How marks are earned

1. By applying the definition of tangent for the observed angle of elevation, we write tan 30°=height of the tree/20 m as the foundational trigonometric relation. (1 mark)
2. Since tan 30° equals 1/√3, we set 1/√3=h/20 and solve algebraically to find that h=20/√3 metres represents the tree's height. (1 mark)
3. Rationalizing the denominator of the expression 20/√3 yields the exact value h=(20√3)/3 metres, which is the final height of the tree. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 9, §9.2 Summary point 2: Heights and distances can be determined using trigonometric ratios.

Section D

Question 32

5 marks

Long Answer

Answer both connected parts. Show the required method, explanation and units.

(a) Explain how a tangent at a point on a circle can be viewed as the limiting position of a secant and state the key distinguishing feature of a tangent compared to a secant. [2 marks]

Answer

A tangent is obtained when a secant line through a circle is rotated so that its two intersection points merge into one point of contact on the circle. Unlike a secant which intersects the circle in two distinct points, a tangent touches the circle in exactly one point.

Step by step

1. **Describe the moving secant becoming a tangent.:** Take a secant that meets the circle at two points A and B. Keep A fixed and rotate the line about A so that B moves along the circle toward A. The two intersection points come closer until they coincide at A, and the line is then the tangent at A. Why: The tangent is the limiting position of the secant as its two intersection points merge into one, which is why a tangent is treated as a special case of a secant (NCERT Class 10 Maths, Ch. 10, §10.2).
2. **State the distinguishing feature.:** A secant meets the circle at two distinct points, while a tangent meets it at exactly one point, the point of contact. Why: The number of common points separates the two lines. Two merged points make one point of contact, so the tangent does not enter the circle.

Topic and concept

Tangent to a circle; tangent at any point is perpendicular to the radius (theorem + proof). A secant cuts a circle at two points. When those two points merge into one by rotating the secant, the line becomes a tangent, which touches the circle at exactly one point (NCERT Class 10 Maths, Ch. 10, §10.2).

How marks are earned

1. A tangent is obtained when a secant line through a circle is rotated so that its two intersection points merge into one point of contact on the circle. (1 mark)
2. Unlike a secant which intersects the circle in two distinct points, a tangent touches the circle in exactly one point. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 10, §10.2 Tangent to a Circle: A tangent is a secant whose two points of intersection coincide.

(b) Prove that the tangents drawn at the endpoints of a diameter of a circle are parallel. [3 marks]

Answer

Let AB be a diameter of the circle with centre O; then OA and OB are collinear along the diameter. The tangent at A is perpendicular to OA and the tangent at B is perpendicular to OB, which lie on the same straight line. Since two lines perpendicular to the same line are parallel to each other, the tangents at A and B are parallel.

Step by step

1. **Draw and label the diameter, centre and tangents.:** Draw a circle with centre O and diameter AB. Let PQ be the tangent at A and RS the tangent at B. Since O lies on AB, the radii OA and OB lie along the same straight line AB. Why: Naming the tangents and noting that the two radii form one line sets up a single transversal AB, which the parallel-lines argument needs.
2. **Apply the tangent–radius perpendicularity at both endpoints.:** The tangent at the point of contact is perpendicular to the radius, so $\angle OAP = 90°$ ($PQ \perp OA$) and $\angle OBS = 90°$ ($RS \perp OB$). Why: At the point of contact, a tangent is perpendicular to the radius drawn there (NCERT Class 10 Maths, Ch. 10, §10.4), giving two right angles on the transversal AB.
3. **Conclude parallelism using angles on the transversal.:** Taking AB as transversal, $\angle PAB = 90°$ and $\angle SBA = 90°$ are alternate interior angles and are equal, so $PQ \parallel RS$. Why: If alternate angles formed by a transversal are equal, the two lines are parallel. Equivalently, two lines perpendicular to the same line AB are parallel.

Topic and concept

Tangent to a circle; tangent at any point is perpendicular to the radius (theorem + proof). The tangent at any point of a circle is perpendicular to the radius at that point. For a diameter AB, the radii at both ends lie on one line, so both tangents are perpendicular to that same line and are therefore parallel (NCERT Class 10 Maths, Ch. 10, §10.4).

How marks are earned

1. Let AB be a diameter of the circle with centre O; then OA and OB are collinear along the diameter. (1 mark)
2. The tangent at A is perpendicular to OA and the tangent at B is perpendicular to OB, which lie on the same straight line. (1 mark)
3. Since two lines perpendicular to the same line are parallel to each other, the tangents at A and B are parallel. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 10, §10.4 Summary: A tangent at any point is perpendicular to the radius through that point, so tangents at endpoints of a diameter are parallel.

Question 33

5 marks

Long Answer

Answer both connected parts. Show the required method, explanation and units.

(a) State the trigonometric ratio sin A in terms of the sides of a right triangle. [2 marks]

Answer

sin A is defined as the ratio of the length of the side opposite ∠A to the length of the hypotenuse. Thus, sin A = (opposite side)/(hypotenuse) in any right triangle.

Step by step

1. **Identify the sides:** In a right triangle, first identify the acute angle A, its opposite side, and the hypotenuse opposite the right angle. Why: The labels depend on the chosen angle, while the hypotenuse remains the longest side.
2. **State the sine ratio:** The sine ratio is sin A = length of the side opposite A divided by length of the hypotenuse. Why: This is the defining right-triangle ratio for sine.

Topic and concept

Values at 0°, 30°, 45°, 60°, 90°. The sine of an acute angle in a right triangle is opposite side divided by hypotenuse. Identify the angle first, then label its opposite side and the hypotenuse.

How marks are earned

1. sin A is defined as the ratio of the length of the side opposite ∠A to the length of the hypotenuse. (1 mark)
2. Thus, sin A = (opposite side)/(hypotenuse) in any right triangle. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 8, §8.2: sin A is defined as the ratio of the side opposite angle A to the hypotenuse.

(b) In right triangle PQR, ∠Q = 90°, PQ = 12 cm and PR = 13 cm. Calculate tan P showing all steps. [3 marks]

Answer

In right triangle PQR with ∠Q equal to 90°, applying the Pythagorean theorem gives QR = √(PR² − PQ²) = √(13² − 12²) = √(169 − 144) = 5 cm, thus determining the length of the remaining side. The tangent of angle P is defined as the ratio of the length of the side opposite P to the length of the side adjacent to P, so tan P = QR/PQ = 5/12. Hence, the value of tan P is 5/12, which is a pure number without any units, representing the trigonometric ratio for angle P.

