ShunyaSaarthi
Class X practice studio

CLASS X · 2026-27 · MODEL PAPER

Class X Mathematics Standard (Code 041) Model Paper 1

Worked answers & explanations

Maximum marks: 80

Time: 3 hours

Subject code: 041

Practice, understand, improve.

Instructions

All 38 questions are compulsory, with one selected option for each internal choice.

Section A has 18 MCQs and 2 assertion-reason questions; Section E has 1+1+2-mark case subparts.

Calculators are not allowed; draw labelled figures where needed.

Section A

Question 1

1 mark

Mcq

Choose the correct answer.

(a) At the point of contact, a tangent to a circle is perpendicular to which line? [1 mark]

A. Diameter
B. Chord through the point of contact
C. Radius drawn to the point of contact
D. Secant through the point of contact

Answer

C. Radius drawn to the point of contact

Step by step

1. **Select and justify the option:** The tangent at a point on the circle is perpendicular to the radius drawn to that point. Why: The tangent at any point of a circle is perpendicular to the radius at that point.

Topic and concept

Tangent to a circle; tangent at any point is perpendicular to the radius (theorem + proof). A tangent to a circle is a line that touches the circle at exactly one point, unlike a secant which meets it at two points or a non-intersecting line which meets it at none.

How marks are earned

1. The tangent at a point on the circle is perpendicular to the radius drawn to that point. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 10, §10.2: The tangent at any point of a circle is perpendicular to the radius at that point.

Question 2

1 mark

Mcq

Choose the correct answer.

(a) In triangle ABC, a line DE is drawn parallel to BC meeting AB at D and AC at E. Which relation must hold? [1 mark]

A. AD/DB = AE/EC

B. AD/AB = BC/DE

C. BD/DC = BE/EC

D. AB/AC = DE/BC

Answer

A. By the Basic Proportionality Theorem, AD/DB = AE/EC.

Step by step

1. **Apply the proportionality theorem:** Because DE is parallel to BC in triangle ABC, the Basic Proportionality Theorem gives AD/DB = AE/EC, which is option A. Why: A line parallel to one side divides the other two sides in the same ratio.

Topic and concept

Basic Proportionality Theorem — proof, application and use of its converse. The Basic Proportionality Theorem gives AD/DB = AE/EC when DE∥BC, with D on AB and E on AC.

How marks are earned

1. Select unique option A and use DE∥BC with the Basic Proportionality Theorem to infer AD/DB=AE/EC. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 6, §6.5 Summary: A line parallel to one side of a triangle divides the other two sides proportionally.

Question 3

1 mark

Mcq

Choose the correct answer.

(a) Which of the following lists of numbers is an arithmetic progression? [1 mark]

A. 2, 4, 6, 8, ...

B. 3, 6, 12, 24, ...

C. –1, 0, 1, 3, ...

D. 1, 4, 9, 16, ...

Answer

A. 2, 4, 6, 8, ...

Step by step

1. **Select and justify the option:** Only in option A the difference between consecutive terms is constant (4 − 2 = 2, 6 − 4 = 2, …). Why: In an AP, the difference between successive terms is the same.

Topic and concept

Definition and examples of AP; common difference. An arithmetic progression is a sequence where each term after the first is obtained by adding a constant called the common difference.

How marks are earned

1. Only in option A the difference between consecutive terms is constant (4 − 2 = 2, 6 − 4 = 2, …). (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 5, §5.2 Arithmetic Progressions: In an AP, the difference between successive terms is the same.

Question 4

1 mark

Mcq

Choose the correct answer.

(a) What term is used for the angle between the observer's line of sight and the horizontal when looking down at an object below the horizontal level? [1 mark]

A. Angle of depression

B. Angle of elevation

C. Vertical angle

D. Angle of incidence

Answer

A. Angle of depression

Step by step

1. **Select and justify the option:** The angle of depression is the angle formed by the line of sight with the horizontal when the object is below the horizontal level. Why: Angle of depression is defined when the line of sight is below the horizontal level.

Topic and concept

Angle of elevation and angle of depression; applications to heights and distances. The angle of depression is the angle between the horizontal line through the observer's eye and the line of sight when looking at a point below the horizontal.

How marks are earned

1. The angle of depression is the angle formed by the line of sight with the horizontal when the object is below the horizontal level. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 9, §9.1 Heights and Distances: Angle of depression is defined when the line of sight is below the horizontal level.

Question 5

1 mark

Mcq

Choose the correct answer.

(a) What is the distance between A(4, 0) and B(9, 0)? [1 mark]

A. 13 units

B. 5 units

C. 36 units

D. 2 units

Answer

B. 5 units

Step by step

1. **Select and justify the option:** For two points on the x-axis, distance = $|x_2 - x_1|$, so $|9 - 4| = 5$ units. Why: If two points lie on the x-axis, their distance is the absolute difference of their x-coordinates.

Topic and concept

Distance formula between two points. If two points lie on the same axis, their distance is the absolute difference of their corresponding coordinates.

How marks are earned

1. For two points on the x-axis, distance = $|x_2 - x_1|$, so $|9 - 4| = 5$ units. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 7, §7.2 Distance Formula: If two points lie on the x-axis, their distance is the absolute difference of their x-coordinates.

Question 6

1 mark

Mcq

Choose the correct answer.

(a) What is cos 60°? [1 mark]

A. 1/2

B. √3/2

C. √2/2

D. 0

Answer

A. 1/2

Step by step

1. **Build a 30°-60°-90° triangle and read cos 60° as adjacent over hypotenuse:** Take an equilateral triangle of side 2 and draw an altitude. It splits the triangle into a 30°-60°-90° right triangle with hypotenuse 2, the shorter side 1 (half the base), and the other side $\sqrt{(2^2 - 1^2)} = \sqrt{3}$, so the sides are 1 : √3 : 2. For the 60° angle the adjacent side is 1 and the hypotenuse is 2, so cos 60° = 1/2, which is option A. Why: Cosine is defined as adjacent side divided by hypotenuse in a right triangle, and the 30°-60°-90° triangle fixes these sides, so the value follows from the geometry rather than from memory. Option B (√3/2) is cos 30°, which comes from using the angle 30° instead of 60°, so A is the only correct choice.

Topic and concept

Proof and application of $\sin^2\theta + \cos^2\theta = 1$. For an acute angle in a right triangle, cos θ = adjacent side / hypotenuse. The 30°-60°-90° triangle, obtained by halving an equilateral triangle of side 2, has sides 1 : √3 : 2. From it cos 60° = 1/2, sin 60° = √3/2 and cos 30° = √3/2.

How marks are earned

1. cos 60° = 1/2 as derived from a 30°-60°-90° right triangle. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 8, §8.3: In a 30°-60°-90° triangle, cos 60° equals one-half.

Question 7

1 mark

Mcq

Choose the correct answer.

(a) What is the abscissa of the point P(−3, 8)? [1 mark]

A. −3
B. 8
C. 3
D. −8

Answer

A. −3

Step by step

1. **Read the signed x-coordinate:** For P(−3, 8), the ordered pair has x = −3 and y = 8. The abscissa is the first coordinate, so the answer is −3 (option A). Why: The abscissa is the signed x-coordinate. A point three units left of the y-axis has x = −3; its ordinary distance from the axis is 3, but the coordinate keeps the leftward sign.

Topic and concept

Using coordinate axes to locate points. The abscissa of (x, y) is its signed x-coordinate. Positive x lies to the right of the y-axis, negative x to the left. For P(−3, 8), the abscissa is −3; the unsigned distance to the y-axis is 3.

How marks are earned

1. The abscissa is the x-coordinate of the point, which for P(−3, 8) is −3. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 7, §7.1 Introduction: The abscissa of a point is its distance from the y-axis, given by the x-coordinate.

Question 8

1 mark

Mcq

Choose the correct answer.

(a) Given the pair of equations 2x + 3y – 5 = 0 and 4x + 6y – 12 = 0, which statement best describes the solution of the system? [1 mark]

A. The lines intersect and there is a unique solution.
B. The lines coincide and there are infinitely many solutions.
C. The lines are parallel and there is no solution because 2/4 ≠ 3/6.
D. The lines are parallel and there is no solution because 2/4 = 3/6 ≠ 5/12.

Answer

D. The lines are parallel and there is no solution because 2/4 = 3/6 ≠ 5/12.

Step by step

1. **Select and justify the option:** Here $a_1/a_2 = 2/4 = 1/2$ and $b_1/b_2 = 3/6 = 1/2$, while $c_1/c_2 = 5/12 \neq 1/2$, so the lines are parallel and the system is inconsistent. Why: Parallel lines occur when $a_1/a_2 = b_1/b_2 \neq c_1/c_2$, giving no solution.

Topic and concept

Graphical representation of pair of linear equations; consistent, inconsistent, dependent conditions. A pair of linear equations in two variables can be represented graphically as two lines in the plane.

How marks are earned

1. Here $a_1/a_2 = 2/4 = 1/2$ and $b_1/b_2 = 3/6 = 1/2$, while $c_1/c_2 = 5/12 \neq 1/2$, so the lines are parallel and the system is inconsistent. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 3, §3.2: Parallel lines occur when $a_1/a_2 = b_1/b_2 \neq c_1/c_2$, giving no solution.

Question 9

1 mark

Mcq

Choose the correct answer.

(a) In right triangle ABC with $\angle C = 90°$, relative to angle A, which side is adjacent? [1 mark]

A. BC

B. AC

C. AB

D. The hypotenuse

Answer

B. AC

Step by step

1. **Select and justify the option:** For angle A, the side AC is next to A (other than the hypotenuse), so it is adjacent. Why: The side adjacent to an acute angle is the one next to it apart from the hypotenuse.

Topic and concept

Trigonometric ratios — sin, cos, tan, cot, sec, cosec — defined for acute angles in a right triangle. In a right triangle, for an acute angle θ, sin θ, cos θ and tan θ are defined as ratios of specific sides, and sec θ, cosec θ, cot θ are their reciprocals.

How marks are earned

1. For angle A, the side AC is next to A (other than the hypotenuse), so it is adjacent. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 8, §8.2: The side adjacent to an acute angle is the one next to it apart from the hypotenuse.

Question 10

1 mark

Mcq

Choose the correct answer.

(a) The roots of the equation $x^2 - x - 6 = 0$ found by factorisation are: [1 mark]

A. 2 and 3

B. −2 and 3

C. −2 and −3

D. 2 and −3

Answer

B. −2 and 3

Step by step

1. **Select and justify the option:** $x^2 - x - 6$ factorises as $(x - 3)(x + 2) = 0$, giving roots $x = 3$ and $x = -2$. Why: Splitting and factorising the middle term gives linear factors whose zeros are the roots.

Topic and concept

Solution by factorisation. If $ax^2 + bx + c$ can be expressed as the product of two linear factors, then each factor set to zero gives the roots of the equation.

How marks are earned

1. $x^2 - x - 6$ factorises as $(x - 3)(x + 2) = 0$, giving roots $x = 3$ and $x = -2$. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 4, §4.3 Solution of a Quadratic Equation by Factorisation: Splitting and factorising the middle term gives linear factors whose zeros are the roots.

Question 11

1 mark

Mcq

Choose the correct answer.

(a) Which option shows a solid formed by joining exactly two basic solids—a cube, cuboid, sphere,

hemisphere, cylinder or cone? [1 mark]

A. Sphere + Hemisphere + Cone

B. Cylinder + Cube

C. Cone + Frustum + Cube

D. Three hemispheres joined together

Answer

B. A cylinder joined to a cube is formed from exactly two listed basic solids.

Step by step

1. **Count the listed solids in each option:** B joins a cylinder and a cube, exactly two of the basic solids named in the question. A combines sphere, hemisphere and cone; C combines cone, frustum and cube; D combines three hemispheres. Why: The question asks for exactly two from the named list, leaving only B.

Topic and concept

Combinations are restricted to any two cubes, cuboids, spheres, hemispheres and right circular cylinders or cones. A composite can join two basic solids. Count the component solids in each choice and check that both belong to the list in the prompt.

How marks are earned

1. Select B and explain that a cylinder and a cube are exactly two listed basic solids joined into one composite. (1 mark)

Source check: NCERT Mathematics X, Ch. 12, combinations of two familiar solids: this question specifies exactly two basic solids.

Question 12

1 mark

Mcq

Choose the correct answer.

(a) Which of the following is the prime factorization of 84? [1 mark]

A. 2 × 2 × 3 × 7

B. 2 × 3 × 7

C. 2 × 2 × 5 × 7

D. 2 × 2 × 3 × 3 × 7

Answer

A. 2 × 2 × 3 × 7

Step by step

1. **Build the prime factorisation of 84 by a division ladder, then match it to option A and rule out B and D:** Divide by the smallest prime each time: 84÷2=42, 42÷2=21, 21÷3=7, and 7 is prime, so 84=2×2×3×7. Check: 2×2=4, 4×3=12, 12×7=84. Option B gives 2×3×7=42, option C gives 2×2×5×7=140, and option D gives 2×2×3×3×7=252, so only A equals 84. Why: Every integer greater than 1 is a product of primes in exactly one way apart from order, so the ladder must end at the factors 2, 2, 3, 7. Option B is the nearest plausible error: it stops one division early and drops a factor of 2. Option D adds an extra 3 that never appears in the ladder.

Topic and concept

Fundamental Theorem of Arithmetic — statement, illustration and applications through prime factorisation. Every integer greater than 1 is either prime or a product of primes, and that product is unique apart from order. For 84, the repeated-division ladder gives 2, 2, 3, 7, and no other set of primes multiplies to 84. Multiplying the factors back is a quick check on any option.

How marks are earned

1. 84=2×2×3×7 by repeatedly dividing by the smallest prime factors. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 1, §1.2: Every integer greater than 1 can be expressed as a unique product of primes.