Step by step

1. **Identify the hypotenuse and find the missing side QR using Pythagoras.:** Since ∠Q = 90°, PR is the hypotenuse, so QR = √(PR² − PQ²) = √(13² − 12²) = √(169 − 144) = √25 = 5 cm, and the check is 12² + 5² = 144 + 25 = 169 = 13². Why: In a right triangle, the side opposite the right angle is the hypotenuse, and the Pythagorean relation connects the hypotenuse and the two legs, allowing the unknown leg to be found before forming the trigonometric ratio.
2. **Identify the opposite and adjacent sides for angle P and form tan P.:** For angle P, QR = 5 cm is the opposite side and PQ = 12 cm is the adjacent side, so tan P = opposite/adjacent = QR/PQ = 5 cm/12 cm = 5/12. Why: The tangent ratio in a right triangle is defined using the side opposite the chosen angle divided by the adjacent non-hypotenuse side, so PR is not used directly in tan P.
3. **State the final value with the correct unit interpretation.:** Hence, tan P = 5/12, which is a pure number without any units because the centimetre units cancel in the ratio 5 cm/12 cm. Why: Trigonometric ratios compare two lengths of the same unit, so the units cancel and only the numerical ratio remains.

Topic and concept

Values at 0°, 30°, 45°, 60°, 90°. In this right triangle, the given right angle first identifies PR as the hypotenuse; after QR is found, tan P is calculated as the ratio of the side opposite angle P to the side adjacent to angle P.

How marks are earned

1. In right triangle PQR with ∠Q equal to 90°, applying the Pythagorean theorem gives QR = √(PR² − PQ²) = √(13² − 12²) = √(169 − 144) = 5 cm, thus determining the length of the remaining side. (1 mark)
2. The tangent of angle P is defined as the ratio of the length of the side opposite P to the length of the side adjacent to P, so tan P = QR/PQ = 5/12. (1 mark)
3. Hence, the value of tan P is 5/12, which is a pure number without any units, representing the trigonometric ratio for angle P. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 8, §8.2: tan A is defined as the ratio of the side opposite to the side adjacent to angle A in a right triangle.

Question 34

5 marks

Long Answer

Attempt one option.

Option A

Answer both connected parts. Show the required method, explanation and units.

(a) Calculate the sum of the first 10 terms of the AP 2, 5, 8, …. [2 marks]

Answer

The AP has first term a = 2, common difference d = 3 and n = 10, so using $S_n = n/2(2a + (n-1)d)$ gives $S_{10} = 10/2[2\times2 + 9\times3]$. Thus, $S_{10} = 5[4 + 27] = 5\times31 = 155$ is the sum of the first ten terms.

Step by step

1. **Identify a, d and n from the AP, then substitute into the sum formula:** The first term is a = 2. The common difference is d = 5 − 2 = 3 (and 8 − 5 = 3, so it is constant). We need n = 10. Using $S_n = n/2\ [2a + (n-1)d]$, $S_{10} = 10/2\ [2\times2 + 9\times3]$. Why: The sum formula needs a, d and n, and d is the same fixed number added to each term in an AP, which is why two consecutive differences are checked. Reading these quantities correctly before substituting prevents errors such as using n − 1 = 10.
2. **Evaluate the expression and verify with the alternative formula:** $S_{10} = 5[4 + 27] = 5 \times 31 = 155$. Check: the last term is $l = a + 9d = 2 + 27 = 29$, so $S_{10} = n/2\ (a + l) = 10/2 \times (2 + 29) = 5 \times 31 = 155$. Why: The two forms of the sum formula (NCERT Class 10 Maths Ch. 5, §5.4) come from pairing terms from the start and end, so they must agree. Getting 155 from both confirms the arithmetic, and no unit is needed because the AP is a list of plain numbers.

Topic and concept

Sum of first n terms: $S_n = n/2(2a+(n-1)d) = n/2(a+l)$. The sum of the first n terms of an AP is $S_n = n/2\ [2a + (n-1)d]$, which also equals $n/2\ (a + l)$. For 2, 5, 8, … with a = 2, d = 3 and n = 10, it gives 155 without adding the ten terms one by one (NCERT Class 10 Maths Ch. 5, §5.4).

How marks are earned

1. The AP has first term a = 2, common difference d = 3 and n = 10, so using $S_n = n/2(2a + (n-1)d)$ gives $S_{10} = 10/2[2\times2 + 9\times3]$. (1 mark)
2. Thus, $S_{10} = 5[4 + 27] = 5\times31 = 155$ is the sum of the first ten terms. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 5, §5.4: The sum of the first n terms of an AP is given by $S_n = n/2(2a + (n-1)d)$.

(b) Derive the formula for the sum S_n of the first n terms of an arithmetic progression by adding the series written forwards and backwards. Present each step clearly. [3 marks]

Answer

Let $S = a + (a + d) + \ldots + [a + (n-1)d]$, and write in reverse as $S = [a + (n-1)d] + \ldots + a$. Adding yields $2S = n[2a + (n-1)d]$ since each of the n pairs sums to $2a + (n-1)d$. Therefore, $S_n = n/2\ [2a + (n-1)d]$, which is the required sum formula.

Step by step

1. **Write the AP sum forwards and then backwards:** Let $S = a + (a + d) + (a + 2d) + \ldots + [a + (n-1)d]$. Writing the same terms in reverse order gives $S = [a + (n-1)d] + [a + (n-2)d] + \ldots + (a + d) + a$. Why: The reverse listing contains exactly the same n terms, so it equals the same sum S. Placing the two lines one under the other lets every term be paired with the term in the same position, which is the idea NCERT §5.4 uses to derive the formula.

2. **Add the two lines term by term and count the pairs:** The first pair is a + [a + (n−1)d] = 2a + (n−1)d. The second pair is (a + d) + [a + (n−2)d] = 2a + (n−1)d. Every one of the n pairs gives 2a + (n−1)d, so 2S = n × [2a + (n−1)d]. Why: In an AP each step forward adds d to one line and removes d from the other, so the d's cancel in each pair and every pair has the same value. A constant pair-sum repeated n times can be written as a product, which is the heart of the proof.
3. **Divide by 2 to get the formula:** S = n[2a + (n−1)d] / 2, so S_n = n/2 [2a + (n−1)d]. Check: for n = 1 this gives S_n = a, the first term, as it should. Why: We added S twice, once forwards and once backwards, so 2S is double the required sum. Dividing by 2 recovers S, and the n = 1 check confirms the formula is consistent.

Topic and concept

Sum of first n terms: S_n = n/2(2a+(n−1)d) = n/2(a+l). Pairing the first and last terms, the second and second-last, and so on gives equal pair-sums in an AP because the common difference d cancels in each pair. This turns a long addition into n × (pair-sum) / 2. The result is S_n = n/2[2a + (n−1)d], which also equals n/2(a + l) when the last term l is known (NCERT Ch. 5, §5.4).

How marks are earned

1. Let S = a + (a + d) + … + [a + (n−1)d], and write in reverse as S = [a + (n−1)d] + … + a. (1 mark)
2. Adding yields 2S = n[2a + (n−1)d] since each of the n pairs sums to 2a + (n−1)d. (1 mark)
3. Therefore, S_n = n/2 [2a + (n−1)d], which is the required sum formula. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 5, §5.4: Sum of an AP is derived by adding the series forwards and backwards term-wise.

OR

Option B

Answer both connected parts. Show the required method, explanation and units.

(a) A ladder has rungs whose lengths decrease uniformly by 2 cm from bottom to top. If the bottom rung is 45 cm long, what is the length of the 8th rung from the bottom? [2 marks]

Answer

We identify that the lengths of the ladder rungs form an arithmetic progression with first term a = 45 cm and common difference d = -2 cm, so using the general term formula a_8 = a + (8−1)d gives a_8 = 45 + 7×(−2) cm. On simplifying the expression 45 + 7×(−2) we obtain 45 − 14 = 31 cm; therefore, the length of the eighth rung from the bottom is 31 centimetres.

Step by step

1. **Recognise the rung lengths as an AP and fix a and d:** The rungs shrink by the same 2 cm each step, so the lengths 45, 43, 41, ... form an AP with first term a = 45 cm and common difference d = −2 cm, and the 8th rung is a_8 = a + (8−1)d = 45 + 7×(−2) cm. Why: A constant change between consecutive terms is exactly what defines an AP. The change is a decrease, so d is negative (−2), not +2. The nth term formula a + (n−1)d applies only because the difference is constant.

2. **Simplify and check with units:** 7×(−2) = −14, so a_8 = 45 − 14 = 31 cm. Check by listing terms: 45, 43, 41, 39, 37, 35, 33, 31, where 31 cm is the 8th term. Why: To reach the 8th term from the 1st you add d seven times (n−1 = 7), not eight times. The term-by-term list confirms the formula, and the answer carries the unit cm because a and d are lengths in cm.

Topic and concept

Definition and examples of AP; common difference. An AP is a sequence in which each term after the first is got by adding a fixed common difference. The ladder rungs fit this because each is 2 cm shorter than the one below. Any term can then be found with a + (n−1)d, with no need to write out every rung.

How marks are earned

1. We identify that the lengths of the ladder rungs form an arithmetic progression with first term a = 45 cm and common difference d = -2 cm, so using the general term formula a_8 = a + (8−1)d gives a_8 = 45 + 7×(−2) cm. (1 mark)
2. On simplifying the expression 45 + 7×(−2) we obtain 45 − 14 = 31 cm; therefore, the length of the eighth rung from the bottom is 31 centimetres. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 5, §5.1: Lengths of ladder rungs form an AP decreasing by 2 cm each time.

(b) Reena's annual salary starts at ₹8000 and increases by ₹500 each year. Calculate her salary in the 30th year of employment. Include all steps and the formula used. [3 marks]

Answer

Reena's annual earnings form an arithmetic progression with first term a = ₹8,000 and a constant annual increase of d = ₹500 over each preceding year, up to the thirtieth term. To compute her salary in the thirtieth year we substitute a = ₹8,000, d = ₹500, and n = 30 into the nth term formula a_{30} = a + (n−1)d, yielding a_{30} = 8,000 + 29×500. Evaluating 29×500 = 14,500 and adding to 8,000 gives a_{30} = 22,500; therefore, Reena's salary in the thirtieth year of employment is ₹22,500.

Step by step

1. **Identify the AP and its a and d:** Her salary is ₹8000 in year 1, ₹8500 in year 2, ₹9000 in year 3, and so on. Each year adds the same ₹500, so the salaries form an AP with a = ₹8000 and d = ₹500. Why: A fixed yearly increase means a constant difference between consecutive terms, which is the definition of an AP. This is what allows the nth term formula to be used and rules out repeated addition.
2. **State the formula and substitute:** The nth term is a_n = a + (n−1)d. For the 30th year, n = 30, so a_{30} = 8000 + (30−1)×500 = 8000 + 29×500. Why: The first-year salary already includes no increment. By year 30, only 29 increments have been added, so the multiplier is n−1 = 29, not 30. This is why the formula uses (n−1)d.
3. **Evaluate and check:** 29×500 = 14,500, so a_{30} = 8000 + 14,500 = ₹22,500. Check: 29 increments of ₹500 is ₹14,500, which added to ₹8000 gives ₹22,500. Why: Multiplication gives the total increase over 29 years, and adding it to the starting salary gives the 30th-year salary. The answer is in rupees, the unit of a and d.

Topic and concept

nth term formula: $a_n = a + (n-1)d$. To find a particular term of an AP, put the known a, d and n into $a_n = a + (n-1)d$. Here a = 8000, d = 500 and n = 30. This replaces adding ₹500 repeatedly, and the formula works because the 30th term is the first term plus 29 equal increments.

How marks are earned

1. Reena's annual earnings form an arithmetic progression with first term a = ₹8,000 and a constant annual increase of d = ₹500 over each preceding year, up to the thirtieth term. (1 mark)
2. To compute her salary in the thirtieth year we substitute a = ₹8,000, d = ₹500, and n = 30 into the nth term formula $a_{30} = a + (n-1)d$, yielding $a_{30} = 8{,}000 + 29\times500$. (1 mark)
3. Evaluating $29\times500 = 14{,}500$ and adding to 8,000 gives $a_{30} = 22{,}500$; therefore, Reena's salary in the thirtieth year of employment is ₹22,500. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 5, §5.3: Salary increments form an AP whose terms are given by the nth term formula.

Question 35

5 marks

Long Answer

Attempt one option.

Option A

Answer both connected parts. Show the required method, explanation and units.