Question 13

1 mark

Mcq

Choose the correct answer.

(a) How many tangents can be drawn from a point lying inside a circle? [1 mark]

A. Zero
B. One
C. Two
D. Infinitely many

Answer

A. Zero

Step by step

1. **Decide whether a tangent is possible:** A tangent touches a circle at exactly one point but a line through a point inside the circle crosses the circle at two points, so no tangent can be drawn from an interior point. Why: The interior point cannot serve as the external start of a tangent segment; choose zero, option A.

Topic and concept

Number of tangents from an external point; lengths of two tangents from an external point are equal (theorem + proof). From a point inside a circle, every line through the point meets the boundary in two places, so zero tangents can be drawn. A tangent touches the circle at only one point.

How marks are earned

1. No tangents can be drawn from a point inside the circle. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 10, §10.3: No tangent can be drawn from a point lying inside the circle.

Question 14

1 mark

Mcq

Choose the correct answer.

(a) The tangent at point P on a circle can be viewed as the limiting position of which of these lines through P? [1 mark]

A. Secant through P

B. Chord through P

C. Diameter through P

D. Non-intersecting line through P

Answer

A. Secant through P

Step by step

1. **Select and justify the option:** A tangent is the limiting case of a secant whose two intersection points on the circle merge into one. Why: By rotating a secant about a point P, its second intersection moves toward P until it coincides, yielding the tangent.

Topic and concept

Tangent to a circle; tangent at any point is perpendicular to the radius (theorem + proof). A tangent can be viewed as the limiting position of a secant line whose two intersection points on the circle merge into one.

How marks are earned

1. A tangent is the limiting case of a secant whose two intersection points on the circle merge into one. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 10, §10.2: By rotating a secant about a point P, its second intersection moves toward P until it coincides, yielding the tangent.

Question 15

1 mark

Mcq

Choose the correct answer.

(a) When a fair coin is tossed once, which of the following represents its sample space? [1 mark]

A. {H, T}
B. {H}
C. {T}
D. Ø

Answer

A. {H, T}

Step by step

1. **Select and justify the option:** The sample space for a fair coin toss includes both head (H) and tail (T), so {H, T}. Why: A fair coin has two equally likely outcomes: head or tail.

Topic and concept

Classical definition of probability — P(E) = number of favourable outcomes / total outcomes. Classical probability applies only when all elementary outcomes have the same chance of occurring.

How marks are earned

1. The sample space for a fair coin toss includes both head (H) and tail (T), so {H, T}. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 14, §Probability — A Theoretical Approach: A fair coin has two equally likely outcomes: head or tail.

Question 16

1 mark

Mcq

Choose the correct answer.

(a) If two angles of ΔABC are respectively equal to two angles of ΔDEF, by which criterion are they similar? [1 mark]

A. SSS
B. SAS
C. AA
D. RHS

Answer

C. AA

Step by step

1. **Select and justify the option:** When two pairs of corresponding angles are equal, triangles are similar by AA criterion. Why: AA criterion: Two angles of one triangle equal two angles of another implies similarity.

Topic and concept

Criteria for similarity of triangles — AA, SSS, SAS. If two angles of one triangle are equal to two angles of another, the triangles are similar (AA criterion).

How marks are earned

1. When two pairs of corresponding angles are equal, triangles are similar by AA criterion. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 6, §6.5 Summary: AA criterion: Two angles of one triangle equal two angles of another implies similarity.

Question 17

1 mark

Mcq

Choose the correct answer.

(a) Which principle is used in the proof that a tangent is perpendicular to the radius at the point of contact? [1 mark]

A. The shortest distance from a point to a line is along the perpendicular

B. Opposite angles in a cyclic quadrilateral sum to 180°

C. Equal chords subtend equal angles at the center

D. The angle in a semicircle is a right angle

Answer

A. The shortest distance from a point to a line is along the perpendicular

Step by step

1. **Select option A by recalling the tangent proof: pick a point Q on the tangent other than P and compare OQ with OP:** Let the tangent touch the circle at P with centre O, and take any other point Q on the tangent. Q lies outside the circle, so OQ is greater than the radius OP. This holds for every Q on the line, so OP is the shortest distance from O to the tangent. The shortest distance from a point to a line is along the perpendicular, so OP is perpendicular to the tangent. Option A states exactly this principle. Why: The tangent meets the circle at only one point, so every other point of the line is outside the circle and farther from O than P. The perpendicular is the unique shortest segment from a point to a line, so OP being shortest forces the right angle at P. Options B, C and D are circle results used in other proofs, and none of them compares OQ with OP.

Topic and concept

Tangent to a circle; tangent at any point is perpendicular to the radius (theorem + proof). At the point of contact, the tangent to a circle is perpendicular to the radius drawn to that point (NCERT Class 10 maths, Ch. 10, §10.4). The proof is indirect. Every point Q of the tangent other than P is outside the circle, so OQ > OP. OP is therefore the shortest distance from O to the line, and the shortest distance is the perpendicular one.

How marks are earned

1. The proof uses the fact that the shortest distance from the centre to the tangent line is along the perpendicular from the centre. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 10, §10.2: In the proof, the radius to the point of contact is the shortest distance to the tangent, establishing a right angle.

Question 18

1 mark

Mcq

Choose the correct answer.

(a) In the mode formula Mode = $l + ((f_1 - f_0)/(2f_1 - f_0 - f_2)) \times h$, what does f_0 denote? [1 mark]

A. frequency of modal class

B. frequency of class just before modal class

C. total frequency

D. class width

Answer

B. frequency of class just before modal class

Step by step

1. **Select and justify the option:** f_0 represents the frequency of the class immediately preceding the modal class. Why: In the mode formula, f_0 is defined as the frequency of the class before the modal class.

Topic and concept

Mode of grouped data — modal class; mode formula. The mode in grouped data is estimated within the modal class by applying Mode = $l+((f_1-f_0)/(2f_1-f_0-f_2))\times h$.

How marks are earned

1. f_0 represents the frequency of the class immediately preceding the modal class. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 13, §13.3: In the mode formula, f_0 is defined as the frequency of the class before the modal class.

Question 19

1 mark

Assertion Reason

Choose the correct answer.

(a) Assertion (A): The distance between P(1, 2) and Q(4, 6) is |(4 − 1) + (6 − 2)| units. Reason (R): In the distance formula, each difference in coordinates must be squared. [1 mark]

A. Both A and R true and R explains A

B. Both A and R true but R does not explain A

C. A true, R false

D. A false, R true

Answer

D. A false, R true

Step by step

1. **Evaluate the true distance and test A and R:** For P(1, 2) and Q(4, 6), the differences are 4 − 1 = 3 and 6 − 2 = 4. Distance PQ = $\sqrt{(3^2 + 4^2)} = \sqrt{(9 + 16)} = \sqrt{25} = 5$ units. The assertion gives |3 + 4| = 7, and 7 ≠ 5, so A is false. R says each coordinate difference is squared, which is exactly what the formula does (3^2 and 4^2), so R is true. Hence the answer is D: A false, R true. Why: The distance formula comes from the Pythagoras theorem on the right triangle with legs $|x_2 - x_1|$ and $|y_2 - y_1|$. The distance is the hypotenuse, so the legs must be squared, added and then square-rooted, and adding the legs directly (3 + 4 = 7) is not valid. R is a true statement, but it does not make A true because A uses the wrong method, so options A and B are ruled out and C is wrong since R is true.

Topic and concept

Distance formula between two points. The distance between (x_1, y_1) and (x_2, y_2) is $\sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$. It is the hypotenuse of a right triangle whose legs are the coordinate differences. Simply adding the differences does not give the distance, except when the points lie on the same horizontal or vertical line.

How marks are earned

1. Assertion is false because the correct distance is $\sqrt{[(4-1)^2+(6-2)^2]}$, and Reason is true as each coordinate difference must be squared before summing. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 7, §7.2 Distance Formula: Each difference in coordinates is squared and summed under the square root for distance calculation.

Question 20

1 mark

Assertion Reason

Choose the correct answer.

(a) Assertion (A): If the discriminant of a quadratic equation is negative, the equation has no real roots. Reason (R): A negative discriminant implies the square root term in the quadratic formula is not real. [1 mark]

A. Both A and R true and R explains A

B. Both A and R true but R does not explain A

C. A true, R false

D. A false, R true

Answer

A. Both A and R true and R explains A

Step by step

1. **Check A and R separately, then decide whether R explains A:** The roots of $ax^2 + bx + c = 0$ are $x = (-b \pm \sqrt{D})/2a$, where $D = b^2 - 4ac$. If $D < 0$, then $\sqrt{D}$ is the square root of a negative number and is not real, so R is true. Then x is not real, so the equation has no real roots, and A is true. For example, $x^2 + 1 = 0$ has $D = 0 - 4 = -4 < 0$, and $x^2 = -1$ has no real solution. R is the direct cause of A, so option A is correct. Why: The discriminant is the expression under the root sign in the quadratic formula, so its sign decides whether the roots are real. R states the mechanism that produces the conclusion in A, which makes it the explanation and not just another true statement. That rules out option B, which applies when both are true but unrelated.

Topic and concept

Nature of roots from discriminant (b^2-4ac): two distinct real, two equal, no real roots. The discriminant $D = b^2 - 4ac$ indicates root type: $D > 0$ gives two distinct real roots, $D = 0$ gives two equal real roots, and $D < 0$ gives no real roots (NCERT Class 10 maths, Ch. 4, §4.4). The quadratic formula $x = (-b \pm \sqrt{D})/2a$ shows why: $D < 0$ makes $\sqrt{D}$ non-real.

How marks are earned

1. Both Assertion and Reason are true, and Reason explains why a negative discriminant yields no real roots. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 4, §4.4 Nature of Roots: When $b^2 - 4ac < 0$, the square root term is not real, so no real roots exist.

Section B

Question 21

2 marks

Short Answer

Attempt one option.

Option A

Answer the question with working and reasons where needed.

(a) List the sample space when a fair coin is tossed two times. [2 marks]

Answer

When a fair coin is tossed twice, the total number of possible outcomes is 2^2 = 4, representing all combinations of heads and tails. Therefore, the sample space for two tosses of a fair coin can be listed explicitly as S = {HH, HT, TH, TT}, showing all possible outcomes.

Step by step

1. **Count the outcomes by combining the two tosses:** The first toss has 2 outcomes (H or T). For each of these the second toss again has 2 outcomes (H or T). By the multiplication principle the total number of outcomes is 2 × 2 = 2² = 4, and a tree diagram with two branches at each stage shows the same count. Why: The tosses are independent, so every result of the first toss can pair with every result of the second. The total is therefore the product of the individual counts, which gives the size of the sample space before any outcomes are written out.
2. **List the ordered outcomes along the tree branches:** Reading the four tree paths gives HH, HT, TH and TT, so S = {HH, HT, TH, TT}. This matches the count of 4 from the previous step. HT (head first, tail second) and TH (tail first, head second) are listed separately. Why: The tosses happen in order, so HT and TH are different outcomes and each is a separate element of the sample space. Writing only 3 outcomes by merging them would be wrong. A fair coin makes the four outcomes equally likely, which is the condition for classical probability.

Topic and concept

Classical definition of probability — P(E) = number of favourable outcomes / total outcomes. The sample space is the set of all possible outcomes of an experiment. For a fair coin tossed twice, each toss has 2 outcomes, giving 2² = 4 ordered outcomes. Because the coin is fair, these outcomes are equally likely, which is what classical probability requires.

How marks are earned

1. When a fair coin is tossed twice, the total number of possible outcomes is 2^2 = 4, representing all combinations of heads and tails. (1 mark)

2. Therefore, the sample space for two tosses of a fair coin can be listed explicitly as S = {HH, HT, TH, TT}, showing all possible outcomes. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 14, §14.1: The sample space of a random experiment is the set of all possible outcomes when outcomes are equally likely.

OR

Option B

Answer the question with working and reasons where needed.

(a) A fair six-sided die is rolled once. Find the probability of obtaining a prime number. [2 marks]

Answer

The favourable outcomes are {2, 3, 5}, so there are 3 prime faces out of 6. Thus P(prime) = 3/6 = 1/2 using P(E) = favourable outcomes/total outcomes.

Step by step

1. **List the equally likely outcomes and pick out the primes:** A fair die has 6 equally likely faces: 1, 2, 3, 4, 5, 6, so the total number of outcomes is 6. A prime has exactly two factors, 1 and itself. 2, 3 and 5 are prime. 1 is not prime because it has only one factor, and 4 and 6 are composite. So the favourable outcomes are {2, 3, 5}, which is 3 faces out of 6. Why: Classical probability needs the sample space and the favourable set counted correctly, and it applies only when every outcome is equally likely, as on a fair die. The definition of a prime decides which faces count, so excluding 1 matters.
2. **Apply P(E) = favourable outcomes / total outcomes and simplify:** P(prime) = 3/6. Dividing the numerator and denominator by 3 gives 3/6 = 1/2. The answer lies between 0 and 1, as a probability must. The complement is {1, 4, 6}, which also has 3 outcomes, so the two probabilities 1/2 + 1/2 sum to 1. Why: Theoretical probability is the ratio of favourable to total equally likely outcomes. Reducing the fraction gives the simplest form of the answer. The complement check confirms that the count of 3 is right.

Topic and concept

Probability of sure and impossible events; 0 ≤ P(E) ≤ 1. Classical probability applies when all outcomes are equally likely, as on a fair die. P(E) = number of favourable outcomes / total number of outcomes, and it always lies between 0 and 1 (NCERT Class 10 maths, Ch. 14, §14.1 and §14.2).

How marks are earned

1. The favourable outcomes are {2, 3, 5}, so there are 3 prime faces out of 6. (1 mark)
2. Thus P(prime) = 3/6 = 1/2 using P(E) = favourable outcomes/total outcomes. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 14, §14.2: The theoretical probability is the ratio of the number of favourable outcomes to the total number of equally likely outcomes.