(a) Calculate the discriminant of the equation $x^2 - 6x + 9 = 0$ and state the nature of its roots. [2 marks]

Answer

Compute the discriminant $D = b^2 - 4ac = (-6)^2 - 4\cdot1\cdot9 = 36 - 36 = 0$. Since D = 0, conclude that the equation has two equal real roots.

Step by step

1. **Identify a, b, c and compute D:** Comparing $x^2 - 6x + 9 = 0$ with $ax^2 + bx + c = 0$ gives a = 1, b = −6, c = 9. So $D = b^2 - 4ac = (-6)^2 - 4(1)(9) = 36 - 36 = 0$. Why: The discriminant is the expression under the square root in the quadratic formula, so it decides what that root does. Squaring −6 gives +36, and the sign of b is irrelevant after squaring. The correct sign in $b^2 - 4ac$ is minus.
2. **Interpret D = 0 and verify:** Since D = 0, the equation has two equal real roots. Check: $x^2 - 6x + 9 = (x - 3)^2$, so $(x - 3)^2 = 0$ gives x = 3 twice. Why: With D = 0 the quadratic formula gives $x = (-b \pm 0)/2a$, so both roots coincide. The factorised form $(x - 3)^2$ confirms a repeated real root, so the answer from D agrees with factorisation.

Topic and concept

Nature of roots from discriminant (b²−4ac): two distinct real, two equal, no real roots. The discriminant D = b² − 4ac shows the nature of roots without solving. D > 0 gives two distinct real roots, D = 0 gives two equal real roots, and D < 0 gives no real roots. Here D = 0, so the roots are real and equal.

How marks are earned

1. Compute the discriminant D = b² - 4ac = (-6)² - 4·1·9 = 36 - 36 = 0. (1 mark)
2. Since D = 0, conclude that the equation has two equal real roots. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 4, §4.4 Nature of Roots: The discriminant D = b² - 4ac decides the number and type of real roots of ax² + bx + c = 0.

(b) For the quadratic equation 3x² + 6x + k = 0, determine the value of k for which the roots are equal and find the common root. [3 marks]

Answer

For the quadratic 3x² + 6x + k = 0 to have equal roots, its discriminant D = b² - 4ac must be zero, so we set D = 6² - 4·3·k = 0 which gives 36 - 12k = 0. Rearranging the equation 36 - 12k = 0 yields 12k = 36, and dividing both sides by 12 gives the required value k = 3 for equal roots. Substituting k = 3 into the formula for equal roots x = -b/(2a) gives x = -6/(2·3) = -6/6 = -1, so the common root is -1.

Step by step

1. **Set D = 0 for equal roots:** For 3x² + 6x + k = 0, a = 3, b = 6, c = k. Equal roots need D = b² − 4ac = 0, so 6² − 4(3)(k) = 0, which gives 36 − 12k = 0. Why: Equal roots occur exactly when the discriminant is zero. Putting D = 0 turns the condition on roots into an equation in the single unknown k.
2. **Solve for k:** 36 − 12k = 0 gives 12k = 36, so k = 36/12 = 3. Why: This is a linear equation in k, solved by moving 12k across and dividing both sides by 12, the coefficient of k. It gives the only value of k that makes the roots equal.
3. **Find the common root and verify:** With D = 0 the quadratic formula x = (−b ± √D)/2a becomes x = −b/2a = −6/(2×3) = −6/6 = −1. Check: 3(−1)² + 6(−1) + 3 = 3 − 6 + 3 = 0. Why: When √D = 0 the ± part vanishes, so both roots equal −b/2a. Substituting x = −1 into the equation giving 0 confirms it is a root.

Topic and concept

Nature of roots from discriminant (b²−4ac): two distinct real, two equal, no real roots. A quadratic has equal real roots exactly when D = b² − 4ac = 0. Setting D = 0 fixes k = 3. The quadratic formula then reduces to x = −b/2a, which gives the repeated root. Here 3x² + 6x + 3 = 3(x + 1)², so x = −1 twice.

How marks are earned

1. For the quadratic 3x² + 6x + k = 0 to have equal roots, its discriminant D = b² - 4ac must be zero, so we set D = 6² - 4·3·k = 0 which gives 36 - 12k = 0. (1 mark)

2. Rearranging the equation 36 - 12k = 0 yields 12k = 36, and dividing both sides by 12 gives the required value k = 3 for equal roots. (1 mark)
3. Substituting k = 3 into the formula for equal roots x = -b/(2a) gives x = -6/(2·3) = -6/6 = -1, so the common root is -1. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 4, §4.4 Nature of Roots: When b² - 4ac = 0, a quadratic equation has two equal real roots.

OR

Option B

Answer both connected parts. Show the required method, explanation and units.

(a) A rectangular garden has an area of 200 m². Its length exceeds its breadth by 5 metres. If the breadth is x metres, form and simplify the quadratic equation satisfied by x. [2 marks]

Answer

Express the breadth as x metres and the length as x + 5 metres, then equate their product to the given area to form x(x + 5) = 200. Expand the left side to get x² + 5x and then subtract 200 from both sides to simplify the equation to x² + 5x - 200 = 0.

Step by step

1. **Define variables and form the area equation:** Let the breadth be x m. The length exceeds it by 5 m, so the length is (x + 5) m. Area = length × breadth gives x(x + 5) = 200. Why: Area of a rectangle is length times breadth, and the given area of 200 m² is the value this product must equal. Defining x first lets both dimensions be written in one unknown, and x > 0 since it is a length.
2. **Expand and simplify to standard form:** x(x + 5) = x² + 5x, so x² + 5x = 200. Subtracting 200 from both sides gives x² + 5x − 200 = 0. Why: A quadratic equation is written in standard form ax² + bx + c = 0 with zero on one side. Here a = 1, b = 5, c = −200. The terms are in m², so subtracting 200 m² is consistent.

Topic and concept

Situational problems based on quadratic equations in day-to-day contexts. A real situation with fixed area gives a quadratic when one dimension is expressed through the other. Writing breadth as x and length as x + 5 makes the area x(x + 5), which expands to a degree-2 expression in x. Setting it equal to 200 and moving 200 across gives the quadratic.

How marks are earned

1. Express the breadth as x metres and the length as x + 5 metres, then equate their product to the given area to form x(x + 5) = 200. (1 mark)
2. Expand the left side to get x² + 5x and then subtract 200 from both sides to simplify the equation to x² + 5x - 200 = 0. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 4, §4.1 Introduction: Real-life dimensions of rectangular areas lead to quadratic equations when area is fixed.