Question 22

2 marks

Short Answer

Attempt one option.

Option A

Answer the question with working and reasons where needed.

(a) In the proof that √2 is irrational, after assuming √2 = p/q in lowest terms, show how a contradiction about p and q arises. [2 marks]

Answer

Squaring gives $p^2 = 2q^2$, so p^2 is even and hence p must be even by unique prime factorisation. Substituting p = 2k yields $2q^2 = 4k^2$ so q^2 is even and q is even, contradicting that p and q have no common factor.

Step by step

1. **Square the assumption and apply the prime-divisor theorem:** Assume √2 = p/q with p and q coprime integers, q ≠ 0. Squaring gives $2 = p^2/q^2$, so $p^2 = 2q^2$. Thus 2 divides p^2. By the theorem 'if a prime divides a^2, then it divides a', 2 divides p, so p is even. Why: The theorem follows from unique prime factorisation. If 2 does not appear in the prime factorisation of p, it cannot appear in that of p^2, so $2 \mid p^2$ forces $2 \mid p$. This step is needed because $p^2 = 2q^2$ only shows that p^2 is even, and the contradiction needs the same conclusion about p itself.
2. **Substitute p = 2k and reach the contradiction:** Since p is even, write p = 2k for some integer k. Then $p^2 = 4k^2$, so $2q^2 = 4k^2$, which simplifies to $q^2 = 2k^2$. So 2 divides q^2, and by the same theorem 2 divides q, so q is even. Then both p and q have 2 as a common factor, which contradicts the assumption that p and q are coprime. So √2 cannot be rational, and √2 is irrational. Why: The proof is by contradiction: an assumption that leads to an impossible conclusion must be false. We began with p/q in lowest terms, which means no common factor other than 1, but we showed that 2 divides both. The only assumption made was that √2 is rational, so this assumption is wrong.

Topic and concept

Proofs of irrationality of √2, √3, √5 and similar surds. To prove √2 irrational, assume √2 = p/q in lowest terms. Then $p^2 = 2q^2$ makes p even, and substituting p = 2k makes q even too. A common factor 2 contradicts co-primality, so the assumption fails and √2 is irrational.

How marks are earned

1. Squaring gives $p^2 = 2q^2$, so p^2 is even and hence p must be even by unique prime factorisation. (1 mark)
2. Substituting p = 2k yields $2q^2 = 4k^2$ so q^2 is even and q is even, contradicting that p and q have no common factor. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 1, §1.3: Assuming √2 rational leads to both numerator and denominator being even, contradicting co-primality.

OR

Option B

Answer the question with working and reasons where needed.

(a) Use prime factorisation to find the least common multiple (LCM) of 84 and 126. [2 marks]

Answer

By applying successive divisions, we obtain the prime factorization of 84 as 84 = 2^2 × 3^1 × 7^1, clearly listing each prime factor with its exponent. By comparing the highest exponent of each prime in 84 = 2^2 × 3^1 × 7^1 and 126 = 2^1 × 3^2 × 7^1, we compute the LCM as 2^2 × 3^2 × 7^1 = 252.

Step by step

1. **Find the prime factorisations of both 84 and 126 by successive division:** 84 ÷ 2 = 42, 42 ÷ 2 = 21, 21 ÷ 3 = 7, and 7 is prime, so 84 = $2^2 \times 3 \times 7$. For 126: 126 ÷ 2 = 63, 63 ÷ 3 = 21, 21 ÷ 3 = 7, and 7 is prime, so 126 = $2 \times 3^2 \times 7$. Multiplying back checks the results: 4 × 3 × 7 = 84 and 2 × 9 × 7 = 126. Why: Every composite number can be written uniquely as a product of primes, so the exponents of each prime can be compared reliably. Both numbers must be factorised so that each prime's power in each number is known before they are compared.
2. **Take the highest power of each prime and multiply:** The prime 2 has powers 2^2 and 2^1, so the highest is 2^2. The prime 3 has powers 3^1 and 3^2, so the highest is 3^2. The prime 7 has power 7^1 in both numbers. LCM = $2^2 \times 3^2 \times 7$ = 4 × 9 × 7 = 252. Check: HCF uses the lowest powers, 2 × 3 × 7 = 42, and HCF × LCM = 42 × 252 = 10584 = 84 × 126. Why: The LCM must be divisible by both numbers, so it needs each prime to at least the highest power found in either number. A smaller power would fail to be divisible by one of them. HCF × LCM = product of the two numbers holds for two numbers and confirms the result.

Topic and concept

Fundamental Theorem of Arithmetic — statement, illustration and applications through prime factorisation. To find the HCF and LCM of two numbers, compare their prime-factor powers. The HCF takes the minimum exponents and the LCM takes the maximum exponents (NCERT Class 10 maths, Ch. 1, §1.2). For 84 and 126 this gives LCM = 252.

How marks are earned

1. By applying successive divisions, we obtain the prime factorization of 84 as 84 = 2^2 × 3^1 × 7^1, clearly listing each prime factor with its exponent. (1 mark)
2. By comparing the highest exponent of each prime in 84 = 2^2 × 3^1 × 7^1 and 126 = 2^1 × 3^2 × 7^1, we compute the LCM as 2^2 × 3^2 × 7^1 = 252. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 1, §1.2: A composite number can be expressed uniquely as a product of primes, which we use for LCM.

Question 23

2 marks

Short Answer

Answer the question with working and reasons where needed.

(a) A person standing atop a 50 m tall building observes a car on the ground at an angle of depression of 30°. Identify the type of angle formed, and state its relation with the angle of elevation from the car to the top of the building. [2 marks]

Answer

The angle formed by the person's line of sight looking downwards from the building to the car is called the angle of depression. This angle of depression is equal in measure to the angle of elevation of the building's top as observed from the car on the ground.

Step by step

1. **Name the angle formed when the observer looks down:** The person on the 50 m building looks down at the car. The angle between the horizontal line through the observer's eye and the line of sight to the car is the angle of depression, which is 30° here. Why: The angle of depression is measured downwards from the horizontal when the observer looks at an object below eye level. It is not measured from the vertical or from the ground, which is why the angle in this question is called the angle of depression.
2. **Relate it to the angle of elevation using alternate angles:** Draw a diagram with the building, the car on the ground and the line of sight. The horizontal through the observer is parallel to the ground. The line of sight is a transversal of these parallel lines, so the angle of depression (30°) and the angle of elevation from the car to the top of the building are alternate interior angles, and so they are equal. The angle of elevation is also 30°. As an extension, tan 30° = 50/d gives d = 50√3 m. Why: Alternate interior angles formed by a transversal across two parallel lines are equal. The horizontal at the observer and the ground are parallel, so the equality is a result of geometry and not a coincidence. It lets us use the depression angle as the angle of the right triangle at the car.

Topic and concept

Angle of elevation and angle of depression; applications to heights and distances. The angle of depression is measured downwards from the horizontal, and the angle of elevation is measured upwards from the horizontal. The observer's horizontal is parallel to the ground, so the two angles are alternate angles and are equal. This equality lets us use tan θ = height / horizontal distance in the right triangle formed.

How marks are earned

1. The angle formed by the person's line of sight looking downwards from the building to the car is called the angle of depression. (1 mark)
2. This angle of depression is equal in measure to the angle of elevation of the building's top as observed from the car on the ground. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 9, §9.1: Angle of depression is defined as the angle between the horizontal and the line of sight when looking down.

Question 24

2 marks

Short Answer

Answer the question with working and reasons where needed.

(a) A bag contains 4 red balls and 2 green balls. One ball is drawn at random. Find the probability that it is green. [2 marks]

Answer

A bag contains a total of 4 red and 2 green balls, giving a sample space size of 4 + 2 = 6 distinct equally likely outcomes when one ball is drawn. Therefore, the probability of drawing a green ball is the ratio of green balls to total balls, P(green) = 2/6, which simplifies to the fraction 1/3.

Step by step

1. **Count the total equally likely outcomes:** The bag has 4 red and 2 green balls, so the total number of balls is 4 + 2 = 6. Drawing one ball gives 6 possible outcomes, and these are equally likely because the balls are identical in size and the draw is at random. Why: Classical probability applies only if every outcome has the same chance. A random draw of identical balls ensures this, so each ball has chance 1/6 and the sample space size is 6. Without this assumption, counting balls would not give a valid probability.
2. **Apply P(E) = favourable / total and check:** Favourable outcomes for green = 2, so P(green) = 2/6 = 1/3. Check: P(red) = 4/6 = 2/3, and 1/3 + 2/3 = 1, as it must be since the ball is either red or green. Why: The probability of an event is the number of favourable outcomes divided by the total number of equally likely outcomes. The sum of the probabilities of all outcomes of the experiment is 1, so the check confirms that 1/3 is consistent.

Topic and concept

Probability of sure and impossible events; 0 ≤ P(E) ≤ 1. For an experiment with equally likely outcomes, P(E) = number of favourable outcomes / total number of outcomes. Here, green balls are the favourable outcomes out of 6 balls, so P(green) = 2/6 = 1/3. Every probability lies between 0 and 1, and the probabilities of complementary events add to 1.

How marks are earned

1. A bag contains a total of 4 red and 2 green balls, giving a sample space size of 4 + 2 = 6 distinct equally likely outcomes when one ball is drawn. (1 mark)
2. Therefore, the probability of drawing a green ball is the ratio of green balls to total balls, P(green) = 2/6, which simplifies to the fraction 1/3. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 14, §14.2: Probability of an event is the ratio of favourable outcomes to the total possible outcomes, assuming all are equally likely.

Question 25

2 marks

Short Answer

Answer the question with working and reasons where needed.

(a) Akhila spent a total of ₹20 on Giant-Wheel rides at ₹3 each and Hoopla games at ₹4 each, playing Hoopla half as many times as she rode. Define suitable variables and write the two equations to model this situation. [2 marks]

Answer

Let x represent the number of Giant-Wheel rides and y represent the number of Hoopla games, and given that Hoopla games are half the number of rides, we have y = x/2. Using the costs ₹3 per ride and ₹4 per game, the total expenditure equation becomes 3x + 4y = 20, which models the given spending situation exactly.

Step by step

1. **Define variables with meaning and translate the 'half as many' condition:** Let x = number of Giant-Wheel rides and y = number of Hoopla games. Since Akhila played Hoopla half as many times as she rode, y = x/2, which can also be written as x − 2y = 0. Why: A word problem becomes algebra only when each unknown is named with its meaning. The phrase 'half as many' is a ratio relation, so the quantity being compared (games) equals one half of the other (rides).
2. **Translate the total-spending statement into a cost equation:** Cost of rides = ₹3 × x = 3x and cost of games = ₹4 × y = 4y. The total spent is ₹20, so 3x + 4y = 20. Check: the two equations together make a pair of linear equations in x and y, with the amounts in rupees. Why: Total cost is the sum of (price per item × number of items) for each activity. Writing it this way gives a linear equation in two variables, and together with y = x/2 it forms the pair of linear equations that NCERT Ch. 3 solves.

Topic and concept

Formulation and solution of word problems using pair of linear equations. To model a situation with a pair of linear equations, name each unknown quantity and turn each separate statement of the problem into one equation. Here one statement gives a ratio relation (y = x/2) and the other gives a total-cost relation (3x + 4y = 20).

How marks are earned

1. Let x represent the number of Giant-Wheel rides and y represent the number of Hoopla games, and given that Hoopla games are half the number of rides, we have y = x/2. (1 mark)
2. Using the costs ₹3 per ride and ₹4 per game, the total expenditure equation becomes 3x + 4y = 20, which models the given spending situation exactly. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 3, §3.1: Translate a real-world scenario into two linear equations by assigning variables.

Section C

Question 26

3 marks

Short Answer

Answer the question with working and reasons where needed.

(a) Determine the area of the minor segment of a circle of radius 10 cm when the central angle is 90°. Use π = 3.14. Show all steps. [3 marks]

Answer

Compute the area of the 90° sector: A_sector = (90/360) × π × 10² = 78.5 cm². Calculate the area of the right triangle formed by the radii and chord: A_triangle = (1/2) × 10 × 10 = 50 cm². Subtract to find the segment area: A_segment = 78.5 − 50 = 28.5 cm².

Step by step

1. **Find the area of the 90° sector:** Sector area = (θ/360) × πr² = (90/360) × 3.14 × 10² = 0.25 × 3.14 × 100 = 78.5 cm². Why: A sector with central angle θ is the fraction θ/360 of the whole circle, so its area is that fraction of πr². 90/360 = 1/4 gives one quarter of the circle.
2. **Find the area of the triangle formed by the two radii and the chord:** The central angle is 90°, so the two radii are perpendicular. Each radius is 10 cm, so the base and height of the right triangle are both 10 cm. Area = ½ × 10 × 10 = 50 cm². Why: The triangle's two equal sides are radii, and the angle between them is the central angle of 90°. A right angle lets one radius serve as the base and the other as the height, so ½ × base × height applies directly.
3. **Subtract the triangle from the sector to get the segment:** Segment area = sector area − triangle area = 78.5 − 50 = 28.5 cm². The area of the minor segment is 28.5 cm². Why: The sector is the minor segment plus the triangle, so removing the triangle leaves only the region between the chord and the arc. Check: 28.5 is positive and less than the sector area 78.5, as it should be.

Topic and concept

Area of segment of a circle. The minor segment is the part of the sector between the chord and the arc. Because the sector is made of the segment and the triangle formed by the two radii and the chord, segment area = sector area − triangle area.

How marks are earned

1. Compute the area of the 90° sector: A_sector = (90/360) × π × 10² = 78.5 cm². (1 mark)
2. Calculate the area of the right triangle formed by the radii and chord: A_triangle = (1/2) × 10 × 10 = 50 cm². (1 mark)
3. Subtract to find the segment area: A_segment = 78.5 − 50 = 28.5 cm². (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 11, §11.2 Summary: The summary states that segment area = sector area − triangle area.

Question 27

3 marks

Short Answer

Answer the question with working and reasons where needed.

(a) Verify that for the numbers 36 and 50, the product of their HCF and LCM equals the product of the numbers. [3 marks]

Answer

We perform prime factorisation to write 36 = 2^2 × 3^2 and 50 = 2^1 × 5^2, clearly identifying each prime factor and its exponent. Taking minimum exponents of common primes gives HCF = 2^1 = 2, and taking maximum exponents gives LCM = 2^2 × 3^2 × 5^2 = 900. Multiplying the HCF and LCM yields 2 × 900 = 1800, which exactly equals the product 36 × 50 = 1800, thus verifying the required property.

Step by step

1. **Prime-factorise both numbers:** 36 = 2 × 2 × 3 × 3 = $2^2 \times 3^2$ and 50 = 2 × 5 × 5 = $2^1 \times 5^2$. Exponent table for primes 2, 3, 5: for 36 they are (2, 2, 0), and for 50 they are (1, 0, 2). Why: The Fundamental Theorem of Arithmetic says every composite number has a unique prime factorisation. Writing both numbers over the same primes lets us compare exponents prime by prime.
2. **Take minimum exponents for HCF and maximum exponents for LCM:** HCF takes the minimum exponent of each prime: $2^1 \times 3^0 \times 5^0 = 2$. LCM takes the maximum exponent: $2^2 \times 3^2 \times 5^2 = 4 \times 9 \times 25 = 900$. Why: The HCF must divide both numbers, so it can use only as much of each prime as both contain (the minimum). The LCM must be a multiple of both, so it needs as much of each prime as either contains (the maximum).
3. **Multiply HCF and LCM and compare with the product of the numbers:** HCF × LCM = 2 × 900 = 1800 and 36 × 50 = 1800. They are equal, so the property is verified. Why: For each prime, min exponent + max exponent = sum of the two exponents. Therefore the primes in HCF × LCM match exactly the primes in the product of the two numbers, which is why the products agree.

Topic and concept

Fundamental Theorem of Arithmetic — statement, illustration and applications through prime factorisation. For two numbers written as products of primes, the HCF uses the smaller power of each prime and the LCM uses the larger power. Since the smaller and larger powers together add up to the two original powers, HCF × LCM = product of the two numbers.

How marks are earned

1. We perform prime factorisation to write 36 = 2^2 × 3^2 and 50 = 2^1 × 5^2, clearly identifying each prime factor and its exponent. (1 mark)

2. Taking minimum exponents of common primes gives HCF = 2^1 = 2, and taking maximum exponents gives LCM = 2^2 × 3^2 × 5^2 = 900. (1 mark)
3. Multiplying the HCF and LCM yields 2 × 900 = 1800, which exactly equals the product 36 × 50 = 1800, thus verifying the required property. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 1, §1.2: HCF×LCM of two numbers equals their product when found by prime factorisation.

Question 28

3 marks

Short Answer

Attempt one option.

Option A

Answer the question with working and reasons where needed.

(a) Using the fact that the full circle area is πr², derive the formula for the area of a sector with central angle θ. Present your reasoning step by step. [3 marks]

Answer

The full circle has a total central angle of 360° and its area is given by the standard formula A = πr² for a circle of radius r, establishing the basis for proportional reasoning. By applying the unitary method, the area corresponding to one degree of central angle is found by dividing the full circle area πr² by 360, giving πr²/360, and hence for θ degrees the area becomes θ×πr²/360. Combining these results yields the general formula for a sector of angle θ: A = (θ/360)×πr², which expresses the sector's area as the appropriate fraction of the full circle area.

Step by step

1. **State the whole-circle starting point and the proportionality:** The full circle has central angle 360° and area πr². A sector's area is directly proportional to its central angle, so a sector of 360° has area πr². Why: Equal central angles cut off equal portions of the same circle, so area grows in the same proportion as the angle. This direct proportion is what allows the unitary method to be used.
2. **Apply the unitary method to find the area for 1°:** 360° → πr². Dividing both by 360: 1° → πr²/360. Why: The unitary method finds the value for one unit first. Dividing the total area by the total number of degrees gives the area contributed by each degree of central angle.
3. **Multiply up to θ degrees and check with a full angle:** θ° → θ × πr²/360, so the area of the sector is A = (θ/360) × πr². Check: for θ = 360° the formula gives (360/360) × πr² = πr², the full circle. Why: Multiplying the one-degree area by θ gives the area for θ degrees. The check shows the formula is consistent with the starting point, since a sector of 360° is the whole circle.

Topic and concept

Area of a sector and length of an arc. A sector with central angle θ occupies the fraction θ/360 of the circle, so its area is that fraction of the full area πr². The formula follows from the unitary method because area is directly proportional to central angle.

How marks are earned

1. The full circle has a total central angle of 360° and its area is given by the standard formula A = πr² for a circle of radius r, establishing the basis for proportional reasoning. (1 mark)
2. By applying the unitary method, the area corresponding to one degree of central angle is found by dividing the full circle area πr² by 360, giving πr²/360, and hence for θ degrees the area becomes θ×πr²/360. (1 mark)
3. Combining these results yields the general formula for a sector of angle θ: A = (θ/360)×πr², which expresses the sector's area as the appropriate fraction of the full circle area. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 11, §11.1: The excerpt applies the unitary method to derive the sector area from the full circle area.

OR

Option B

Answer the question with working and reasons where needed.

(a) If the area of a sector of a circle is 154 cm² and its central angle is 70°, determine the radius of the circle. Use π = 22/7. [3 marks]

Answer

Using A=(θ/360)πr² with A=154 cm², θ=70° and π=22/7 gives 154=(11/18)r². Hence r²=154×18/11=252 cm² and the positive radius is r=√252=6√7 cm≈15.87 cm.

Step by step

1. **Substitute the given values into the sector-area formula:** The formula is A = (θ/360) × πr². Substitute A = 154 cm², θ = 70° and π = 22/7 to get 154 = (70/360) × (22/7) × r². Now 70/360 = 7/36, and (7/36) × (22/7) = 22/36 = 11/18. So 154 = (11/18) r². Why: A sector of angle θ occupies θ/360 of the full circle, so its area is that fraction of πr². Reducing the constant factor to 11/18 before solving avoids arithmetic slips.
2. **Rearrange the equation to find r²:** Multiply 154 = (11/18)r² by 18/11 on both sides. Then r² = 154 × 18/11 = 14 × 18 = 252 cm². Why: Multiplying by the reciprocal 18/11 isolates r². Cancelling 154 ÷ 11 first makes the arithmetic simple and preserves the square-centimetre unit.
3. **Take the positive square root and verify in the formula:** r = √252 = √(36 × 7) = 6√7 cm. Taking √7 ≈ 2.6458 gives r ≈ 15.87 cm. Check: r² = 252, so (11/18) × 252 = 11 × 14 = 154 cm², which equals the given area. Why: A radius is a length, so only the positive root is taken. Splitting 252 = 36 × 7 extracts the perfect square 36. Substituting back into A = (θ/360)πr² confirms the answer.

Topic and concept

Area of a sector and length of an arc. The area of a sector with central angle θ in a circle of radius r is (θ/360°) × πr² (NCERT Class 10 maths, Ch. 11, §11.1 and §11.2). To find r, substitute the data, solve for r², and then take the positive square root.

How marks are earned

1. Substitute A=154 cm², θ=70° and π=22/7 into A=(θ/360)πr², obtaining 154=(11/18)r². (1 mark)
2. Rearrange the equation correctly to get r²=154×18/11=252 cm². (1 mark)
3. Take the positive square root and report r=6√7 cm≈15.87 cm with a check in the sector-area formula. (1 mark)

Source check: Substitution check: (70/360)(22/7)(6√7)²=(11/18)×252=154 cm², matching the given area.

Question 29

3 marks

Short Answer

Answer the question with working and reasons where needed.

(a) In a circle of radius 10 cm, calculate (a) the length of the arc and (b) the area of the sector subtended by a central angle of 144°. Use π = 3.14. Show your steps. [3 marks]

Answer

Arc length: l = (144/360) × 2π × 10 = 0.4 × 2 × 3.14 × 10 = 25.12 cm. Sector area: A = (144/360) × π × 10² = 0.4 × 3.14 × 100 = 125.6 cm². Thus, the arc length is 25.12 cm and the sector area is 125.6 cm².

Step by step

1. **Write the formula, simplify the angle fraction and find the arc length:** Arc length l = (θ/360) × 2πr. With θ = 144° and r = 10 cm, 144/360 = 2/5 = 0.4. So l = 0.4 × 2 × 3.14 × 10 = 0.4 × 62.8 = 25.12 cm. Why: An arc is the fraction θ/360 of the whole circumference 2πr, because the central angle fixes what part of the circle the arc covers. Simplifying 144/360 to 2/5 makes the arithmetic easier.
2. **Write the formula and find the sector area:** Sector area A = (θ/360) × πr² = 0.4 × 3.14 × 10² = 0.4 × 314 = 125.6 cm². Why: The same fraction θ/360 applies to the area, since the sector is that part of the whole circle's area πr². Using r² gives a result in cm², not cm.
3. **Verify the two results with a relation and state them with units:** Check: A = ½ × l × r = ½ × 25.12 × 10 = 125.6 cm², which matches. So the arc length is 25.12 cm and the sector area is 125.6 cm². Why: A sector can be viewed as a thin triangle with base equal to the arc length and height equal to the radius, so ½ × l × r is an independent check. Agreement of the two answers confirms the arithmetic.

Topic and concept

Area of a sector and length of an arc. For a central angle θ, both the arc length and the sector area are the same fraction θ/360 of the circumference 2πr and the area πr² respectively. So one fraction (here 2/5) is used in both formulas.

How marks are earned

1. Arc length: l = (144/360) × 2π × 10 = 0.4 × 2 × 3.14 × 10 = 25.12 cm. (1 mark)
2. Sector area: A = (144/360) × π × 10² = 0.4 × 3.14 × 100 = 125.6 cm². (1 mark)
3. Thus, the arc length is 25.12 cm and the sector area is 125.6 cm². (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 11, §11.2 Summary: The summary lists formulas for both arc length and sector area.

Question 30

3 marks

Short Answer

Answer the question with working and reasons where needed.

(a) Given points P(2,−2) and Q(6,2), find (a) the midpoint M of PQ and (b) the distance from the origin to M. [3 marks]

Answer

Using the midpoint formula for P(2,−2) and Q(6,2), we calculate M = ((2 + 6)/2, (−2 + 2)/2) = (4, 0) as the midpoint of PQ on the Cartesian plane. To find the distance from the origin O(0,0) to M(4,0), we apply the distance formula OM = √[(4 − 0)² + (0 − 0)²] = √16, which yields the square root of the sum of squared coordinate differences. Therefore, the computed distance from the origin to the midpoint M is OM = 4 units, as determined by the evaluation of √16.

Step by step

1. **Apply the midpoint formula:** M = $((x_1 + x_2)/2, (y_1 + y_2)/2)$ = ((2 + 6)/2, (−2 + 2)/2) = (8/2, 0/2) = (4, 0). Why: The midpoint divides PQ in the ratio 1:1, which is the section formula with m = n = 1. That makes each coordinate of M the average of the corresponding coordinates of P and Q.
2. **Apply the distance formula from the origin to M:** With O(0, 0) and M(4, 0), OM = √[(4 − 0)² + (0 − 0)²] = √(16 + 0) = √16. Why: The distance between two points is the length of the hypotenuse of a right triangle with legs Δx and Δy, by the Pythagoras theorem. So the distance is √(Δx² + Δy²).
3. **Evaluate and interpret the result:** √16 = 4, so OM = 4 units. Check: M lies on the x-axis, so its distance from the origin is just its x-coordinate, 4. Why: Since the y-coordinates of O and M are both 0, Δy = 0 and the distance reduces to the horizontal gap. This agrees with the formula result, which confirms the answer.

Topic and concept

Midpoint as a special case of internal division. The midpoint is the average of the coordinates, $((x_1+x_2)/2, (y_1+y_2)/2)$. The distance between two points then comes from the distance formula $\sqrt{((x_2-x_1)^2 + (y_2-y_1)^2)}$. Here the midpoint is found first and then used in the distance formula.

How marks are earned

1. Using the midpoint formula for P(2,−2) and Q(6,2), we calculate M = ((2 + 6)/2, (−2 + 2)/2) = (4, 0) as the midpoint of PQ on the Cartesian plane. (1 mark)
2. To find the distance from the origin O(0,0) to M(4,0), we apply the distance formula OM = $\sqrt{[(4 - 0)^2 + (0 - 0)^2]}$ = $\sqrt{16}$, which yields the square root of the sum of squared coordinate differences. (1 mark)
3. Therefore, the computed distance from the origin to the midpoint M is OM = 4 units, as determined by the evaluation of $\sqrt{16}$. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 7, §7.4 Summary: Midpoint formula gives average of coordinates.

Question 31

3 marks

Short Answer

Attempt one option.

Option A

Answer the question with working and reasons where needed.

(a) Given that sin A = 3/5 for an acute angle A, find cos A, tan A and sec A, showing all steps. [3 marks]

Answer

Using the Pythagorean identity $\sin^2 A + \cos^2 A = 1$ for an acute angle A, we have $\cos A = \sqrt{(1 - \sin^2 A)} = \sqrt{(1 - (3/5)^2)} = \sqrt{(1 - 9/25)} = \sqrt{(16/25)} = 4/5$, giving the required cosine value. The tangent of angle A is defined as the ratio of its sine to its cosine, therefore $\tan A = \sin A/\cos A = (3/5)/(4/5) = 3/4$, which provides the exact value of tan A. The secant of angle A is the reciprocal of the cosine, so $\sec A = 1/\cos A = 1/(4/5) = 5/4$, yielding the exact value of sec A for the given angle.