(b) Solve the quadratic equation $6x^2 + x - 2 = 0$ by splitting the middle term and factorisation. [3 marks]

Answer

Compute a·c = 6·(-2) = -12 and choose numbers 4 and -3 whose product is -12 and sum is 1. Rewrite the middle term as 4x - 3x and factor by grouping: $6x^2 + 4x - 3x - 2 = 2x(3x + 2) -1(3x + 2) = (3x + 2)(2x - 1)$. Set each factor equal to zero, giving roots x = -2/3 and x = 1/2.

Step by step

1. **Find the splitting numbers from ac and b:** For $6x^2 + x - 2 = 0$, a = 6, b = 1, c = −2, so ac = 6×(−2) = −12. We need two numbers with product −12 and sum 1. The pair 4 and −3 works, since 4×(−3) = −12 and 4 + (−3) = 1. Why: If the middle term bx is split into px + qx, then p + q = b. The product pq must be ac so that the four terms can be grouped into common factors. These two conditions identify the split.
2. **Split the middle term and factorise by grouping:** $6x^2 + x - 2 = 6x^2 + 4x - 3x - 2 = 2x(3x + 2) - 1(3x + 2) = (3x + 2)(2x - 1)$. Why: Grouping the first two and last two terms gives the same bracket (3x + 2) in both parts. Taking it out as a common factor writes the quadratic as a product of two linear factors.
3. **Equate each factor to zero and verify:** 3x + 2 = 0 gives x = −2/3, and 2x − 1 = 0 gives x = 1/2. Check x = 1/2: 6(1/4) + 1/2 − 2 = 1.5 + 0.5 − 2 = 0. Check x = −2/3: 6(4/9) − 2/3 − 2 = 8/3 − 2/3 − 2 = 0. Why: A product is zero only if at least one factor is zero, so each linear factor set to zero gives a root. Substituting both roots back to get 0 confirms the factorisation.

Topic and concept

Solution by factorisation. A quadratic that splits into two linear factors has roots found by setting each factor to zero. Splitting the middle term needs two numbers whose sum is b and whose product is ac. For $6x^2 + x - 2$ these are 4 and −3, giving (3x + 2)(2x − 1).

How marks are earned

1. Compute a·c = 6·(-2) = -12 and choose numbers 4 and -3 whose product is -12 and sum is 1. (1 mark)
2. Rewrite the middle term as 4x - 3x and factor by grouping: $6x^2 + 4x - 3x - 2 = 2x(3x + 2) -1(3x + 2) = (3x + 2)(2x - 1)$. (1 mark)
3. Set each factor equal to zero, giving roots x = -2/3 and x = 1/2. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 4, §4.3 Solution of a Quadratic Equation by Factorisation: Splitting the middle term and grouping lead to linear factors whose zeros are the equation's roots.

Section E

Question 36

4 marks

Case Based

In a botanical garden, a circular pond has centre O and radius 8 m. A straight rod AB lies along the tangent to the pond at its contact point P. A point Q lies 15 m from O outside the pond. From Q, two ropes touch the pond at T_1 and T_2, and $QT_1 = QT_2$.

(a) According to the passage, what is a tangent to a circle? [1 mark]

Answer

A tangent to a circle is a straight line that touches the circle at exactly one point.

Step by step

1. **State the definition and contrast it with other line positions:** A tangent to a circle is a straight line that touches the circle at exactly one point. A secant meets the circle at two points, and a line that misses the circle meets it at no point. Why: The number of common points separates the three possible positions of a line and a circle. Exactly one common point is what defines a tangent, and this is why a secant or a non-intersecting line is not a tangent (NCERT Ch. 10, §10.1). The passage does not define the term, so the answer rests on the NCERT definition.

Topic and concept

Tangent to a circle; tangent at any point is perpendicular to the radius (theorem + proof). A line and a circle in a plane can have 0, 1 or 2 common points. With 0 the line does not intersect the circle. With 2 it is a secant. With exactly 1 it is a tangent, and that single point is the point of contact.

How marks are earned

1. A tangent to a circle is a straight line that touches the circle at exactly one point. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 10, §10.2: A tangent to a circle intersects it at exactly one point.

(b) What relationship between the rod AB at P and the radius OP is described by the theorem referenced in the passage? [1 mark]

Answer

Since AB is tangent to the pond at P, AB is perpendicular to radius OP; the angle between AB and OP is 90°.

Step by step

1. **Name the tangent-radius perpendicularity theorem and apply it to rod AB and radius OP:** Rod AB lies along the tangent to the pond at the contact point P, and OP is the radius drawn to that same point P. By the theorem that a tangent is perpendicular to the radius at the point of contact, AB ⊥ OP, so the angle between AB and OP is 90°. Why: The tangent touches the circle only at P, so every other point of AB lies outside the circle and is farther from O than P is. That makes OP the shortest distance from O to the line AB, and the shortest distance from a point to a line is the perpendicular. This is why the angle at P must be 90°.

Topic and concept

Tangent to a circle; tangent at any point is perpendicular to the radius (theorem + proof). A tangent meets a circle at exactly one point, P. Every other point of the tangent lies outside the circle, so the radius OP is the shortest segment from the centre O to the tangent line. The shortest segment to a line is perpendicular to it, so the tangent is perpendicular to the radius at the point of contact. This is the tangent-radius perpendicularity theorem of Class X Circles.

How marks are earned

1. State that AB, a tangent at P, is perpendicular to radius OP at P, giving a 90° angle. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 10, Summary: At the point of contact, the tangent is perpendicular to the radius drawn to that point.

Attempt one of the final two-mark options.

Option A

(c) (i) How many tangents can be drawn from point Q to the pond, and what are they named in the passage? (ii) Which property of the lengths QT_1 and QT_2, mentioned in the passage, illustrates a theorem about tangents from an external point? [2 marks]

Answer

From the external point Q, exactly two tangents can be drawn to the pond, named QT_1 and QT_2. The lengths of the two tangents drawn from the external point Q to the pond are equal ($QT_1 = QT_2$).