Step by step

1. **Use $\sin^2A + \cos^2A = 1$ to find cos A:** $\cos^2A = 1 - \sin^2A = 1 - (3/5)^2 = 1 - 9/25 = 16/25$. A is acute, so cos A is positive and $\cos A = \sqrt{(16/25)} = 4/5$. Alternatively, sin A = 3/5 gives a right triangle with opposite side 3 and hypotenuse 5, so the adjacent side is $\sqrt{(25 - 9)} = 4$ and cos A = 4/5. Why: The identity $\sin^2A + \cos^2A = 1$ links the two ratios, so one gives the other. The positive root is chosen because cos A > 0 for an acute angle. The 3-4-5 triangle gives the same result by the Pythagoras theorem.

2. **Use tan A = sin A / cos A to find tan A:** tan A = (3/5) ÷ (4/5) = (3/5) × (5/4) = 3/4. In the 3-4-5 triangle, tan A = opposite/adjacent = 3/4, which agrees. Why: tan A is opposite over adjacent. Dividing sin A by cos A cancels the common hypotenuse, so tan A = sin A / cos A. Having cos A from step 1, we only need to divide the two values.
3. **Use sec A = 1/cos A to find sec A:** sec A = 1 ÷ (4/5) = 5/4. In the triangle, sec A = hypotenuse/adjacent = 5/4. Check: 1 + tan²A = 1 + 9/16 = 25/16 = (5/4)² = sec²A. Why: sec A is defined as the reciprocal of cos A, so it follows from the value of cos A without a new identity. The check with 1 + tan²A = sec²A confirms that all three values fit together.

Topic and concept

Determining other trigonometric ratios when one is known. If one trigonometric ratio of an acute angle is known, the others follow from the definitions and Pythagorean identities (NCERT Class 10 maths, Ch. 8, §8.4 and §8.5). Here sin A = 3/5 gives cos A = 4/5, tan A = 3/4 and sec A = 5/4.

How marks are earned

1. Using the Pythagorean identity sin² A + cos² A = 1 for an acute angle A, we have cos A = √(1 − sin² A) = √(1 − (3/5)²) = √(1 − 9/25) = √(16/25) = 4/5, giving the required cosine value. (1 mark)
2. The tangent of angle A is defined as the ratio of its sine to its cosine, therefore tan A = sin A/cos A = (3/5)/(4/5) = 3/4, which provides the exact value of tan A. (1 mark)
3. The secant of angle A is the reciprocal of the cosine, so sec A = 1/cos A = 1/(4/5) = 5/4, yielding the exact value of sec A for the given angle. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 8, §8.4: The identity sin² A + cos² A = 1 allows finding cos A given sin A.

OR

Option B

Answer the question with working and reasons where needed.

(a) In right triangle PQR, ∠Q = 90°, PQ = 12 cm and PR = 13 cm. Calculate tan P showing all steps. [3 marks]

Answer

In right triangle PQR with ∠Q equal to 90°, applying the Pythagorean theorem gives QR = √(PR² − PQ²) = √(13² − 12²) = √(169 − 144) = 5 cm, thus determining the length of the remaining side. The tangent of angle P is defined as the ratio of the length of the side opposite P to the length of the side adjacent to P, so tan P = QR/PQ = 5/12. Hence, the value of tan P is 5/12, which is a pure number without any units, representing the trigonometric ratio for angle P.

Step by step

1. **Find the missing side QR using the Pythagorean theorem:** Since ∠Q = 90°, PR = 13 cm is the hypotenuse, so QR = √(PR² − PQ²) = √(13² − 12²) = √(169 − 144) = √25 = 5 cm; check: 5² + 12² = 25 + 144 = 169 = 13². Why: In a right triangle the square of the hypotenuse equals the sum of the squares of the other two sides, so the unknown leg can be found from the two known lengths before any ratio is formed.
2. **Identify the sides opposite and adjacent to P and write tan P:** For angle P, the side opposite is QR = 5 cm (across from P) and the side adjacent is PQ = 12 cm (it touches P and is not the hypotenuse), so tan P = QR/PQ = 5/12. Why: tan A is defined as the ratio of the side opposite angle A to the side adjacent to angle A, so the sides must be assigned relative to P and not relative to R.
3. **State the final value and its nature:** Hence tan P = 5/12 (about 0.417); the two lengths are both in cm, so the units cancel and the answer is a pure number. Why: A trigonometric ratio compares two lengths of the same kind, so it is dimensionless and no unit is attached to it.

Topic and concept

Values at 0°, 30°, 45°, 60°, 90°. In a right triangle, the trigonometric ratios of an acute angle are fixed by the sides relative to that angle. When two sides are known, the third follows from the Pythagorean theorem, and the ratios can then be written. Here QR is found first and then tan P = opposite/adjacent is formed.

How marks are earned

1. In right triangle PQR with ∠Q equal to 90°, applying the Pythagorean theorem gives QR = √(PR² − PQ²) = √(13² − 12²) = √(169 − 144) = 5 cm, thus determining the length of the remaining side. (1 mark)
2. The tangent of angle P is defined as the ratio of the length of the side opposite P to the length of the side adjacent to P, so tan P = QR/PQ = 5/12. (1 mark)
3. Hence, the value of tan P is 5/12, which is a pure number without any units, representing the trigonometric ratio for angle P. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 8, §8.2: tan A is defined as the ratio of the side opposite to the side adjacent to angle A in a right triangle.

Section D

Question 32

5 marks

Long Answer

Answer both connected parts. Show the required method, explanation and units.

(a) For the linear polynomial p(x)=4x−8, determine its zero and the coordinates of the point where the graph of y=p(x) meets the x-axis. [2 marks]

Answer

By applying the definition of a zero of the polynomial, set p(x)=0 which yields the equation 4x−8=0; solving this gives x=2 as the unique zero of p(x). Because the zero of the function corresponds to its x–intercept on the Cartesian plane, the graph of y=4x−8 meets the x–axis at the coordinate point (2,0).

Step by step

1. **Find the zero of p(x)=4x−8 by setting p(x)=0 and solving:** A zero k satisfies p(k)=0, so 4x−8=0. Add 8 to both sides to get 4x=8, then divide by 4 to get x=2. Check: p(2)=4×2−8=8−8=0. Why: A real number k is a zero of p(x) exactly when substituting x=k makes p(k)=0. So the zero comes from solving p(x)=0. A linear polynomial has one zero, which here is x=2. The substitution check confirms it.
2. **Convert the zero into the coordinates of the x-axis intersection point:** Every point on the x-axis has y=0. The graph y=4x−8 meets the x-axis where y=p(x)=0, which happens at x=2. So the point is (2,0). Check: putting x=2 in y=4x−8 gives y=0. Why: The x-intercepts of y=ax+b are exactly the zeros of the polynomial. The x-coordinate of the meeting point is therefore the zero, and its y-coordinate is 0 because it lies on the x-axis. A line with non-zero slope meets the x-axis at only one point, so (2,0) is the unique intersection.

Topic and concept

Zeros of a polynomial; geometric meaning of zeros as x-intercepts. A zero of p(x) is a value k with p(k)=0. Geometrically, the graph of y=p(x) crosses the x-axis at the point (k,0). For the linear polynomial 4x−8, the zero x=2 therefore gives the single x-intercept (2,0).

How marks are earned

1. By applying the definition of a zero of the polynomial, set p(x)=0 which yields the equation 4x−8=0; solving this gives x=2 as the unique zero of p(x). (1 mark)
2. Because the zero of the function corresponds to its x–intercept on the Cartesian plane, the graph of y=4x−8 meets the x–axis at the coordinate point (2,0). (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 2, §2.2 Geometrical Meaning of the Zeroes of a Polynomial: The zero of ax+b is the x-coordinate of the point where the graph of y=ax+b intersects the x-axis.

(b) Prove algebraically that any non-zero linear polynomial p(x)=ax+b, with a≠0, has exactly one zero. [3 marks]

Answer

The zero is x=−b/a. Since a≠0, substitution gives p(−b/a)=a(−b/a)+b=0, so it exists. If u and v are zeros, then au+b=av+b=0, hence a(u−v)=0 and u=v. Therefore the zero is unique.

Step by step

1. **Find the candidate zero:** Set ax+b=0 and divide by the non-zero coefficient a to obtain x=−b/a. Why: Division is valid because the question states a≠0.
2. **Verify existence:** Substitute x=−b/a into p(x): a(−b/a)+b=−b+b=0. Why: This substitution proves that the candidate is an actual zero.
3. **Prove uniqueness:** If u and v are both zeros, then au+b=0 and av+b=0; subtraction gives a(u−v)=0, so u=v because a≠0. Why: Any two supposed zeros must coincide, completing the algebraic proof without requiring a graph.

Topic and concept

Zeros of a polynomial; geometric meaning of zeros as x-intercepts. A zero is a value x for which p(x)=0. Solving ax+b=0 gives x=−b/a; substitution proves it exists and subtraction of two zero equations proves it is unique.

How marks are earned

1. Set ax+b=0 and solve x=−b/a, noting that a≠0 permits the division. (1 mark)
2. Substitute x=−b/a to show p(−b/a)=−b+b=0, establishing existence. (1 mark)
3. Assume u and v are zeros; subtract their zero equations to get a(u−v)=0 and infer u=v because a≠0. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 2, §2.2 Geometrical Meaning of the Zeroes of a Polynomial: The graph of y=ax+b intersects the x-axis at a single point whose x-coordinate is −b/a.

Question 33

5 marks

Long Answer

Answer both connected parts. Show the required method, explanation and units.

(a) In ΔABC, a line DE ∥ BC meets AB at D and AC at E. If AD = DB, use the Basic Proportionality

Theorem to find the ratio AE : EC. [2 marks]

Answer

Applying the Basic Proportionality Theorem in triangle ABC with DE parallel to BC, we deduce the ratio of segments AD to DB is equal to the ratio of segments AE to EC. Since AD equals DB, the ratio AD/DB equals 1, which implies that AE/EC = 1 and therefore AE is congruent in length to EC.

Step by step

1. **Apply the Basic Proportionality Theorem to the parallel line DE in triangle ABC.:** Since DE ∥ BC in ΔABC, the Basic Proportionality Theorem gives AD/DB = AE/EC. Why: The direct Basic Proportionality Theorem says that a line drawn parallel to one side of a triangle divides the other two sides in the same ratio.
2. **Use the given equality AD = DB to determine the required ratio on AC.:** Because AD = DB, AD/DB = 1/1, so AE/EC = 1/1 and therefore AE : EC = 1 : 1. Why: Equal segments on one divided side make that side's ratio 1 : 1, and the proportional division by the parallel line forces the corresponding ratio on the other side to be the same.

Topic and concept

Basic Proportionality Theorem — proof, application and use of its converse. This question uses the direct Basic Proportionality Theorem: if a line is drawn parallel to one side of a triangle, it divides the other two sides in the same ratio. Here, DE ∥ BC, so the divided segments on AB and AC satisfy AD/DB = AE/EC.

How marks are earned

1. Applying the Basic Proportionality Theorem in triangle ABC with DE parallel to BC, we deduce the ratio of segments AD to DB is equal to the ratio of segments AE to EC. (1 mark)
2. Since AD equals DB, the ratio AD/DB equals 1, which implies that AE/EC = 1 and therefore AE is congruent in length to EC. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 6, §6.3: If a line is drawn parallel to one side of a triangle, it divides the other two sides proportionally (AD/DB = AE/EC).

(b) In ΔABC, a line through D on AB parallel to BC meets AC at E. If AD = 3 cm, DB = 5 cm and AE = 4.8 cm, find EC. [3 marks]

Answer

By applying the Basic Proportionality Theorem to triangle ABC with DE parallel to BC, we can establish that the ratio of AD to DB is equal to the ratio of AE to EC. Substituting the given values into the proportional relationship yields 3/5 = 4.8/EC, which sets up the equation for finding EC. Cross-multiplying and solving the equation results in EC = (5 × 4.8)/3, which simplifies to EC = 8 cm.

Step by step

1. **Apply the Basic Proportionality Theorem to relate the divided side lengths.:** Since DE ∥ BC in ΔABC, the Basic Proportionality Theorem gives AD/DB = AE/EC. Why: A line parallel to one side of a triangle divides the other two sides proportionally, so the ratio of the two parts of AB equals the ratio of the two corresponding parts of AC.
2. **Substitute the given lengths into the proportional relationship.:** Using AD = 3 cm, DB = 5 cm, and AE = 4.8 cm in AD/DB = AE/EC gives 3 cm/5 cm = 4.8 cm/EC, or 3/5 = 4.8/EC. Why: Ratios compare like units, so the centimetre units cancel inside the proportion while the unknown EC remains a length to be found in centimetres.
3. **Cross-multiply and solve for EC with units.:** From 3/5 = 4.8/EC, cross-multiplication gives 3 × EC = 5 × 4.8 = 24, so EC = 24/3 = 8 cm; check: 3/5 = 0.6 and 4.8/8 = 0.6. Why: In a true proportion, the cross-products are equal, and the check confirms that the computed length gives the same ratio on AC as the given ratio on AB.

Topic and concept

Basic Proportionality Theorem — proof, application and use of its converse. This question uses the direct Basic Proportionality Theorem because the line through D is given parallel to BC. Therefore the side segments are proportional: AD/DB = AE/EC, allowing EC to be calculated from the known lengths.

How marks are earned

1. By applying the Basic Proportionality Theorem to triangle ABC with DE parallel to BC, we can establish that the ratio of AD to DB is equal to the ratio of AE to EC. (1 mark)
2. Substituting the given values into the proportional relationship yields 3/5 = 4.8/EC, which sets up the equation for finding EC. (1 mark)
3. Cross-multiplying and solving the equation results in EC = (5 × 4.8)/3, which simplifies to EC = 8 cm. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 6, §6.3: A line parallel to one side of a triangle divides the other sides in the same ratio.

Question 34

5 marks

Long Answer

Attempt one option.

Option A

Answer both connected parts. Show the required method, explanation and units.

(a) In a single toss of a fair coin, let E be the event ‘head appears’. Calculate P(E), P(not E), and verify

that P(E)+P(not E)=1. [2 marks]

Answer

For a single toss of a fair coin, the probability of the event E, defined as obtaining a head, is P(E) = 1/2, and consequently P(not E) is calculated as 1 − 1/2 = 1/2. By summing these values, P(E) + P(not E) = 1/2 + 1/2 = 1, which verifies the fundamental complementary rule in probability theory.

Step by step

1. **List the sample space of one toss and compute P(E) and P(not E) by favourable over total:** The sample space is {H,T}, so there are 2 equally likely outcomes. E='head' has 1 favourable outcome, H. Then P(E)=1/2. The outcome not in E is T, so P(not E)=1/2. Check by the complement formula: 1−1/2=1/2. Why: For equally likely outcomes, probability is the number of favourable outcomes divided by the total number of outcomes. A fair coin makes H and T equally likely. The complement 'not E' contains every outcome outside E, so P(not E) can be found by counting or by subtracting from 1. Both give the same result.
2. **Add the two probabilities to verify P(E)+P(not E)=1:** P(E)+P(not E)=1/2+1/2=2/2=1. This agrees with the outcome count, where 1 favourable plus 1 unfavourable makes 2 outcomes out of 2. Why: E and not-E together cover all possible outcomes with no overlap, so their probabilities must add up to the total probability of the sample space, which is 1. The sum of 1 is a consistency check on the calculation.

Topic and concept

Elementary events and sum of probabilities. With equally likely outcomes, P(E) equals favourable outcomes divided by total outcomes. The complement 'not E' covers all other outcomes, so P(E)+P(not E)=1. For one toss of a fair coin, the two outcomes H and T each have probability 1/2.

How marks are earned

1. For a single toss of a fair coin, the probability of the event E, defined as obtaining a head, is P(E) = 1/2, and consequently P(not E) is calculated as 1 − 1/2 = 1/2. (1 mark)
2. By summing these values, P(E) + P(not E) = 1/2 + 1/2 = 1, which verifies the fundamental complementary rule in probability theory. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 14, §14.2: For any event E, P(E) + P(not E) = 1.

(b) In a single throw of a fair die, let F be the event ‘an even number appears’. Find P(F), P(not F) and verify that P(F)+P(not F)=1. [3 marks]

Answer

On throwing a fair six-faced die once, the even-numbered outcomes constitute the set F = {2, 4, 6}, so there are three favourable outcomes out of six possible results. Hence, the probability of the event F is P(F) = number of favourable outcomes / total outcomes = 3/6 = 1/2, and the complement P(not F) equals 1 − 1/2 = 1/2. Adding P(F) and P(not F) gives 1/2 + 1/2 = 1, which confirms the complementary rule by demonstrating that an event and its complement together account for all possible outcomes.

Step by step

1. **List the sample space and the favourable set F for the even-number event:** One throw of a fair die gives the sample space {1,2,3,4,5,6}, which has 6 equally likely outcomes. The even numbers are F={2,4,6}, so there are 3 favourable outcomes. Why: Probability by counting needs the complete list of equally likely outcomes. A fair die makes all six faces equally likely. Listing F explicitly prevents miscounting the favourable outcomes.
2. **Compute P(F) and P(not F), listing the complement set to cross-check:** P(F)=3/6=1/2. The complement 'not F' is the set {1,3,5}, which has 3 outcomes, so P(not F)=3/6=1/2. Check with the subtraction rule: 1−1/2=1/2. Why: Counting the complement directly and using 1−P(F) must agree because not-F contains exactly the outcomes outside F. Getting the same value both ways confirms the count. Simplifying 3/6 to 1/2 keeps the answer in lowest terms.
3. **Verify that P(F)+P(not F)=1 using fractions and using outcome counts:** P(F)+P(not F)=1/2+1/2=1. Directly, 3/6+3/6=6/6=1. The sets {2,4,6} and {1,3,5} together make all 6 outcomes, with no outcome in both. Why: Complementary events cover the sample space with no overlap, so their probabilities sum to 1. The direct count 3+3=6 out of 6 shows why the rule holds for this die, and it does not rely on the rule alone.

Topic and concept

Elementary events and sum of probabilities. For equally likely outcomes, P(F) is favourable outcomes divided by total outcomes. The event and its complement split the sample space into two parts that do not overlap, so their probabilities add to 1. On a die, the even faces {2,4,6} and the odd faces {1,3,5} split the six outcomes in this way.

How marks are earned

1. On throwing a fair six-faced die once, the even-numbered outcomes constitute the set F = {2, 4, 6}, so there are three favourable outcomes out of six possible results. (1 mark)
2. Hence, the probability of the event F is P(F) = number of favourable outcomes / total outcomes = 3/6 = 1/2, and the complement P(not F) equals 1 − 1/2 = 1/2. (1 mark)
3. Adding P(F) and P(not F) gives 1/2 + 1/2 = 1, which confirms the complementary rule by demonstrating that an event and its complement together account for all possible outcomes. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 14, §14.2: For any event E, the probabilities of E and its complement add up to 1.

OR

Option B

Answer both connected parts. Show the required method, explanation and units.

(a) State the probability of a sure event and the probability of an impossible event. [2 marks]

Answer

In probability theory, a sure event is defined as one that will occur under all possible circumstances, and its probability is assigned the value 1. By contrast, an impossible event is one that cannot occur under any circumstance whatsoever, and its probability is assigned the value 0.

Step by step

1. **Give the probability of a sure event and of an impossible event with reasons from favourable over total:** If an experiment has n equally likely outcomes, a sure event has all n outcomes favourable, so P=n/n=1. An example is getting a number ≤6 on a die, which gives 6/6=1. An impossible event has 0 favourable outcomes, so P=0/n=0. An example is getting 7 on a die, which gives 0/6=0. Why: Probability is favourable outcomes divided by total outcomes, so the number of favourable outcomes fixes the value. All outcomes favourable gives 1, and none favourable gives 0. Since the favourable count lies between 0 and n, every probability satisfies 0≤P(E)≤1.
2. **State the two values and link them to the complement rule:** The probability of a sure event is 1 and the probability of an impossible event is 0. These are consistent with P(E)+P(not E)=1: the complement of a sure event is impossible, and 1+0=1. Why: Complementary events together cover all outcomes, so their probabilities add to 1. Since the complement of the sure event is the impossible event, the pair (1, 0) shows the rule is consistent at the two extreme values.

Topic and concept

Complementary events and their probability relation. Probability by counting is favourable outcomes divided by total outcomes, so it always lies between 0 and 1. The sure event takes the upper limit 1 because every outcome is favourable. The impossible event takes the lower limit 0 because no outcome is favourable.

How marks are earned

1. In probability theory, a sure event is defined as one that will occur under all possible circumstances, and its probability is assigned the value 1. (1 mark)
2. By contrast, an impossible event is one that cannot occur under any circumstance whatsoever, and its probability is assigned the value 0. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 14, §14.2: The probability of a sure event is 1 and that of an impossible event is 0.

(b) One fair coin is tossed and one fair die is rolled. List the sample space and find the probability that the result is a head on the coin and a 4 on the die. [3 marks]

Answer

The sample space has 2×6 = 12 equally likely outcomes: S = {(H,1),(H,2),(H,3),(H,4),(H,5),(H,6),(T,1),(T,2),(T,3),(T,4),(T,5),(T,6)}. The favourable outcome for {head,4} is just (H,4), so there is 1 favourable outcome. Therefore, P(head and 4) = 1/12 by dividing favourable outcomes by total outcomes.

Step by step

1. **Use the multiplication principle to build the sample space of the coin and die together:** The coin has 2 outcomes (H,T) and the die has 6 (1 to 6), so the number of combined outcomes is 2×6=12. Pairing each coin result with each die result gives S={(H,1),(H,2),(H,3),(H,4),(H,5),(H,6),(T,1),(T,2),(T,3),(T,4),(T,5),(T,6)}. Why: The sample space of combined independent experiments is the set of all ordered pairs of individual outcomes. Each coin outcome can occur with each die outcome, so the counts multiply. Writing ordered pairs (coin, die) keeps the 12 outcomes distinct. Because the coin and die are fair, all 12 pairs are equally likely.
2. **Identify the favourable outcomes for 'head on the coin and 4 on the die':** The event needs the coin to show H and the die to show 4. Only the pair (H,4) satisfies both conditions, so there is 1 favourable outcome. (T,4) fails on the coin and (H,1) to (H,6) other than (H,4) fail on the die. Why: 'And' means both conditions must hold in the same pair, so each pair is checked against both. Counting only pairs that satisfy both avoids mixing in outcomes where just one condition holds.
3. **Divide favourable by total outcomes to get the probability:** P(head and 4)=favourable/total=1/12. Check: 1/12 equals (1/2)×(1/6), the coin probability times the die probability. Why: With 12 equally likely outcomes, probability is favourable outcomes over total outcomes. The product check agrees because the coin and die are independent, so it supports the count of 12. The result lies between 0 and 1, as any probability must.

Topic and concept

Classical definition of probability — P(E) = number of favourable outcomes / total outcomes. For two experiments performed together, the sample space is the set of all ordered pairs of individual outcomes, and the number of pairs is the product of the individual counts. When every pair is equally likely, the probability is favourable pairs divided by total pairs. Here 1 favourable pair out of 2×6=12 gives 1/12.

How marks are earned

1. The sample space has 2×6 = 12 equally likely outcomes: S = {(H,1),(H,2),(H,3),(H,4),(H,5),(H,6),(T,1),(T,2),(T,3),(T,4),(T,5),(T,6)}. (1 mark)
2. The favourable outcome for {head,4} is just (H,4), so there is 1 favourable outcome. (1 mark)
3. Therefore, P(head and 4) = 1/12 by dividing favourable outcomes by total outcomes. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 14, §14.1: The sample space of combined independent experiments is the set of all ordered pairs of individual outcomes.

Question 35

5 marks

Long Answer

Attempt one option.

Option A

Answer both connected parts. Show the required method, explanation and units.

(a) For the quadratic polynomial $p(x)=2x^2-8x+6$, find (i) the sum of its zeros, and (ii) the product of its zeros. [2 marks]

Answer

Sum of zeros = −(coefficient of x)/(coefficient of x^2) = −(−8)/2 = 4. Product of zeros = (constant term)/(coefficient of x^2) = 6/2 = 3.

Step by step

1. **Identify a, b, c and apply sum of zeros = −b/a, keeping the sign of b:** Compare $p(x)=2x^2-8x+6$ with ax^2+bx+c to get $a=2$, $b=-8$, $c=6$. Then $\alpha+\beta = -b/a = -(-8)/2 = 8/2 = 4$. Why: For a quadratic the sum of zeros is −b/a, so the minus sign in the formula must act on the coefficient b, which is itself negative here. The two negatives combine to give a positive sum.
2. **Apply product of zeros = c/a:** Using $c=6$ and $a=2$, $\alpha\beta = c/a = 6/2 = 3$. Check: $p(x)=2(x-1)(x-3)$ has zeros 1 and 3, whose sum is 1+3=4 and product is 1×3=3, matching both results. Why: Factorising ax^2+bx+c as $a(x-\alpha)(x-\beta)$ and comparing coefficients gives $\alpha\beta=c/a$ with no sign change. The check with the actual zeros confirms the coefficient-based values.

Topic and concept

Relationship between zeros and coefficients for quadratic polynomials. For a quadratic ax^2+bx+c with zeros α and β, comparing it with $a(x-\alpha)(x-\beta)$ gives $\alpha+\beta=-b/a$ and $\alpha\beta=c/a$. The zeros' sum and product can therefore be read off from the coefficients without solving the equation. Here $a=2$, $b=-8$, $c=6$.

How marks are earned

1. Sum of zeros = −(coefficient of x)/(coefficient of x^2) = −(−8)/2 = 4. (1 mark)
2. Product of zeros = (constant term)/(coefficient of x^2) = 6/2 = 3. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 2, §2.3: For ax^2+bx+c, the sum of zeros $\alpha+\beta$ equals $-b/a$ and their product $\alpha\beta$ equals c/a.

(b) If α and β are the zeros of $ax^2+bx+c=0$, find the equation whose zeros are $\alpha+1$ and $\beta+1$. [3 marks]

Answer

For the transformed zeros $\alpha+1$ and $\beta+1$, their sum equals $(\alpha+\beta)+2$, which by using $\alpha+\beta=-b/a$ simplifies to $(-b+2a)/a$ as the new sum. The product of the new zeros is $(\alpha+1)(\beta+1)=\alpha\beta+(\alpha+\beta)+1$, which substitutes to $(c/a)+(-b/a)+1=(c-b+a)/a$ when using the original relations. Therefore, the monic polynomial with zeros $\alpha+1$ and $\beta+1$ is $x^2-[(-b+2a)/a]x+(c-b+a)/a=0$; multiplying through by a yields the required equation $ax^2-(-b+2a)x+(c-b+a)=0$.

Step by step

1. **Find the sum of the new zeros:** New sum = $(\alpha+1)+(\beta+1) = (\alpha+\beta)+2$. Using $\alpha+\beta = -b/a$, this is $-b/a + 2 = (-b+2a)/a$. Why: Adding the two shifted zeros adds 1 twice, so the new sum is the old sum plus 2. The old sum is known from the coefficient relation $\alpha+\beta=-b/a$, so no zero has to be found.