Step by step

1. **Count the tangents from the external point Q and name them:** Q is 15 m from O, and the radius is 8 m, so Q lies outside the pond because 15 > 8. From an external point exactly two tangents can be drawn to the circle. In the passage they are QT_1 and QT_2, touching at T_1 and T_2. Why: A point inside a circle has no tangent and a point on the circle has one. A point outside the circle has exactly two (NCERT Ch. 10, §10.3). The passage gives OQ > radius, so Q is external and the count is two.

2. **Show that the two tangent lengths are equal:** Join OT_1 and OT_2. Each radius is perpendicular to its tangent, so $\angle OT_1Q = \angle OT_2Q = 90°$. In right triangles OT_1Q and OT_2Q, $OT_1 = OT_2 = 8$ m and OQ is common, so by RHS the triangles are congruent. Hence $QT_1 = QT_2$. Check by Pythagoras: $QT_1 = \sqrt{(15^2 - 8^2)} = \sqrt{(225 - 64)} = \sqrt{161}$ m, and QT_2 gives the same value. Why: The tangent is perpendicular to the radius at the point of contact, which gives two right triangles with equal radii and a common hypotenuse OQ. RHS congruence makes the corresponding sides QT_1 and QT_2 equal. This is the theorem that the two tangents from an external point are equal in length (NCERT Ch. 10, §10.4).

Topic and concept

Number of tangents from an external point; lengths of two tangents from an external point are equal (theorem + proof). From an external point there are exactly two tangents, and their lengths are equal. The equality follows from the tangent being perpendicular to the radius at the point of contact, which makes the two right triangles formed with the centre congruent by RHS. Pythagoras gives the common length, here $\sqrt{161}$ m.

How marks are earned

1. From the external point Q, exactly two tangents can be drawn to the pond, named QT_1 and QT_2. (1 mark)
2. The lengths of the two tangents drawn from the external point Q to the pond are equal ($QT_1 = QT_2$). (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 10, §10.3: A point outside the circle has exactly two tangents to the circle. Check the reasoning against NCERT Class 10 maths, Ch. 10, Summary: The lengths of the two tangents from an external point to a circle are equal.

OR

Option B

(c) Calculate the length of each rope QT_1 and QT_2 from the external point Q. [2 marks]

Answer

A radius to a tangent is perpendicular at contact, so OT_1Q is right-angled and $QT_1^2 = OQ^2 - OT_1^2$. $QT_1 = \sqrt{(15^2 - 8^2)} = \sqrt{161}$ m; tangents from Q are equal, so $QT_2 = \sqrt{161}$ m too.

Step by step

1. **Justify the right angle at T_1 and write the Pythagoras relation in triangle OT_1Q:** QT_1 is a tangent and OT_1 is the radius to the point of contact T_1, so $\angle OT_1Q = 90°$. In right triangle OT_1Q, the hypotenuse is OQ = 15 m and $OT_1 = 8$ m, so $OQ^2 = OT_1^2 + QT_1^2$, which gives $QT_1^2 = OQ^2 - OT_1^2$. Why: The tangent is perpendicular to the radius at the point of contact, so the triangle formed by O, T_1 and Q has its right angle at T_1. The side OQ is opposite that right angle, so it is the hypotenuse. The Pythagoras theorem can be used only after this right angle is established.

2. **Substitute the values, simplify, and use equal tangents for the second rope:** $QT_1^2 = 15^2 - 8^2 = 225 - 64 = 161$ m², so $QT_1 = \sqrt{161}$ m ≈ 12.69 m, since $12.69^2 \approx 161.04$. The two tangents from the external point Q are equal, so $QT_2 = QT_1 = \sqrt{161}$ m ≈ 12.69 m. Why: The theorem on tangents from an external point says both tangent segments have the same length. The second rope therefore needs no separate calculation. The check $12.69^2 \approx 161$ confirms the square root, and the length is given in metres because the distances were in metres.

Topic and concept

Number of tangents from an external point; lengths of two tangents from an external point are equal (theorem + proof). Two ideas combine here. First, the radius to the point of contact is perpendicular to the tangent, which makes triangle OTQ right-angled at T. Second, the two tangents from an external point are equal in length. Pythagoras then gives the tangent length from the distance OQ and the radius.

How marks are earned

1. A radius to a tangent is perpendicular at contact, so OT_1Q is right-angled and $QT_1^2 = OQ^2 - OT_1^2$. (1 mark)
2. $QT_1 = \sqrt{(15^2 - 8^2)} = \sqrt{161}$ m; tangents from Q are equal, so $QT_2 = \sqrt{161}$ m too. (1 mark)

Source check: Check the passage facts and NCERT Class 10 maths, Ch. 10, §10.2 against both steps.

Question 37

4 marks

Case Based

In an experiment, two unbiased coins are tossed together. The possible outcomes of this random experiment are equally likely. Consider events based on this experiment.

(a) List the sample space for the experiment described in the passage. [1 mark]

Answer

The sample space is {HH, HT, TH, TT}.

Step by step

1. **List all ordered outcomes from tossing the first coin and the second coin.:** For each coin, the possible results are H or T. Combining the first coin with the second coin gives: HH, HT, TH, TT. Therefore, the sample space is {HH, HT, TH, TT}. Why: The sample space must contain every possible elementary outcome of the random experiment; HT and TH are written separately because the two coin positions can be distinguished as first coin and second coin.

Topic and concept

Classical definition of probability — P(E) = number of favourable outcomes / total outcomes. The probability of an event E, assuming equally likely outcomes, is the ratio of favourable outcomes to the total number of outcomes.

How marks are earned

1. The sample space is {HH, HT, TH, TT}. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 14, §14.1 Probability — A Theoretical Approach: The sample space of a random experiment is the set of all possible outcomes.

(b) If E is the event “exactly one head appears,” find P(E). [1 mark]

Answer

Using U7-CH14-F001, P(E) = Number of favourable outcomes (2) / Total outcomes (4) = 2/4 = 1/2.

Step by step

1. **List the outcomes and apply the probability ratio:** Total equally likely outcomes of tossing two coins are HH, HT, TH, TT, so there are 4. The favourable outcomes for E (exactly one head) are HT and TH, so there are 2. P(E) = favourable / total = 2/4 = 1/2. Why: When all outcomes are equally likely, the probability of an event is the number of favourable outcomes divided by the total number of outcomes (NCERT Ch. 14, §14.1). Listing the sample space shows that HT and TH are different outcomes, so both are counted. The result 1/2 lies between 0 and 1, as a probability must.