2. **Find the product of the new zeros:** New product = (α+1)(β+1) = αβ + α + β + 1 = αβ + (α+β) + 1. Substituting αβ=c/a and α+β=−b/a gives c/a − b/a + 1 = (c − b + a)/a. Why: Expanding the two brackets turns the product into terms of αβ and α+β, which are the two quantities the coefficients give directly. This is why the shifted product can be found without knowing α and β separately.
3. **Form the polynomial and clear the fraction:** A quadratic with sum S and product P is $x^2 - Sx + P$, so $x^2 - [(2a-b)/a]x + (a-b+c)/a = 0$. Multiplying every term by a gives $ax^2 - (2a-b)x + (a-b+c) = 0$. Check with $2x^2-8x+6$ (zeros 1, 3): a=2, b=−8, c=6 gives $2x^2 - 12x + 16 = 2(x-2)(x-4)$, whose zeros 2 and 4 are the old zeros plus 1. Why: Since $x^2-(\text{sum})x+\text{product}$ has the required sum and product as its zeros' relations, it is the required monic form. Multiplying by the nonzero a keeps the same zeros while removing denominators. The numerical check confirms the zeros shift by 1.

Topic and concept

Relationship between zeros and coefficients for quadratic polynomials. Once the sum and product of the zeros of a quadratic are known, the quadratic is $x^2 - (\text{sum})x + \text{product}$ up to a constant multiple. Here the new sum and product are written in terms of α+β=−b/a and αβ=c/a, so the new equation comes from the original coefficients alone.

How marks are earned

1. For the transformed zeros α+1 and β+1, their sum equals (α+β)+2, which by using α+β=−b/a simplifies to (−b+2a)/a as the new sum. (1 mark)
2. The product of the new zeros is (α+1)(β+1)=αβ+(α+β)+1, which substitutes to (c/a)+(−b/a)+1= (c−b+a)/a when using the original relations. (1 mark)
3. Therefore, the monic polynomial with zeros α+1 and β+1 is $x^2-[(-b+2a)/a]x+(c-b+a)/a=0$; multiplying through by a yields the required equation $ax^2-(-b+2a)x+(c-b+a)=0$. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 2, §2.3 Relationship between Zeroes and Coefficients of a Polynomial: For ax^2+bx+c, α+β=−b/a and αβ=c/a.

OR

Option B

Answer both connected parts. Show the required method, explanation and units.

(a) For the quadratic polynomial $p(x)=2x^2-4x+1$, find the sum and product of its zeros without solving for the zeros. [2 marks]

Answer

Applying the relation α+β=−b/a for the quadratic $p(x)=2x^2-4x+1$, we substitute b=−4 and a=2 to obtain α+β=−(−4)/2=2 as the sum of the zeros. Using the formula αβ=c/a for the same polynomial, where c=1 and a=2, we compute the product of the zeros as 1/2, which confirms the required result.

Step by step

1. **Identify the coefficients and calculate the sum of the zeros.:** Compare $p(x)=2x^2-4x+1$ with ax^2+bx+c. Here, $a=2$, $b=-4$ and $c=1$. If the zeros are α and β, then $\alpha+\beta=-b/a=-(-4)/2=4/2=2$. Therefore, the sum of the zeros is 2. Why: The coefficient relation for a quadratic ax^2+bx+c gives $\alpha+\beta=-b/a$, so the sum can be found directly from a and b without solving the quadratic.
2. **Use the constant and leading coefficients to calculate the product of the zeros.:** Using the same coefficients $a=2$ and $c=1$, $\alpha\beta=c/a=1/2$. Therefore, the product of the zeros is 1/2. Why: For a quadratic ax^2+bx+c, the product of the zeros depends on c/a, which gives the required product directly from the coefficients.

Topic and concept

Relationship between zeros and coefficients for quadratic polynomials. For a quadratic ax^2+bx+c, the sum of its zeros $\alpha+\beta$ equals $-b/a$ and their product $\alpha\beta$ equals c/a.

How marks are earned

1. Applying the relation $\alpha+\beta=-b/a$ for the quadratic $p(x)=2x^2-4x+1$, we substitute $b=-4$ and $a=2$ to obtain $\alpha+\beta=-(-4)/2=2$ as the sum of the zeros. (1 mark)
2. Using the formula $\alpha\beta=c/a$ for the same polynomial, where $c=1$ and $a=2$, we compute the product of the zeros as 1/2, which confirms the required result. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 2, §2.3 Relationship between Zeroes and Coefficients of a Polynomial: For ax^2+bx+c, $\alpha+\beta=-b/a$ and $\alpha\beta=c/a$.

(b) The graph of a quadratic polynomial $y=p(x)$ crosses the x-axis at $x=-2$ and $x=5$ and also passes through the point $(1,-8)$. Determine the explicit form of $p(x)$. [3 marks]

Answer

Since the polynomial crosses the x–axis at $x=-2$ and $x=5$, its factorized form can be written as $p(x)=k(x+2)(x-5)$, where k is a nonzero constant. Substituting the point $(1,-8)$ into this form gives $-8=k(1+2)(1-5)=k\cdot3\cdot(-4)=-12k$, hence solving for k yields $k=(-8)/(-12)=2/3$. Thus, upon expanding, the explicit form of the quadratic polynomial is $p(x)=(2/3)(x+2)(x-5)=(2/3)(x^2-3x-10)=(2/3)x^2-2x-20/3$ as required.

Step by step

1. **Write the factor form from the zeros, including a constant k:** Zeros at $x=-2$ and $x=5$ mean $(x+2)$ and $(x-5)$ are factors, so $p(x)=k(x+2)(x-5)$ with $k\neq0$ a constant. Why: If $p(\alpha)=0$ then $(x-\alpha)$ is a factor of $p(x)$, so each x-intercept gives one linear factor. The zeros fix the factors only up to a constant multiplier k, because multiplying p by any nonzero number leaves its zeros unchanged.
2. **Use the point (1,−8) to find k:** Put $x=1$, $p(1)=-8$: $-8 = k(1+2)(1-5) = k\cdot3\cdot(-4) = -12k$. So $k = (-8)/(-12) = 2/3$. Why: The extra point is needed to fix k, because it ties the curve's height to one value. Substituting it turns the unknown constant into a single linear equation.
3. **Expand to the explicit form and verify:** $p(x) = (2/3)(x+2)(x-5) = (2/3)(x^2 - 3x - 10) = (2/3)x^2 - 2x - 20/3$. Check: $p(1) = 2/3 - 2 - 20/3 = (2 - 6 - 20)/3 = -24/3 = -8$, which matches the given point. Why: The question asks for the explicit polynomial, so the factor form must be multiplied out. Evaluating the result at $x=1$ confirms that k and the expansion are correct.

Topic and concept

Zeros of a polynomial; geometric meaning of zeros as x-intercepts. The zeros of p(x) are the x-coordinates where its graph crosses the x-axis, and each zero α gives a factor (x−α). A quadratic with two known zeros is therefore k(x−α)(x−β), and one more point on the graph determines the constant k.

How marks are earned

1. Since the polynomial crosses the x–axis at x=−2 and x=5, its factorized form can be written as p(x)=k(x+2)(x−5), where k is a nonzero constant. (1 mark)
2. Substituting the point (1,−8) into this form gives −8=k(1+2)(1−5)=k·3·(−4)=−12k, hence solving for k yields k=(−8)/(−12)=2/3. (1 mark)
3. Thus, upon expanding, the explicit form of the quadratic polynomial is p(x)=(2/3)(x+2)(x−5)=(2/3)(x²−3x−10)=(2/3)x²−2x−20/3 as required. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 2, §2.4 Summary: If p(α)=0 then (x−α) is a factor of p(x), so zeros determine linear factors.

Section E

Question 36

4 marks

Case Based

A surveyor wants to determine the height of a tall tree without climbing it. She places a 2 m staff vertically on level ground and measures its shadow as 3 m at a particular time. At that same time, the tree's shadow measures 15 m. Assuming the sun's rays are parallel, she applies the principle of similar triangles to find the tree's height.

(a) Refer to the scenario: A surveyor places a 2 m staff vertically on level ground. At a certain instant, the staff's shadow measures 3 m and the shadow of a nearby tree measures 15 m. Assuming the sun's rays are parallel, identify the two triangles that are similar. [1 mark]

Answer

The right triangle formed by the 2 m staff and its 3 m shadow and the right triangle formed by the tree and its 15 m shadow are similar.

Step by step

1. **Identify the two right triangles and justify similarity by AA:** Triangle 1 is formed by the 2 m staff, its 3 m shadow and the sun's ray; triangle 2 is formed by the tree, its 15 m shadow and the sun's ray. Both the staff and the tree are vertical, so each makes 90° with the ground. The parallel sun rays make the same angle of elevation with the ground, so two angles match and the triangles are similar by AA. Why: If two triangles have equal corresponding angles, their sides are in proportion (NCERT Ch. 6, §6.3), so showing a 90° angle and an equal elevation angle is what justifies similarity.

Topic and concept

Applications of similarity criteria to prove triangles similar and indirect measurement. Similar triangles have equal corresponding angles and proportional corresponding sides. In this shadow set-up, the vertical objects and the parallel sun rays give two equal angles, which is enough to say the triangles are similar.

How marks are earned

1. The right triangle formed by the 2 m staff and its 3 m shadow and the right triangle formed by the tree and its 15 m shadow are similar. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 6, §6.3: If two triangles have equal corresponding angles, their sides are in proportion.

(b) Using the similar triangles from the scenario, write the correct proportional relation connecting the

staff height (2 m), tree height (h), staff shadow (3 m), and tree shadow (15 m). [1 mark]

Answer

2 : h = 3 : 15, i.e., 2/h = 3/15.

Step by step

1. **Match corresponding sides and write the ratio:** The AA angle matching pairs the staff height with the tree height and the staff shadow with the tree shadow. Hence (staff height)/(tree height)= (staff shadow)/(tree shadow), giving 2/h=3/15, i.e. 2 : h = 3 : 15. The equivalent form h/2=15/3 is also valid. Why: In similar triangles, ratios of corresponding sides are equal, and corresponding sides lie opposite equal angles, so height must be paired with height and shadow with shadow, not height with shadow.

Topic and concept

Applications of similarity criteria to prove triangles similar and indirect measurement. When triangles are similar, each side of one corresponds to the side opposite the same angle in the other, and those ratios are equal. This lets us set up an equation with one unknown, the tree height h.

How marks are earned

1. 2 : h = 3 : 15, i.e., 2/h = 3/15. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 6, §6.3: In similar triangles, ratios of corresponding sides are equal.

Attempt one of the final two-mark options.

Option A

(c) (i) Calculate the height of the tree based on the proportionality you have written. (ii) Which concept from Chapter 6 justifies the use of proportional triangles to find the tree's height indirectly? [2 marks]

Answer

From 2/h = 3/15, cross-multiply to get 2×15 = 3×h, so 30 = 3h and h = 10 m. The application of similarity of triangles for indirect measurement (Concept CH6-C08) justifies this method.

Step by step

1. **Cross-multiply and solve for h:** From 2/h=3/15, cross-multiplying gives 2×15=3×h, so 30=3h and h=30/3=10 m. Check: 2/10=0.2 and 3/15=0.2, so both ratios agree. Why: Solving the proportion for the unknown side gives the length, and the check that both ratios are equal confirms the arithmetic.
2. **Name the justifying concept:** The method rests on similarity of triangles (AA criterion): when two triangles have equal corresponding angles, their corresponding sides are proportional, so the unreachable tree height can be found from measurable lengths. Why: Similarity allows indirect measurement of large or unreachable lengths (NCERT Ch. 6, §6.1), which is why the proportional triangles justify finding the tree's height without climbing it.

Topic and concept

Applications of similarity criteria to prove triangles similar and indirect measurement. Similarity of triangles lets us find an unreachable length from a measurable one. Equal angles (AA) give proportional sides, and solving the proportion gives the unknown height.

How marks are earned

1. From 2/h = 3/15, cross-multiply to get 2×15 = 3×h, so 30 = 3h and h = 10 m. (1 mark)
2. The application of similarity of triangles for indirect measurement (Concept CH6-C08) justifies this method. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 6, §6.3: Solving the proportionality gives the unknown length. Check the reasoning against NCERT Class 10 maths, Ch. 6, §6.5: Similarity lets us find lengths indirectly using proportional triangles.

OR

Option B

(c) Another tree casts a 9 m shadow at the same instant. Calculate its height and state why the staff ratio applies. [2 marks]

Answer

Parallel sunlight makes the staff and tree triangles similar, so height/shadow = 2/3 for both. For a 9 m shadow, height = (2/3) × 9 m = 6 m.

Step by step

1. **Explain why the staff and tree triangles are similar and form the ratio:** The sun's rays are parallel, so they make the same angle with the ground for the staff and the tree; both are vertical, giving a 90° angle each, so the triangles are similar by AA and height/shadow = 2 m/3 m = 2/3 for both. Why: If two triangles have equal corresponding angles, their corresponding sides are in proportion, so the height-to-shadow ratio stays the same for every vertical object at that instant.
2. **Apply the ratio to the 9 m shadow and check:** Let the height be h: h/9 = 2/3, so h = (2/3) × 9 m = 18/3 m = 6 m; check: 6/9 = 2/3, matching the staff ratio. Why: Solving the proportion of corresponding sides gives the unknown length, which is the idea of indirect measurement using similar triangles.

Topic and concept

Applications of similarity criteria to prove triangles similar and indirect measurement. Similar triangles allow unreachable heights to be found indirectly. At one instant, parallel sunlight gives each vertical object a right-angled triangle with the same sun angle, so triangles are similar by AA and the height-to-shadow ratio is constant.

How marks are earned

1. Parallel sunlight makes the staff and tree triangles similar, so height/shadow = 2/3 for both. (1 mark)
2. For a 9 m shadow, height = (2/3) × 9 m = 6 m. (1 mark)

Source check: Check the passage facts and NCERT Class 10 maths, Ch. 6, §6.3 against both steps.

Question 37

4 marks

Case Based

In an experiment, the height h(t), in metres, of a ball thrown vertically upward is modelled by the quadratic polynomial $h(t)=-5t^2+20t$, where t is time in seconds.

(a) Find the values of t for which the height function $h(t)=-5t^2+20t$ equals zero. [1 mark]

Answer

Solving $-5t^2+20t=0$ gives $t(-5t+20)=0$, so t=0 or t=4.

Step by step

1. **Factorise h(t) and apply the zero-product property:** Set $-5t^2+20t=0$. Taking out the common factor gives $-5t(t-4)=0$, equivalent to $t(-5t+20)=0$. So t=0 or $t-4=0$, i.e. t=0 s or t=4 s. Check: $h(4) = -5(16)+20(4) = -80+80 = 0$. Why: A zero of the polynomial is a value of t that makes h(t)=0. When a product is zero, at least one factor must be zero, so each factor can be set to zero separately. Substituting t=4 back confirms it is a zero.

Topic and concept

Zeros of a polynomial; geometric meaning of zeros as x-intercepts. A real number k is a zero of a polynomial p if p(k)=0. For $h(t)=-5t^2+20t$, the zeros are found by factorising and setting each factor to zero, giving two values of t, in seconds.

How marks are earned

1. Solving $-5t^2+20t=0$ gives $t(-5t+20)=0$, so t=0 or t=4. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 2, §2.1 Introduction: A real number k is a zero of the polynomial p(x) if p(k)=0.

(b) Explain the physical significance of the times t=0 and t=4 found in this experiment. [1 mark]

Answer

These zeros are the times when the height becomes zero, i.e., at t=0 when the ball is launched and at t=4 s when it returns to ground level.

Step by step

1. **Interpret h(t)=0 in the context of the ball's motion:** Here h(t) is the height above the ground in metres, so h=0 means the ball is at ground level. At t=0 s the ball is at ground level as it is thrown, and at t=4 s it is back at ground level. The flight time is therefore $4-0 = 4$ s. Why: Zeros of h(t) are the times at which the height is zero, that is, the ball is at ground level. A quadratic has at most two zeros, so these two times are the only moments at ground level: the start and the landing.

Topic and concept

Zeros of a polynomial; geometric meaning of zeros as x-intercepts. The zeros of h(t) are the t-values where the graph of height against time meets the t-axis. Because h measures height above the ground, each zero is a moment when the ball is on the ground.

How marks are earned

1. These zeros are the times when the height becomes zero, i.e., at t=0 when the ball is launched and at t=4 s when it returns to ground level. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 2, §2.2 Geometrical Meaning of the Zeroes of a Polynomial: The zeros of p(x) correspond to the x-coordinates where the graph of y=p(x) crosses the x-axis.

Attempt one of the final two-mark options.

Option A

(c) (i) Write h(t) in its factorised form corresponding to its zeros. (ii) Use the coefficients of h(t) to verify that the sum of its zeros equals 4. [2 marks]

Answer

h(t) can be factorised as −5t(t−4), showing factors corresponding to the zeros t=0 and t=4. The sum of zeros is 0+4=4, and −b/a for h(t)=−5t²+20t is −20/−5=4, verifying the relationship.

Step by step

1. **Factorise h(t) including the leading coefficient:** h(t)=−5t²+20t. Taking out the common factor −5t gives h(t)=−5t(t−4). Setting each factor to zero gives t=0 and t=4, so the zeros are 0 and 4. This has the form a(t−α)(t−β) with a=−5, α=0, β=4. Why: A quadratic can be written as a(t−α)(t−β), where a is the leading coefficient and α, β are the zeros, so the factor −5 must be kept in the factorised form.
2. **Verify the sum of zeros using the coefficients:** Compare with at²+bt+c: a=−5, b=20, c=0. Then −b/a=−20/−5=4. The sum of the zeros found is 0+4=4, so the two values match. Why: For a quadratic, the sum of zeros α+β equals −b/a (NCERT Ch. 2, §2.3), so computing −b/a from the coefficients and comparing it to the actual sum checks the factorisation.

Topic and concept

Zeros of a polynomial; geometric meaning of zeros as x-intercepts. A quadratic with zeros α and β can be written as a(x−α)(x−β). Its zeros satisfy α+β=−b/a and αβ=c/a. Here the zeros 0 and 4 can be checked against the coefficients a=−5, b=20, c=0.

How marks are earned

1. h(t) can be factorised as −5t(t−4), showing factors corresponding to the zeros t=0 and t=4. (1 mark)
2. The sum of zeros is 0+4=4, and −b/a for h(t)=−5t²+20t is −20/−5=4, verifying the relationship. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 2, §2.3 Relationship between Zeroes and Coefficients of a Polynomial: Quadratic expressions can be factorised into linear factors corresponding to each zero. Check the reasoning against NCERT Class 10 maths, Ch. 2, §2.3 Relationship between Zeroes and Coefficients of a Polynomial: For ax^2+bx+c, the sum of zeros equals $-b/a$.

OR

Option B

(c) Find the product of the zeros of h(t) from its coefficients and verify it using the two zeros found earlier. [2 marks]

Answer

For $h(t)=-5t^2+20t+0$, the product of zeros is $c/a = 0/(-5) = 0$. The zeros 0 and 4 have product $0\times4 = 0$, agreeing with the coefficient relation.

Step by step

1. **Identify the coefficients and find the product of zeros from them:** Writing $h(t) = -5t^2 + 20t + 0$ gives $a = -5$, $b = 20$, $c = 0$, so the product of zeros = $c/a = 0/(-5) = 0$. Why: For a quadratic $ax^2 + bx + c$, the product of its zeros equals c/a; with $c = 0$ the product must be zero, which means one zero is 0.
2. **Verify using the two zeros found earlier:** The zeros are $t = 0$ and $t = 4$, since $-5t(t - 4) = 0$; their product is $0 \times 4 = 0$, which equals $c/a = 0$, so the check agrees. Why: Multiplying independently found zeros and comparing with c/a confirms the coefficient relation, since any number multiplied by zero gives zero.

Topic and concept

Zeros of a polynomial; geometric meaning of zeros as x-intercepts. For a quadratic $ax^2 + bx + c$ with zeros α and β, the sum is $-b/a$ and the product is c/a. For $h(t) = -5t^2 + 20t$, $c = 0$, so the product of zeros is 0, meaning one zero is $t = 0$ (the instant of throwing); the other is $t = 4$ s.

How marks are earned

1. For $h(t)=-5t^2+20t+0$, the product of zeros is $c/a = 0/(-5) = 0$. (1 mark)
2. The zeros 0 and 4 have product $0\times4 = 0$, agreeing with the coefficient relation. (1 mark)

Source check: Check the passage facts and NCERT Class 10 maths, Ch. 2, §2.1 Introduction against both steps.

Question 38

4 marks

Case Based

A surveyor wants to determine the height of a lighthouse. From point P on level ground, the angle of elevation to the top is 45°. He then moves 15 m closer to the lighthouse to point Q and finds the angle of elevation to be 60°. Let h be the height of the lighthouse and x be the horizontal distance from P to the base of the lighthouse.

(a) What is the numerical value of tan 45°? [1 mark]

Answer

tan 45° = 1.

Step by step

1. **Use the standard tangent value:** In a 45° right triangle the opposite and adjacent legs are equal, so tan 45° = opposite/adjacent = 1. Why: The tangent ratio equals one whenever the two legs around the acute angle have equal length; the passage supplies 45° but not a table of values.

Topic and concept

Angle of elevation and angle of depression; applications to heights and distances. For an angle of elevation θ, tan θ is opposite height divided by horizontal distance. At 45°, these lengths are equal, so tan 45° = 1.

How marks are earned

1. Using a 45° right triangle, opposite and adjacent legs are equal, so tan 45° = 1. (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 9, §9.1: Defines the angle of elevation as the angle between the horizontal and the line of sight to a point above.

(b) Write the expression relating h and x using the measurement at point P. [1 mark]

Answer

Using tan 45°=h/x⇒h=x

Step by step

1. **Identify the right triangle at P, name the opposite and adjacent sides to the 45° angle, and apply tan θ = opposite/adjacent with tan 45° = 1.:** At P the lighthouse (height h) is the side opposite the 45° angle of elevation and the ground distance x is the side adjacent to it, so tan 45° = h/x. Since tan 45° = 1, we get 1 = h/x, hence h = x. Check: both h and x are lengths in metres, and equal sides are consistent with a 45° angle in a right triangle. Why: The angle of elevation is measured from the horizontal, so the vertical height is opposite and the horizontal distance is adjacent; tan θ = height/horizontal distance links them, and the standard value tan 45° = 1 reduces the relation to h = x.

Topic and concept

Angle of elevation and angle of depression; applications to heights and distances. The angle of elevation is the angle between the horizontal and the line of sight to a point above. In the right triangle formed by the lighthouse, the ground and the line of sight, tan θ = height / horizontal distance. At P, θ = 45° and tan 45° = 1, so the height equals the horizontal distance x.

How marks are earned

1. Using tan 45°=h/x⇒h=x (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 9, §9.1: Relates height and horizontal distance through tan θ=height/distance.

Attempt one of the final two-mark options.

Option A

(c) (i) Write the equation that represents tan 60° at point Q in terms of h and x. (ii) Eliminate h to derive an equation in x only. [2 marks]

Answer

$\tan 60° = h/(x-15) \Rightarrow \sqrt{3} = h/(x-15)$ Substitute $h = x$ into $\sqrt{3} = h/(x-15)$ to get $\sqrt{3} = x/(x-15)$

Step by step

1. **Find Q's horizontal distance from the base, then write tan 60° in the right triangle at Q.:** Q is 15 m closer to the lighthouse than P, so its horizontal distance from the base is $x - 15$ (metres). At Q the height h is opposite the 60° angle, so $\tan 60° = h/(x - 15)$. Since $\tan 60° = \sqrt{3}$, we get $\sqrt{3} = h/(x - 15)$. Why: Moving towards the base shortens the horizontal distance by the distance moved, and the angle of elevation relation $\tan \theta$ = height/horizontal distance applies in the new right triangle with the same height h.
2. **Substitute h = x from the measurement at P into the equation from Q to eliminate h.:** From part (b), $h = x$. Replacing h in $\sqrt{3} = h/(x - 15)$ gives $\sqrt{3} = x/(x - 15)$, an equation in x only. (Solving further: $\sqrt{3}(x - 15) = x$, so $x(\sqrt{3} - 1) = 15\sqrt{3}$, giving $x = 15(3 + \sqrt{3})/2 \approx 35.49$ m, though only the equation is required here.) Why: The lighthouse height h is the same in both right triangles, so equating it using the two relations removes h; this combines the two right-triangle relations to leave one unknown.

Topic and concept

Angle of elevation and angle of depression; applications to heights and distances. The same lighthouse height h appears in two right triangles, one at P and one at Q. Using $\tan \theta$ = height / horizontal distance in each gives two equations. Because Q is 15 m closer, its horizontal distance is $x - 15$. Substituting $h = x$ from P into the Q equation leaves an equation in x alone.

How marks are earned

1. $\tan 60° = h/(x-15) \Rightarrow \sqrt{3} = h/(x-15)$ (1 mark)
2. Substitute $h = x$ into $\sqrt{3} = h/(x-15)$ to get $\sqrt{3} = x/(x-15)$ (1 mark)

Source check: Check the reasoning against NCERT Class 10 maths, Ch. 9, §9.1: Angle of elevation uses tan θ=height/horizontal distance. Check the reasoning against NCERT Class 10 maths, Ch. 9, §9.1: Two right-triangle relations can be combined to eliminate height.

OR

Option B

(c) Use h=x and tan 60°=h/(x−15) to find the lighthouse height in exact form. [2 marks]

Answer

Substitution gives √3=x/(x−15), hence (√3−1)x=15√3. The height is h=x=15√3/(√3−1)=(15/2)(3+√3) m.

Step by step

1. **Substitute h = x into the second tangent equation and clear the denominator:** Putting h = x in tan 60° = h/(x − 15) gives √3 = x/(x − 15), so √3(x − 15) = x, i.e. √3x − 15√3 = x, hence (√3 − 1)x = 15√3. Why: Both tangent relations equal the same height over a horizontal distance, so replacing h by x eliminates the height and leaves one equation in x, which is possible because the 15 m shift links the two distances.
2. **Solve for x by rationalising and give the height with units:** x = 15√3/(√3 − 1); multiplying by (√3 + 1)/(√3 + 1) gives 15√3(√3 + 1)/((√3 − 1)(√3 + 1)) = (45 + 15√3)/(3 − 1) = (45 + 15√3)/2 = (15/2)(3 + √3) m; so h = x ≈ 7.5 × 4.732 ≈ 35.5 m. Why: Multiplying numerator and denominator by the conjugate (√3 + 1) uses (a − b)(a + b) = a² − b² to remove the surd from the denominator without changing the value, giving the exact form.

Topic and concept

Angle of elevation and angle of depression; applications to heights and distances. In height and distance problems, tan θ = height/horizontal distance for each angle of elevation. At 45° tan 45° = 1, so h = x. At 60° from the point 15 m nearer, tan 60° = √3 = h/(x − 15). Combining the two equations eliminates h and leads to an exact answer.

How marks are earned

1. Substitution gives √3=x/(x−15), hence (√3−1)x=15√3. (1 mark)
2. The height is h=x=15√3/(√3−1)=(15/2)(3+√3) m. (1 mark)

Source check: Check the passage facts and NCERT Class 10 maths, Ch. 9, §9.1 against both steps.