Topic and concept

Probability of sure and impossible events; 0 ≤ P(E) ≤ 1. For equally likely outcomes, P(E) = (number of favourable outcomes) / (total number of outcomes). Two coins have 4 outcomes: HH, HT, TH, TT. Exactly one head happens in two of them, HT and TH, which gives 1/2.

How marks are earned

1. Using U7-CH14-F001, P(E) = Number of favourable outcomes (2) / Total outcomes (4) = 2/4 = 1/2. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 14, §14.1 Probability — A Theoretical Approach: Classical probability is defined as favourable outcomes over total equally likely outcomes.

Attempt one of the final two-mark options.

Option A

(c) (i) For the event F = “at least one head appears,” determine whether it is a sure event, an impossible event, or neither. (ii) Let G be the event “both tosses show tails.” Find P(G′) and verify that P(G)+P(G′)=1. [2 marks]

Answer

P(F)=3/4, which lies strictly between 0 and 1, so F is neither a sure nor an impossible event. P(G)=1/4, so P(G′)=1−1/4=3/4, and thus P(G)+P(G′)=1/4+3/4=1.

Step by step

1. **Find P(F) and classify the event:** The sample space is {HH, HT, TH, TT}, so there are 4 outcomes. F (at least one head) = {HH, HT, TH}, so there are 3 favourable outcomes. P(F) = 3/4. Since 0 < 3/4 < 1, F is neither a sure event nor an impossible event. Why: A sure event has probability 1 and an impossible event has probability 0 (NCERT Ch. 14, §14.2). F fails to occur in the outcome TT, so it is not sure. It does occur in three outcomes, so it is not impossible.
2. **Find P(G′) and verify that the probabilities add to 1:** G = {TT}, so P(G) = 1/4. G′ means 'not both tails', which is {HH, HT, TH}, so P(G′) = 3/4. Also P(G′) = 1 − 1/4 = 3/4. Check: P(G) + P(G′) = 1/4 + 3/4 = 4/4 = 1. Why: An event and its complement together cover every outcome with no overlap. Their probabilities therefore add to 1, so P(E′) = 1 − P(E). Counting the complement directly gives the same 3/4, which confirms the result.

Topic and concept

Complementary events and their probability relation. The complement G′ of an event G contains all outcomes not in G, so P(G) + P(G′) = 1. An event with probability strictly between 0 and 1 is neither sure nor impossible. Here P(F) = 3/4 and P(G′) = 3/4. Both come from the 4 equally likely outcomes.

How marks are earned

1. P(F)=3/4, which lies strictly between 0 and 1, so F is neither a sure nor an impossible event. (1 mark)
2. P(G)=1/4, so P(G′)=1−1/4=3/4, and thus P(G)+P(G′)=1/4+3/4=1. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 14, §14.2 Summary: A probability lies between 0 and 1; 0 indicates impossible, 1 indicates sure. Check the reasoning against NCERT Class 10 maths, Ch. 14, §14.2 Summary: For any event E, P(E)+P(E′)=1.

OR

Option B

(c) Find the probability of at least one tail, then the probability of its complementary event. [2 marks]

Answer

At least one tail occurs in HT, TH and TT, so its probability is 3/4. The complementary event is no tail, namely HH, with probability 1/4; the two probabilities sum to 1.

Step by step

1. **List the sample space and count outcomes with at least one tail:** Tossing two coins gives the sample space {HH, HT, TH, TT}, which has 4 equally likely outcomes. Outcomes with at least one tail are HT, TH and TT, which is 3 outcomes. So P(at least one tail) = 3/4. Why: Probability for equally likely outcomes is favourable outcomes divided by total outcomes. Each of the four outcomes is equally likely, so 3 favourable outcomes out of 4 give 3/4. Listing the sample space first ensures no outcome is missed or counted twice.

2. **Identify the complementary event and verify with P(E) + P(not E) = 1:** The complement of 'at least one tail' is 'no tail', which is HH only, so P(no tail) = 1/4. Check: 3/4 + 1/4 = 1, and also 1 − 3/4 = 1/4. Why: An event and its complement together cover every outcome with no overlap. Their probabilities therefore add up to 1. The sum equal to 1 confirms the counting is correct.

Topic and concept

Complementary events and their probability relation. For equally likely outcomes, P(E) = favourable outcomes ÷ total outcomes. The complement of E contains every outcome not in E, so P(E) + P(not E) = 1. Here 'at least one tail' and 'no tail' are complementary events on the sample space {HH, HT, TH, TT}.

How marks are earned

1. At least one tail occurs in HT, TH and TT, so its probability is 3/4. (1 mark)
2. The complementary event is no tail, namely HH, with probability 1/4; the two probabilities sum to 1. (1 mark)

Source check: Check the passage facts and NCERT Class 10 maths, Ch. 14, §14.1 Probability — A Theoretical Approach against both steps.

Question 38

4 marks

Case Based

A water storage tank at the fire station has the shape of a right circular cylinder of radius 3 m and height 10 m, with two hemispheres of the same radius attached at both ends. The tank is made of steel and needs painting on its external surface and its internal capacity is used for water storage.

(a) Which parts of the tank's surface will require painting on the outside? [1 mark]

Answer

Only the curved surface of the cylinder and the curved surfaces of the two hemispheres are exposed and require painting.

Step by step

1. **Decide which surfaces are exposed on the outside:** The tank is a cylinder with a hemisphere joined at each end. The flat circular ends of the cylinder are covered by the hemispheres, and the flat circular bases of the hemispheres lie against the cylinder. These circles are hidden inside the joins and are not painted. The outside surface is therefore the curved surface of the cylinder plus the curved surfaces of the two hemispheres. Why: When solids are joined, the faces where they meet become internal and are no longer part of the outer surface. Only exposed curved surfaces count towards the area to be painted, so the total is CSA of cylinder + 2 × CSA of hemisphere (NCERT Ch. 12, §12.2).

Topic and concept

Surface area and volume of combinations — cone + hemisphere, cylinder + hemisphere, etc.. For a composite solid, the outer surface area includes only the faces that remain visible. A cylinder with a hemisphere on each end hides the circular join faces, so only the cylinder's curved surface and the two hemispherical curved surfaces are painted.

How marks are earned

1. Only the curved surface of the cylinder and the curved surfaces of the two hemispheres are exposed and require painting. (1 mark)

Source check: Check the reasoning against NCERT Class 10 mathematics, Ch. 12, §12.2: Only exposed curved surfaces of joined solids contribute to the total surface area.

(b) Write an expression using standard surface-area formulas to compute the tank's total external surface area. [1 mark]

Answer

TSA = CSA_cylinder + 2 × CSA_hemisphere = 2πrh + 2 × (2πr²).

Step by step

1. **Add the curved surface areas of the cylinder and the two hemispheres, then substitute r = 3 m and h = 10 m as a check:** TSA = CSA_cylinder + 2 × CSA_hemisphere = 2πrh + 2 × (2πr²). Check with r = 3 m, h = 10 m: 2πrh = 2π × 3 × 10 = 60π m² and 2 × 2πr² = 4π × 9 = 36π m², so TSA = 60π + 36π = 96π m² ≈ 301.44 m² (using π = 3.14). The two hemispheres together make a sphere, so 2 × 2πr² = 4πr². Why: The hemispherical ends are attached to the cylinder's circular faces, so those flat faces are hidden and only the curved surfaces are exposed to paint. The total external area is therefore the cylinder's curved surface plus the curved surfaces of both hemispheres, and the area unit is m².

Topic and concept

Surface area and volume of combinations — cone + hemisphere, cylinder + hemisphere, etc.. For a cylinder with a hemisphere on each end, only the curved surfaces are on the outside. The cylinder's circular bases are covered by the hemispheres, and the hemispheres' flat bases are covered by the cylinder. So TSA = CSA of cylinder + 2 × CSA of hemisphere. The CSA of a cylinder is 2πrh and the CSA of a hemisphere is 2πr².

How marks are earned

1. TSA = CSA_cylinder + 2 × CSA_hemisphere = 2πrh + 2 × (2πr²). (1 mark)

Source check: Check the reasoning against NCERT Class 10 mathematics, Ch. 12, §12.2: Total surface area of a cylinder with two hemispheres attached is CSA_cylinder + 2×CSA_hemisphere.

Attempt one of the final two-mark options.

Option A

(c) (i) Formulate the expression for the tank's internal volume (its capacity). (ii) Explain why the circular faces at the joins are not counted in the external surface area. [2 marks]

Answer

Volume_total = Volume_cylinder + 2 × Volume_hemisphere = πr²h + 2 × (2/3 πr³). Because those flat faces become internal at the junction, they are not exposed and hence not included in the external surface area.

Step by step

1. **Add the volume of the cylinder to the volumes of the two hemispheres, then substitute the given values:** Volume_total = Volume_cylinder + 2 × Volume_hemisphere = πr²h + 2 × (2/3 πr³). Check with r = 3 m, h = 10 m: πr²h = π × 9 × 10 = 90π m³ and 2 × (2/3)π × 27 = 36π m³, so V = 126π m³ ≈ 396 m³ (π = 22/7). Why: Volume is the space inside a solid. When two solids are joined, the space they enclose adds with no overlap, so the capacity of the tank is the sum of the volumes of its parts. The unit is m³.
2. **Explain why the circular faces at the joins are left out of the external surface area:** The circular faces at the joins lie inside the solid, where the cylinder and each hemisphere meet. They are not exposed, so they are not included in the external surface area. Why: Surface area counts only the outer surface that can be seen or painted. Joining solids hides those faces inside the body, so they do not contribute to the total external area.

Topic and concept

Surface area and volume of combinations — cone + hemisphere, cylinder + hemisphere, etc.. The volume of a composite solid is the sum of the volumes of the solids it is made from. Here these are one cylinder (πr²h) and two hemispheres (each 2/3 πr³). For surface area, faces that are joined together are inside the solid and are not counted, so only the outer curved surfaces contribute.

How marks are earned

1. Volume_total = Volume_cylinder + 2 × Volume_hemisphere = πr²h + 2 × (2/3 πr³). (1 mark)
2. Because those flat faces become internal at the junction, they are not exposed and hence not included in the external surface area. (1 mark)

Source check: Check the reasoning against NCERT Class 10 mathematics, Ch. 12, §12.3: The volume of a composite solid equals the sum of volumes of its parts. Check the reasoning against NCERT Class 10 mathematics, Ch. 12, §12.2: Hidden faces formed by joining solids are not counted in the total surface area.

OR

Option B

(c) Calculate the tank's total external surface area using its given radius and cylinder height. [2 marks]

Answer

The exposed area is $2\pi rh+4\pi r^2$ because the two hemispheres together form one sphere. With r=3 m and h=10 m, area = $2\pi(3)(10)+4\pi(3^2)=96\pi$ m².

Step by step

1. **Decide which surfaces are exposed and write the area formula:** The tank is a cylinder with a hemisphere on each end. The circular joins are hidden, so the external area is the curved surface of the cylinder plus the curved surfaces of two hemispheres. Area = $2\pi rh + 2(2\pi r^2) = 2\pi rh + 4\pi r^2$, because two hemispheres together make one sphere of area $4\pi r^2$. Why: Only exposed curved surfaces count in the external area. The circular faces where the solids join are inside the tank and are not painted. Two hemispheres of equal radius have the same curved area as one complete sphere, so the sum becomes $4\pi r^2$.
2. **Substitute r = 3 m and h = 10 m, then add and give an approximate value:** Cylinder part: $2\pi(3)(10) = 60\pi$ m². Sphere part: $4\pi(3^2) = 36\pi$ m². Total = $60\pi + 36\pi = 96\pi$ m². Using $\pi = 3.14$, area ≈ 96 × 3.14 = 301.44 m². Using $\pi = 22/7$, area ≈ 301.7 m². Why: Both parts contain π, so their coefficients can be added directly to give 96π. Area is in square units, so the answer is in m². The two decimal values agree closely, which checks the arithmetic. The exact value 96π m² is the keyed answer.

Topic and concept

Surface area and volume of combinations — cone + hemisphere, cylinder + hemisphere, etc.. For a composite solid, only the surfaces left exposed after joining count towards the external surface area. A cylinder with two hemispherical ends has area equal to the cylinder's curved surface plus the curved surfaces of both hemispheres. The hemisphere bases are hidden and are not counted.

How marks are earned

1. The exposed area is $2\pi rh+4\pi r^2$ because the two hemispheres together form one sphere. (1 mark)
2. With r=3 m and h=10 m, area = $2\pi(3)(10)+4\pi(3^2)=96\pi$ m². (1 mark)

Source check: Check the passage facts and NCERT Class 10 mathematics, Ch. 12, §12.2 against both steps.